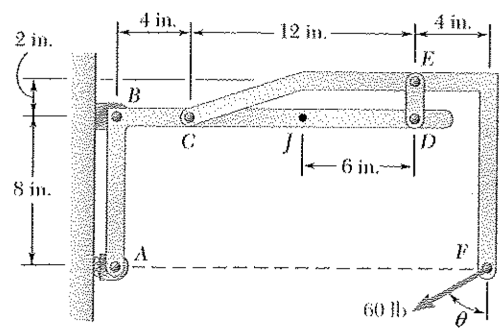
CHAPTER 7



PROBLEM 7.1

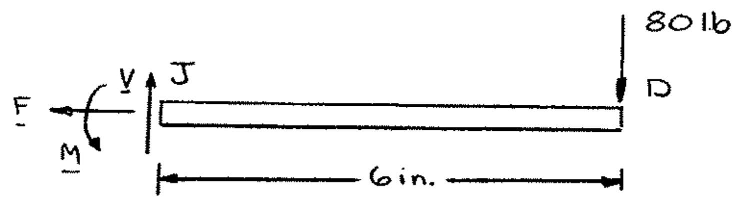
Determine the internal forces (axial force, shearing force, and bending moment) at Point J of the structure indicated.

Frame and loading of Problem 6.79.

SOLUTION

From Problem 6.79: On JD $\quad\quad$ $\mathbf{D} = 80 \text{ lb}\downarrow$

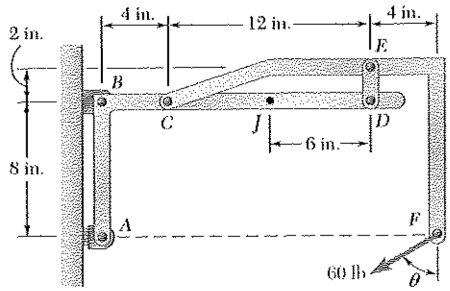
FBD of JD:



$+\uparrow \Sigma F_y = 0$: $\quad V - 80 \text{ lb} = 0$ $\quad\quad$ $\mathbf{V} = 80.0 \text{ lb}\uparrow$ ◀

$\xrightarrow{+} \Sigma F_x = 0$: $\quad F = 0$ $\quad\quad$ $F = 0$ ◀

$+\circlearrowright \Sigma M_J = 0$: $\quad M - (80 \text{ lb})(6 \text{ in.}) = 0$ $\quad\quad$ $\mathbf{M} = 480 \text{ lb} \cdot \text{in.} \circlearrowright$ ◀



PROBLEM 7.2

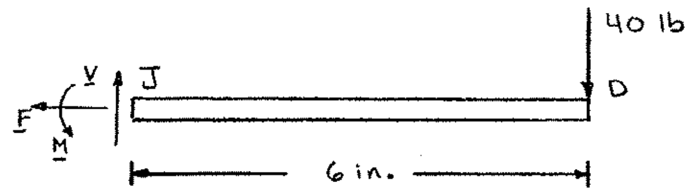
Determine the internal forces (axial force, shearing force, and bending moment) at Point J of the structure indicated.

Frame and loading of Problem 6.80.

SOLUTION

From Problem 6.80: On JD $\quad\quad\quad$ $\mathbf{D} = 40\text{ lb}\downarrow$

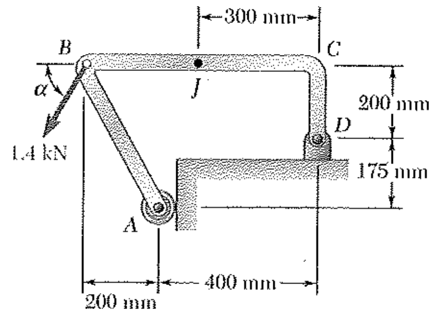
FBD of JD:



$+\uparrow\Sigma F_y = 0:\quad V - 40\text{ lb} = 0$ $\qquad$ $\mathbf{V} = 40.0\text{ lb}\uparrow$ ◀

$\xrightarrow{+}\Sigma F_x = 0:\quad F = 0$ $\qquad$ $F = 0$ ◀

$+\circlearrowleft\Sigma M_J = 0:\quad M - (40\text{ lb})(6\text{ in.}) = 0$ $\qquad$ $\mathbf{M} = 240\text{ lb}\cdot\text{in.}\circlearrowleft$ ◀

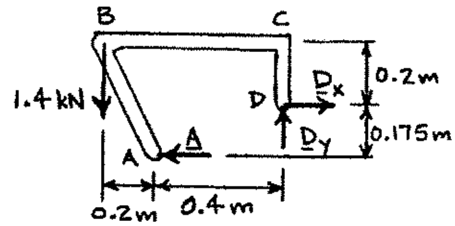


PROBLEM 7.3

Determine the internal forces at Point J when $\alpha = 90°$.

SOLUTION

Free body: Entire bracket



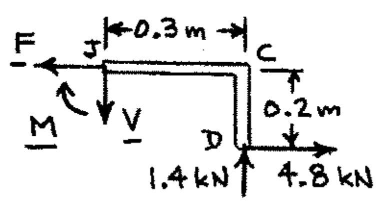
$$+\circlearrowleft \Sigma M_D = 0: \quad (1.4 \text{ kN})(0.6 \text{ m}) - A(0.175 \text{ m}) = 0$$

$$A = +4.8 \text{ kN} \qquad \mathbf{A} = 4.8 \text{ kN} \longleftarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad D_x - 4.8 = 0 \qquad \mathbf{D}_x = 4.8 \text{ kN} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad D_y - 1.4 = 0 \qquad \mathbf{D}_y = 1.4 \text{ kN} \uparrow$$

Free body: JCD

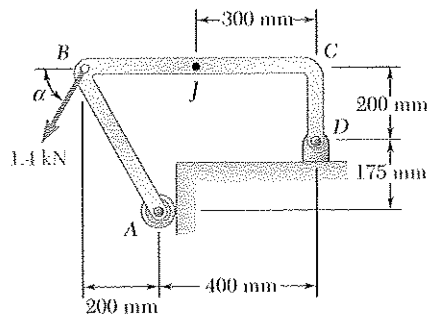


$$\xrightarrow{+} \Sigma F_x = 0: \quad 4.8 \text{ kN} - F = 0 \qquad \mathbf{F} = 4.80 \text{ kN} \longleftarrow ◀$$

$$+\uparrow \Sigma F_y = 0: \quad 1.4 \text{ kN} - V = 0 \qquad \mathbf{V} = 1.400 \text{ kN} \downarrow ◀$$

$$+\circlearrowleft \Sigma M_J = 0: \quad (4.8 \text{ kN})(0.2 \text{ m}) + (1.4 \text{ kN})(0.3 \text{ m}) - M = 0$$

$$M = +1.38 \text{ kN} \cdot \text{m} \qquad \mathbf{M} = 1.380 \text{ kN} \cdot \text{m} \circlearrowleft ◀$$

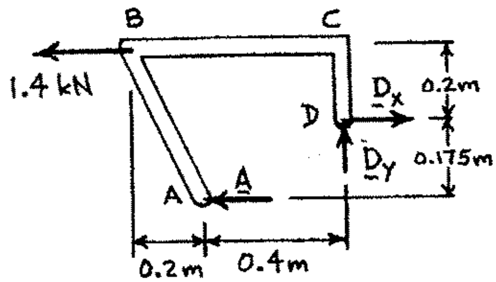


PROBLEM 7.4

Determine the internal forces at Point J when $\alpha = 0$.

SOLUTION

Free body: Entire bracket

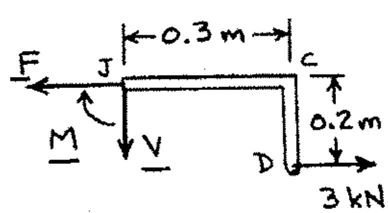


$$+\uparrow \Sigma F_y = 0: \quad D_y = 0 \qquad \mathbf{D}_y = 0$$

$$+\circlearrowleft \Sigma M_A = 0: \quad (1.4 \text{ kN})(0.375 \text{ m}) - D_x(0.175 \text{ m}) = 0$$

$$D_x = +3 \text{ kN} \qquad \mathbf{D}_x = 3 \text{ kN} \longrightarrow$$

Free body: JCD

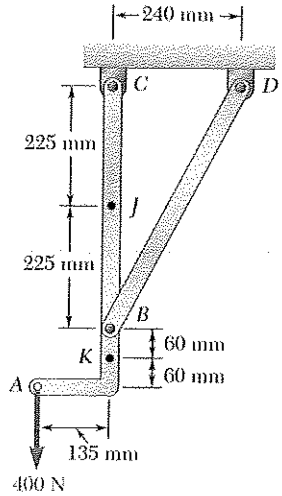


$$\xrightarrow{+} \Sigma F_x = 0: \quad 3 \text{ kN} - F = 0 \qquad \mathbf{F} = 3.00 \text{ kN} \longleftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -V = 0 \qquad V = 0 \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_J = 0: \quad (3 \text{ kN})(0.2 \text{ m}) - M = 0$$

$$M = +0.6 \text{ kN} \cdot \text{m} \qquad \mathbf{M} = 0.600 \text{ kN} \cdot \text{m} \circlearrowright \blacktriangleleft$$

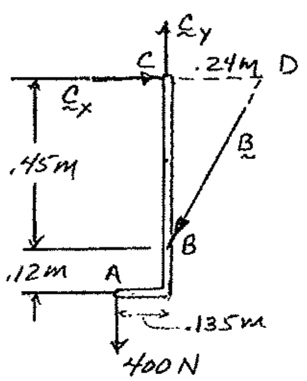


PROBLEM 7.5

Determine the internal forces at Point J of the structure shown.

SOLUTION

FBD ABC:



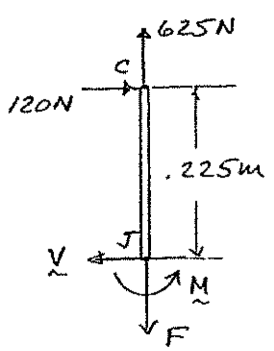
$\circlearrowleft \Sigma M_D = 0\text{:} \quad (0.375 \text{ m})(400 \text{ N}) - (0.24 \text{ m})C_y = 0$

$$\mathbf{C}_y = 625 \text{ N}\uparrow$$

$\circlearrowleft \Sigma M_B = 0\text{:} \quad -(0.45 \text{ m})C_x + (0.135 \text{ m})(400 \text{ N}) = 0$

$$\mathbf{C}_x = 120 \text{ N} \longrightarrow$$

FBD CJ:

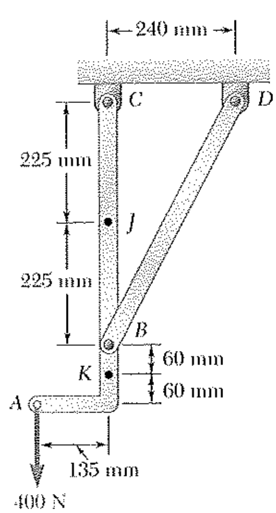


$\uparrow \Sigma F_y = 0\text{:} \quad 625 \text{ N} - F = 0$ $\qquad \mathbf{F} = 625 \text{ N}\downarrow$ ◀

$\longrightarrow \Sigma F_x = 0\text{:} \quad 120 \text{ N} - V = 0$ $\qquad \mathbf{V} = 120.0 \text{ N} \longleftarrow$ ◀

$\circlearrowleft \Sigma M_J = 0\text{:} \quad M - (0.225 \text{ m})(120 \text{ N}) = 0$

$\mathbf{M} = 27.0 \text{ N}\cdot\text{m} \circlearrowright$ ◀

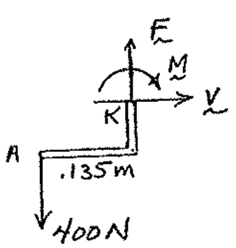


PROBLEM 7.6

Determine the internal forces at Point K of the structure shown.

SOLUTION

FBD AK:



$\xrightarrow{} \Sigma F_x = 0: \quad V = 0$

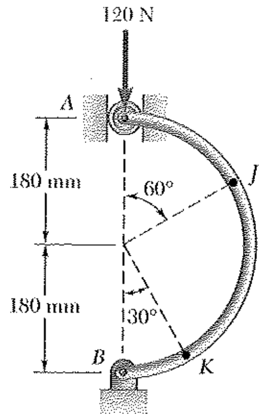
$$\mathbf{V} = 0 \blacktriangleleft$$

$\uparrow \Sigma F_y = 0: \quad F - 400\text{ N} = 0$

$$\mathbf{F} = 400\text{ N} \uparrow \blacktriangleleft$$

$\circlearrowleft \Sigma M_K = 0: \quad (0.135\text{ m})(400\text{ N}) - M = 0$

$$\mathbf{M} = 54.0\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

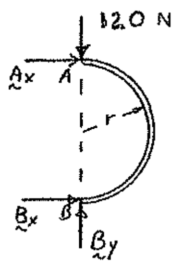


PROBLEM 7.7

A semicircular rod is loaded as shown. Determine the internal forces at Point J.

SOLUTION

FBD Rod:



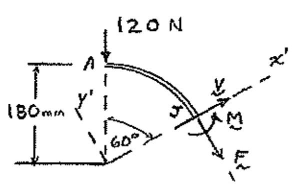
$\circlearrowleft \Sigma M_B = 0: \quad A_x(2r) = 0$

$\mathbf{A}_x = 0$

$\nearrow \Sigma F_{x'} = 0: \quad V - (120\text{ N})\cos 60° = 0$

$\mathbf{V} = 60.0\text{ N} \nearrow$ ◀

FBD AJ:



$\searrow \Sigma F_{y'} = 0: \quad F + (120\text{ N})\sin 60° = 0$

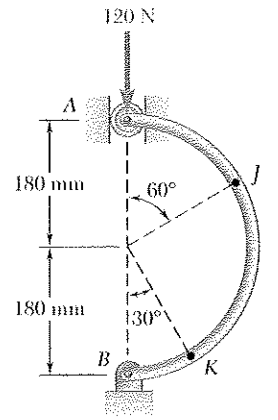
$F = -103.923\text{ N}$

$\mathbf{F} = 103.9\text{ N} \nwarrow$ ◀

$\circlearrowleft \Sigma M_J = 0: \quad M - [(0.180\text{ m})\sin 60°](120\text{ N}) = 0$

$M = 18.7061$

$\mathbf{M} = 18.71 \circlearrowright$ ◀

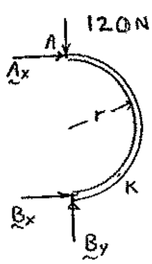


PROBLEM 7.8

A semicircular rod is loaded as shown. Determine the internal forces at Point K.

SOLUTION

FBD Rod:



$$\uparrow \Sigma F_y = 0: \quad B_y - 120\text{ N} = 0 \quad \mathbf{B}_y = 120\text{ N}\uparrow$$

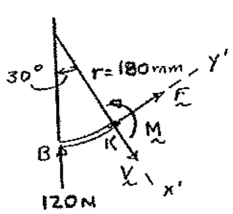
$$\circlearrowleft \Sigma M_A = 0: \quad 2rB_x = 0 \quad \mathbf{B}_x = 0$$

$$\searrow \Sigma F_{x'} = 0: \quad V - (120\text{ N})\cos 30° = 0$$

$$V = 103.923\text{ N}$$

$$\mathbf{V} = 103.9\text{ N} \searrow \blacktriangleleft$$

FBD BK:



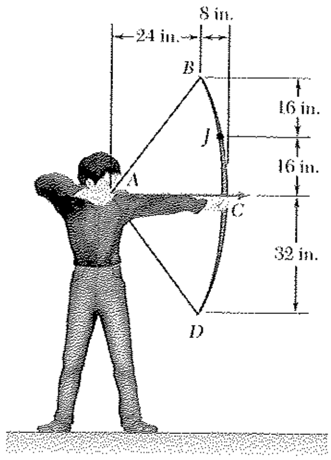
$$\nearrow \Sigma F_{y'} = 0: \quad F + (120\text{ N})\sin 30° = 0$$

$$F = -60\text{ N}$$

$$\mathbf{F} = 60.0\text{ N} \swarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_K = 0: \quad M - [(0.180\text{ m})\sin 30°](120\text{ N}) = 0$$

$$\mathbf{M} = 10.80\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

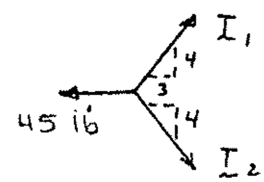


PROBLEM 7.9

An archer aiming at a target is pulling with a 45-lb force on the bowstring. Assuming that the shape of the bow can be approximated by a parabola, determine the internal forces at Point J.

SOLUTION

FBD Point A:

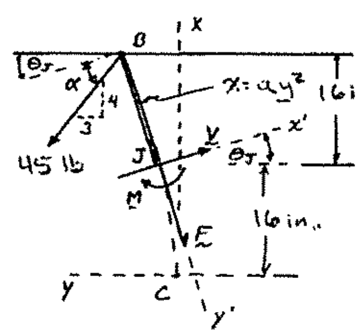


By symmetry $\qquad T_1 = T_2$

$$\xrightarrow{+} \Sigma F_x = 0: \quad 2\left(\frac{3}{5}T_1\right) - 45 \text{ lb} = 0 \quad T_1 = T_2 = 37.5 \text{ lb}$$

Curve CJB is parabolic: $x = ay^2$

FBD BJ:



At B: $x = 8$ in. $\qquad y = 32$ in.

$$a = \frac{8 \text{ in.}}{(32 \text{ in.})^2} = \frac{1}{128 \text{ in.}}$$

$$x = \frac{y^2}{128}$$

$$\text{Slope of parabola} = \tan\theta = \frac{dx}{dy} = \frac{2y}{128} = \frac{y}{64}$$

At J: $$\theta_J = \tan^{-1}\left[\frac{16}{64}\right] = 14.036°$$

So $$\alpha = \tan^{-1}\frac{4}{3} - 14.036° = 39.094°$$

$$\nearrow \Sigma F_{x'} = 0: \quad V - (37.5 \text{ lb})\cos(39.094°) = 0$$

$\mathbf{V} = 29.1 \text{ lb} \nearrow$ ◀

PROBLEM 7.9 (Continued)

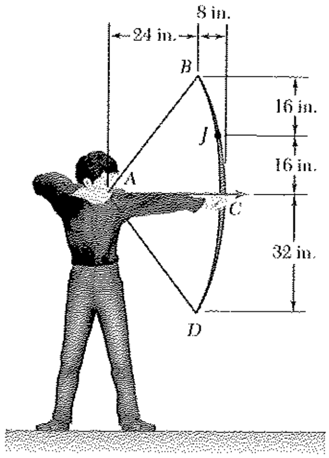
$\searrow \Sigma F_{y'} = 0$: $F + (37.5 \text{ lb})\sin(39.094°) = 0$

$F = -23.647$

F = 23.6 lb $\nwarrow$ ◀

$\circlearrowleft \Sigma M_J = 0$: $M + (16 \text{ in.})\left[\frac{3}{5}(37.5 \text{ lb})\right] + [(8-2) \text{ in.}]\left[\frac{4}{5}(37.5 \text{ lb})\right] = 0$

M = 540 lb·in. $\circlearrowright$ ◀



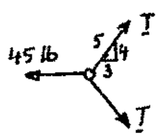
PROBLEM 7.10

For the bow of Problem 7.9, determine the magnitude and location of the maximum (*a*) axial force, (*b*) shearing force, (*c*) bending moment.

PROBLEM 7.9 An archer aiming at a target is pulling with a 45-lb force on the bowstring. Assuming that the shape of the bow can be approximated by a parabola, determine the internal forces at Point *J*.

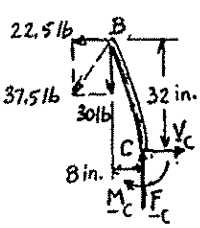
SOLUTION

Free body: Point *A*



$\xrightarrow{+} \Sigma F_x = 0: \quad 2\left(\frac{3}{5}T\right) - 45 \text{ lb} = 0$ $\qquad T = 37.5 \text{ lb}$ ◁

Free body: Portion of bow *BC*



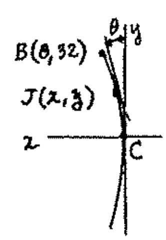
$+\uparrow \Sigma F_y = 0: \quad F_C - 30 \text{ lb} = 0$ $\qquad \mathbf{F}_C = 30 \text{ lb} \uparrow$ ◁

$\xrightarrow{+} \Sigma F_x = 0: \quad V_C - 22.5 \text{ lb} = 0$ $\qquad \mathbf{V}_C = 22.5 \text{ lb} \longrightarrow$ ◁

$\circlearrowleft \Sigma M_C = 0: \quad (22.5 \text{ lb})(32 \text{ in.}) + (30 \text{ lb})(8 \text{ in.}) - M_C = 0$

$\mathbf{M}_C = 960 \text{ lb} \cdot \text{in.} \circlearrowright$ ◁

Equation of parabola



$$x = ky^2$$

At *B*: $\qquad 8 = k(32)^2 \quad k = \frac{1}{128}$

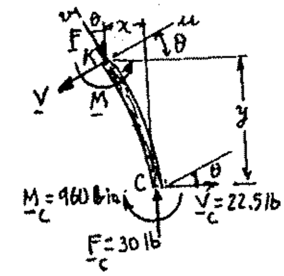
Therefore, equation is $\qquad x = \frac{y^2}{128} \qquad (1)$

The slope at *J* is obtained by differentiating (1):

$$d_x = \frac{2y\,dy}{128}, \quad \tan\theta = \frac{dx}{dy} = \frac{y}{64} \qquad (2)$$

(*a*) Maximum axial force

$$+\nwarrow \Sigma F_V = 0: \quad -F + (30\text{ lb})\cos\theta - (22.5\text{ lb})\sin\theta = 0$$



Free body: Portion bow *CK*

$$F = 30\cos\theta - 22.5\sin\theta$$

F is largest at $C(\theta = 0)$

$F_m = 30.0$ lb at C ◀

(*b*) Maximum shearing force

$$+\nearrow \Sigma F_V = 0: \quad -V + (30\text{ lb})\sin\theta + (22.5\text{ lb})\cos\theta = 0$$

$$V = 30\sin\theta + 22.5\cos\theta$$

V is largest at B (and D)

Where $\quad \theta = \theta_{max} = \tan^{-1}\left(\frac{1}{2}\right) = 26.56°$

$$V_m = 30\sin 26.56° + 22.5\cos 26.56°$$

$V_m = 33.5$ lb at B and D ◀

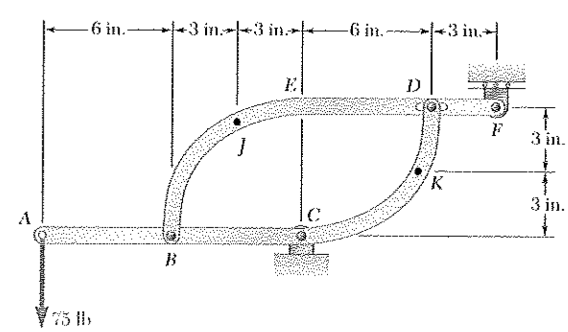
(*c*) Maximum bending moment

$$+\circlearrowleft \Sigma M_K = 0: \quad M - 960\text{ lb}\cdot\text{in.} + (30\text{ lb})x + (22.5\text{ lb})y = 0$$

$$M = 960 - 30x - 22.5y$$

M is largest at C, where $x = y = 0$.

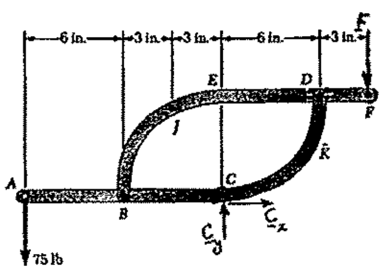
$M_m = 960$ lb·in. at C ◀



PROBLEM 7.11

Two members, each consisting of a straight and a quarter-circular portion of rod, are connected as shown and support a 75-lb load at A. Determine the internal forces at Point J.

SOLUTION



Free body: Entire frame

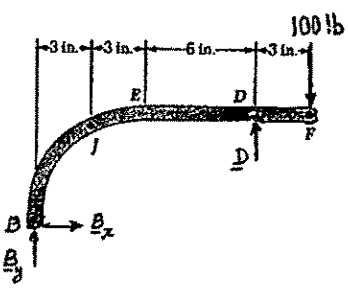
$$+\circlearrowleft \Sigma M_C = 0: \quad (75\text{ lb})(12\text{ in.}) - F(9\text{ in.}) = 0$$

$$\mathbf{F} = 100\text{ lb}\downarrow \lhd$$

$$\underrightarrow{+} \Sigma F_x = 0: \quad C_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 75\text{ lb} - 100\text{ lb} = 0$$

$$C_y = +175\text{ lb} \qquad \mathbf{C} = 175\text{ lb}\uparrow \lhd$$



Free body: Member $BEDF$

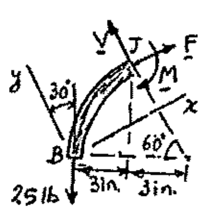
$$+\circlearrowleft \Sigma M_B = 0: \quad D(12\text{ in.}) - (100\text{ lb})(15\text{ in.}) = 0$$

$$\mathbf{D} = 125\text{ lb}\uparrow \lhd$$

$$\underrightarrow{+} \Sigma F_x = 0: \quad B_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad B_y + 125\text{ lb} - 100\text{ lb} = 0$$

$$B_y = -25\text{ lb} \qquad \mathbf{B} = 25\text{ lb}\downarrow \lhd$$



Free body: BJ

$$+\nearrow \Sigma F_x = 0: \quad F - (25\text{ lb})\sin 30° = 0$$

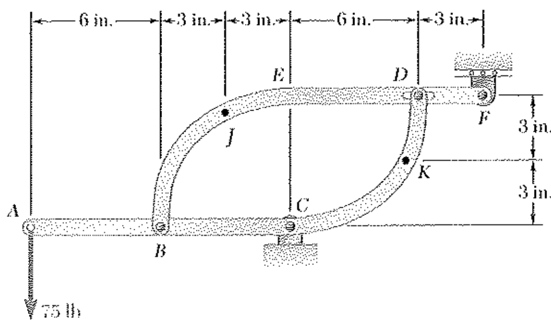
$$\mathbf{F} = 12.50\text{ lb} \measuredangle 30.0° \blacktriangleleft$$

$$+\nwarrow \Sigma F_y = 0: \quad V - (25\text{ lb})\cos 30° = 0$$

$$\mathbf{V} = 21.7\text{ lb} \searrow 60.0° \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -M + (25\text{ lb})(3\text{ in.}) = 0$$

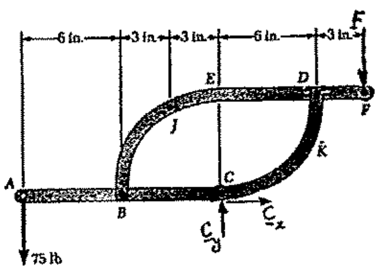
$$\mathbf{M} = 75.0\text{ lb}\cdot\text{in.} \circlearrowright \blacktriangleleft$$



PROBLEM 7.12

Two members, each consisting of a straight and a quarter-circular portion of rod, are connected as shown and support a 75-lb load at A. Determine the internal forces at Point K.

SOLUTION



Free body: Entire frame

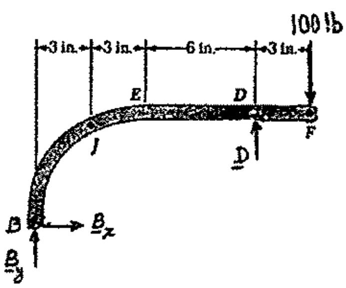
$$+\circlearrowleft \Sigma M_C = 0: \quad (75\text{ lb})(12\text{ in.}) - F(9\text{ in.}) = 0$$

$$\mathbf{F} = 100\text{ lb}\downarrow \triangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 75\text{ lb} - 100\text{ lb} = 0$$

$$C_y = +175\text{ lb} \qquad \mathbf{C} = 175\text{ lb}\uparrow \triangleleft$$



Free body: Member $BEDF$

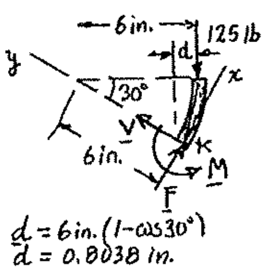
$$+\circlearrowleft \Sigma M_B = 0: \quad D(12\text{ in.}) - (100\text{ lb})(15\text{ in.}) = 0$$

$$\mathbf{D} = 125\text{ lb}\uparrow \triangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad B_y + 125\text{ lb} - 100\text{ lb} = 0$$

$$B_y = -25\text{ lb} \qquad \mathbf{B} = 25\text{ lb}\downarrow \triangleleft$$



Free body: DK

We found in Problem 7.11 that

$$\mathbf{D} = 125\text{ lb}\uparrow \text{ on } BEDF.$$

Thus $\qquad \mathbf{D} = 125\text{ lb}\downarrow \text{ on } DK. \triangleleft$

$$+\nearrow \Sigma F_x = 0: \quad F - (125\text{ lb})\cos 30° = 0$$

$$\mathbf{F} = 108.3\text{ lb} \measuredangle 60.0° \blacktriangleleft$$

PROBLEM 7.12 (Continued)

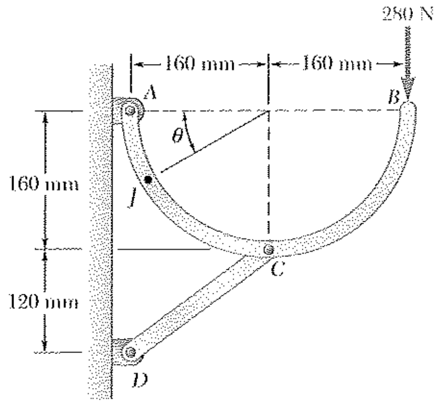
$\nwarrow^{+} \Sigma F_y = 0$: $V - (125 \text{ lb})\sin 30° = 0$

$\mathbf{V} = 62.5 \text{ lb} \searrow 30.0°$ ◀

$+\circlearrowright \Sigma M_K = 0$: $M - (125 \text{ lb})d = 0$

$$M = (125 \text{ lb})d = (125 \text{ lb})(0.8038 \text{ in.})$$
$$= 100.5 \text{ lb} \cdot \text{in.}$$

$\mathbf{M} = 100.5 \text{ lb} \cdot \text{in.} \circlearrowright$ ◀

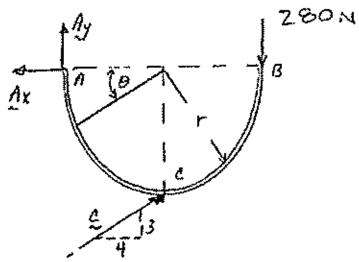


PROBLEM 7.13

A semicircular rod is loaded as shown. Determine the internal forces at Point J knowing that $\theta = 30°$.

SOLUTION

FBD AB:



$$\circlearrowleft \Sigma M_A = 0: \quad r\left(\frac{4}{5}C\right) + r\left(\frac{3}{5}C\right) - 2r(280\text{ N}) = 0$$

$$\mathbf{C} = 400\text{ N} \nearrow$$

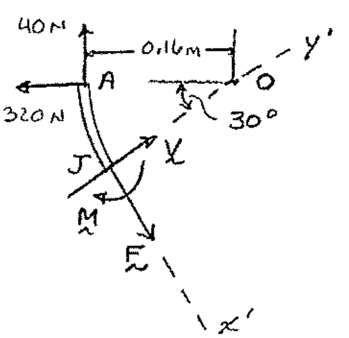
$$\rightarrow \Sigma F_x = 0: \quad -A_x + \frac{4}{5}(400\text{ N}) = 0$$

$$\mathbf{A}_x = 320\text{ N} \longleftarrow$$

$$\uparrow \Sigma F_y = 0: \quad A_y + \frac{3}{5}(400\text{ N}) - 280\text{ N} = 0$$

$$\mathbf{A}_y = 40.0\text{ N} \uparrow$$

FBD AJ:



$$\searrow \Sigma F_{x'} = 0: \quad F - (320\text{ N})\sin 30° - (40.0\text{ N})\cos 30° = 0$$

$$F = 194.641\text{ N}$$

$$\mathbf{F} = 194.6\text{ N} \nwarrow 60.0° \blacktriangleleft$$

$$\nearrow \Sigma F_{y'} = 0: \quad V - (320\text{ N})\cos 30° + (40\text{ N})\sin 30° = 0$$

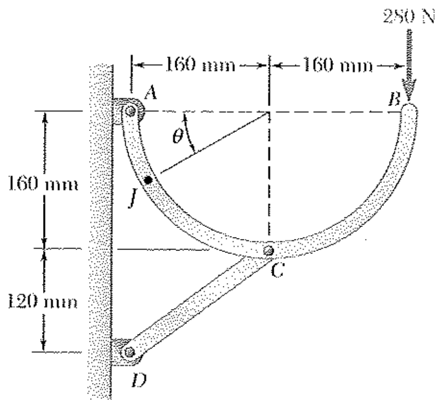
$$V = 257.13\text{ N}$$

$$\mathbf{V} = 257\text{ N} \measuredangle 30.0° \blacktriangleleft$$

$$\circlearrowleft \Sigma M_0 = 0: \quad (0.160\text{ m})(194.641\text{ N}) - (0.160\text{ m})(40.0\text{ N}) - M = 0$$

$$M = 24.743$$

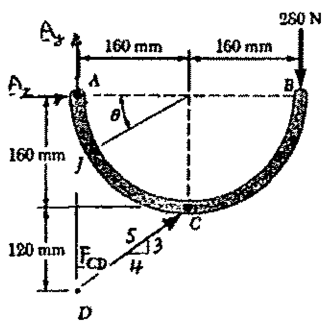
$$\mathbf{M} = 24.7\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$



PROBLEM 7.14

A semicircular rod is loaded as shown. Determine the magnitude and location of the maximum bending moment in the rod.

SOLUTION



Free body: Rod ACB

$$+\circlearrowleft \Sigma M_A = 0: \quad \left(\frac{4}{5}F_{CD}\right)(0.16 \text{ m}) + \left(\frac{3}{5}F_{CD}\right)(0.16 \text{ m}) - (280 \text{ N})(0.32 \text{ m}) = 0$$

$$\mathbf{F}_{CD} = 400 \text{ N} \nearrow \quad \triangleleft$$

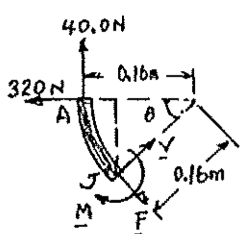
$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + \frac{4}{5}(400 \text{ N}) = 0$$

$$A_x = -320 \text{ N} \qquad \mathbf{A}_x = 320 \text{ N} \longleftarrow \quad \triangleleft$$

$$+\Sigma F_y = 0: \quad A_y + \frac{3}{5}(400 \text{ N}) - 280 \text{ N} = 0$$

$$A_y = +40.0 \text{ N} \qquad \mathbf{A}_y = 40.0 \text{ N} \uparrow \quad \triangleleft$$

Free body: AJ (For $\theta < 90°$)



$$+\circlearrowleft \Sigma M_J = 0: \quad (320 \text{ N})(0.16 \text{ m})\sin\theta - (40.0 \text{ N})(0.16 \text{ m})(1-\cos\theta) - M = 0$$

$$M = 51.2\sin\theta + 6.4\cos\theta - 6.4 \qquad (1)$$

For maximum value between A and C:

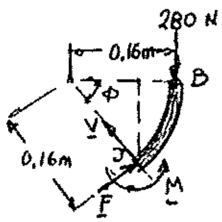
$$\frac{dM}{d\theta} = 0: \quad 51.2\cos\theta - 6.4\sin\theta = 0$$

$$\tan\theta = \frac{51.2}{6.4} = 8 \qquad \theta = 82.87° \quad \triangleleft$$

Carrying into (1):

$$M = 51.2\sin 82.87° + 6.4\cos 82.87° - 6.4 = +45.20 \text{ N}\cdot\text{m} \quad \triangleleft$$

PROBLEM 7.14 (Continued)



Free body: BJ (For $\theta > 90°$)

$$+\circlearrowleft \Sigma M_J = 0: \quad M - (280\text{ N})(0.16\text{ m})(1-\cos\phi) = 0$$

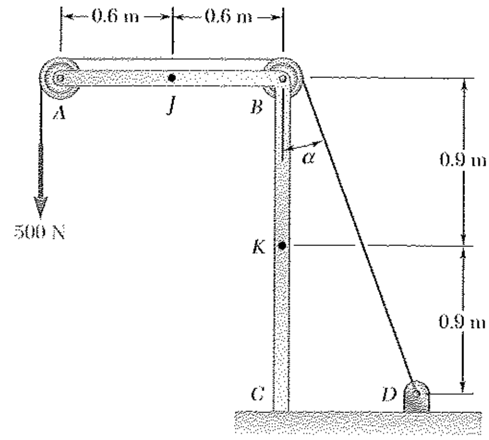
$$M = (44.8\text{ N}\cdot\text{m})(1-\cos\phi)$$

Largest value occurs for $\phi = 90°$, that is, at C, and is

$$M_C = 44.8\text{ N}\cdot\text{m} \quad \triangleleft$$

We conclude that

$$M_{max} = 45.2\text{ N}\cdot\text{m} \quad \text{for} \quad \theta = 82.9° \blacktriangleleft$$



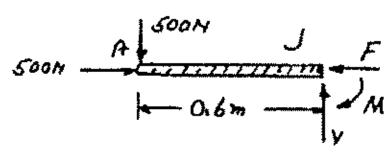
PROBLEM 7.15

Knowing that the radius of each pulley is 150 mm, that $\alpha = 20°$, and neglecting friction, determine the internal forces at (*a*) Point *J*, (*b*) Point *K*.

SOLUTION

Tension in cable = 500 N. Replace cable tension by forces at pins *A* and *B*. Radius does not enter computations: (cf. Problem 6.90)

(*a*) Free body: *AJ*



$\xrightarrow{+} \Sigma F_x = 0: \quad 500\text{ N} - F = 0$

$F = 500\text{ N}$ **F** $= 500$ N ← ◀

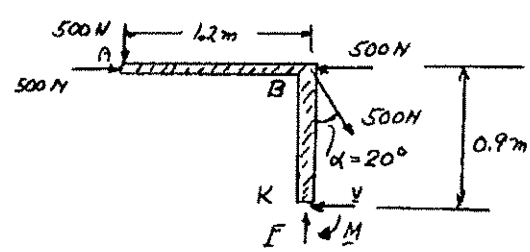
$+\uparrow \Sigma F_y = 0: \quad V - 500\text{ N} = 0$

$V = 500\text{ N}$ **V** $= 500$ N ↑ ◀

$+\circlearrowleft \Sigma M_J = 0: \quad (500\text{ N})(0.6\text{ m}) = 0$

$M = 300\text{ N}\cdot\text{m}$ **M** $= 300$ N · m ↻ ◀

(*b*) Free body: *ABK*



$\xrightarrow{+} \Sigma F_x = 0: \quad 500\text{ N} - 500\text{ N} + (500\text{ N})\sin 20° - V = 0$

$V = 171.01\text{ N}$ **V** $= 171.0$ N ← ◀

PROBLEM 7.15 (Continued)

$$+\uparrow \Sigma F_y = 0: \quad -500\text{ N} - (500\text{ N})\cos 20° + F = 0$$

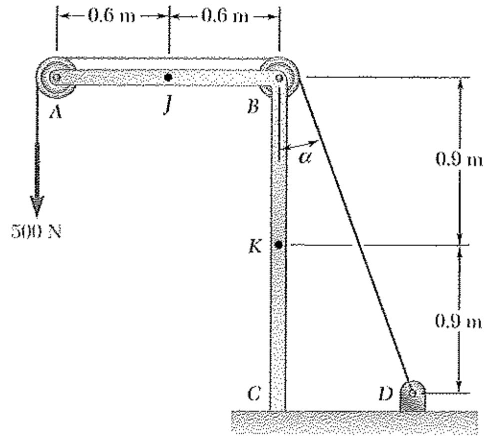
$$F = 969.8\text{ N}$$

$$\mathbf{F} = 970\text{ N}\uparrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_K = 0: \quad (500\text{ N})(1.2\text{ m}) - (500\text{ N})\sin 20°(0.9\text{ m}) - M = 0$$

$$M = 446.1\text{ N}\cdot\text{m}$$

$$\mathbf{M} = 446\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$



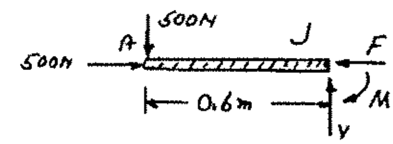
PROBLEM 7.16

Knowing that the radius of each pulley is 150 mm, that $\alpha = 30°$, and neglecting friction, determine the internal forces at (*a*) Point *J*, (*b*) Point *K*.

SOLUTION

Tension in cable = 500 N. Replace cable tension by forces at pins *A* and *B*. Radius does not enter computations: (cf. Problem 6.90)

(*a*) Free body: *AJ*:



$\xrightarrow{+} \Sigma F_x = 0: \quad 500\text{ N} - F = 0$

$F = 500\text{ N}$ $\qquad\qquad$ $\mathbf{F} = 500\text{ N} \longleftarrow$ ◀

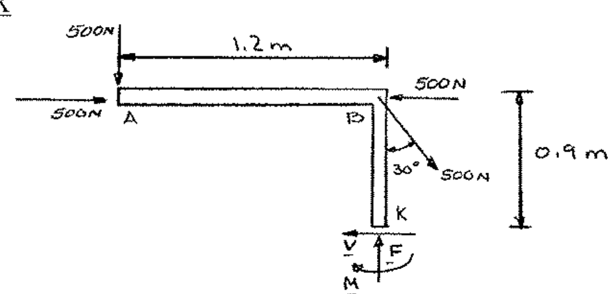
$+\!\uparrow \Sigma F_y = 0: \quad V - 500\text{ N} = 0$

$V = 500\text{ N}$ $\qquad\qquad$ $\mathbf{V} = 500\text{ N}\uparrow$ ◀

$+\circlearrowleft \Sigma M_J = 0: \quad (500\text{ N})(0.6\text{ m}) = 0$

$M = 300\text{ N}\cdot\text{m}$ $\qquad\qquad$ $\mathbf{M} = 300\text{ N}\cdot\text{m} \circlearrowright$ ◀

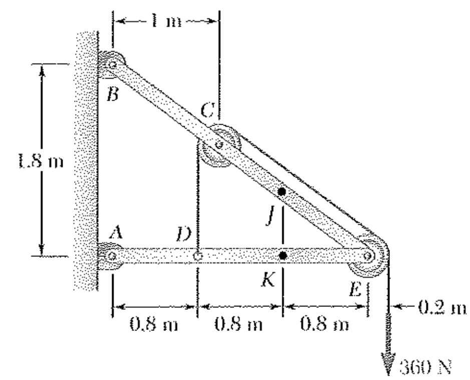
(*b*) FBD: Portion *ABK*



$\xrightarrow{+} \Sigma F_x = 0: \quad 500\text{ N} - 500\text{ N} + (500\text{ N})\sin 30° - V$ $\qquad\qquad$ $\mathbf{V} = 250\text{ N} \longleftarrow$ ◀

$+\!\uparrow \Sigma F_y = 0: \quad -500\text{ N} - (500\text{ N})\cos 30° + F = 0$ $\qquad\qquad$ $\mathbf{F} = 933\text{ N}\uparrow$ ◀

$+\circlearrowleft \Sigma M_K = 0: \quad (500\text{ N})(1.2\text{ m}) - (500\text{ N})\sin 30°(0.9\text{ m}) - M = 0$ $\qquad\qquad$ $\mathbf{M} = 375\text{ N}\cdot\text{m} \circlearrowright$ ◀

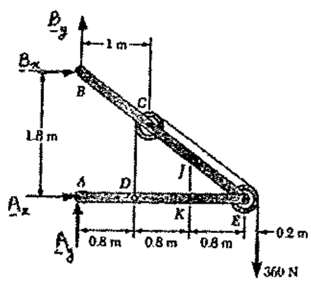


PROBLEM 7.17

Knowing that the radius of each pulley is 200 mm and neglecting friction, determine the internal forces at Point J of the frame shown.

SOLUTION

Free body: Frame and pulleys



$$+\circlearrowleft \Sigma M_A = 0: \quad -B_x(1.8 \text{ m}) - (360 \text{ N})(2.6 \text{ m}) = 0$$

$$B_x = -520 \text{ N} \qquad \mathbf{B}_x = 520 \text{ N} \longleftarrow \triangleleft$$

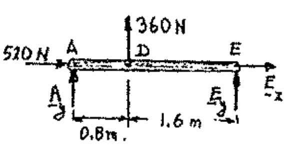
$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - 520 \text{ N} = 0$$

$$A_x = +520 \text{ N} \qquad \mathbf{A}_x = 520 \text{ N} \longrightarrow \triangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + B_y - 360 \text{ N} = 0$$

$$A_y + B_y = 360 \text{ N} \qquad (1)$$

Free body: Member AE



$$+\circlearrowleft \Sigma M_E = 0: \quad -A_y(2.4 \text{ m}) - (360 \text{ N})(1.6 \text{ m}) = 0$$

$$A_y = -240 \text{ N} \qquad \mathbf{A}_y = 240 \text{ N} \downarrow \triangleleft$$

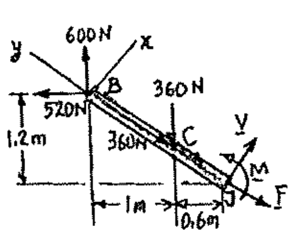
From (1):

$$B_y = 360 \text{ N} + 240 \text{ N}$$

$$B_y = +600 \text{ N} \qquad \mathbf{B}_y = 600 \text{ N} \uparrow \triangleleft$$

Free body: BJ

We recall that the forces applied to a pulley may be applied directly to its axle.



$$\nwarrow + \Sigma F_y = 0: \quad \frac{3}{5}(600 \text{ N}) + \frac{4}{5}(520 \text{ N})$$

$$-360 \text{ N} - \frac{3}{5}(360 \text{ N}) - F = 0$$

$$F = +200 \text{ N} \qquad \mathbf{F} = 200 \text{ N} \searrow \blacktriangleleft$$

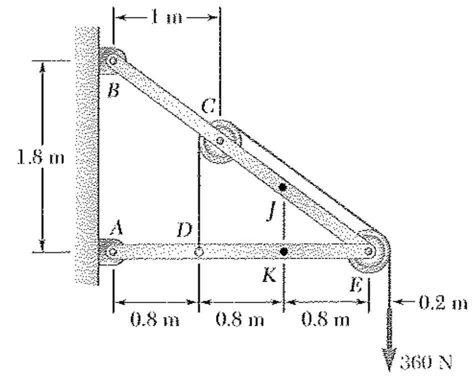
PROBLEM 7.17 (Continued)

$$+\nearrow \Sigma F_x = 0: \quad \frac{4}{5}(600 \text{ N}) - \frac{3}{5}(520 \text{ N}) - \frac{4}{5}(360 \text{ N}) + V = 0$$

$$V = +120.0 \text{ N} \qquad \mathbf{V} = 120.0 \text{ N} \nearrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_J = 0: \quad (520 \text{ N})(1.2 \text{ m}) - (600 \text{ N})(1.6 \text{ m}) + (360 \text{ N})(0.6 \text{ m}) + M = 0$$

$$M = +120.0 \text{ N}\cdot\text{m} \qquad \mathbf{M} = 120.0 \text{ N}\cdot\text{m} \circlearrowleft \blacktriangleleft$$

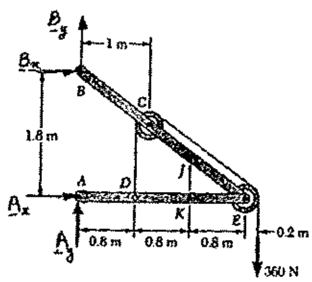


PROBLEM 7.18

Knowing that the radius of each pulley is 200 mm and neglecting friction, determine the internal forces at Point K of the frame shown.

SOLUTION

Free body: Frame and pulleys



$$+\circlearrowleft \Sigma M_A = 0: \quad -B_x(1.8\text{ m}) - (360\text{ N})(2.6\text{ m}) = 0$$

$$B_x = -520\text{ N} \qquad \mathbf{B}_x = 520\text{ N} \leftarrow \triangleleft$$

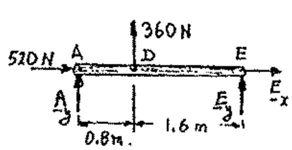
$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - 520\text{ N} = 0$$

$$A_x = +520\text{ N} \qquad \mathbf{A}_x = 520\text{ N} \rightarrow \triangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + B_y - 360\text{ N} = 0$$

$$A_y + B_y = 360\text{ N} \qquad (1)$$

Free body: Member AE



$$+\circlearrowleft \Sigma M_E = 0: \quad -A_y(2.4\text{ m}) - (360\text{ N})(1.6\text{ m}) = 0$$

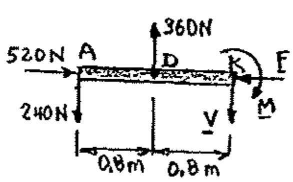
$$A_y = -240\text{ N} \qquad \mathbf{A}_y = 240\text{ N} \downarrow \triangleleft$$

From (1):

$$B_y = 360\text{ N} + 240\text{ N}$$

$$B_y = +600\text{ N} \qquad \mathbf{B}_y = 600\text{ N} \uparrow \triangleleft$$

Free body: AK



$$\xrightarrow{+} \Sigma F_x = 0: \quad 520\text{ N} - F = 0$$

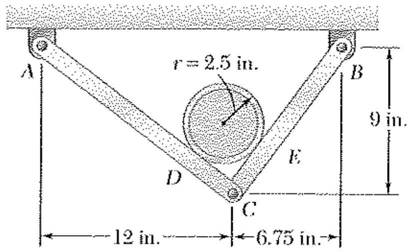
$$F = +520\text{ N} \qquad \mathbf{F} = 520\text{ N} \leftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 360\text{ N} - 240\text{ N} - V = 0$$

$$V = +120.0\text{ N} \qquad \mathbf{V} = 120.0\text{ N} \downarrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_K = 0: \quad (240\text{ N})(1.6\text{ m}) - (360\text{ N})(0.8\text{ m}) - M = 0$$

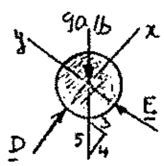
$$M = +96.0\text{ N}\cdot\text{m} \qquad \mathbf{M} = 96.0\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$



PROBLEM 7.19

A 5-in.-diameter pipe is supported every 9 ft by a small frame consisting of two members as shown. Knowing that the combined weight of the pipe and its contents is 10 lb/ft and neglecting the effect of friction, determine the magnitude and location of the maximum bending moment in member *AC*.

SOLUTION



Free body: 10-ft section of pipe

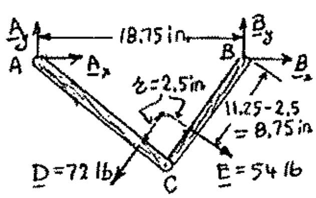
$+\nearrow \Sigma F_x = 0: \quad D - \frac{4}{5}(90 \text{ lb}) = 0 \qquad \mathbf{D} = 72 \text{ lb} \nearrow \triangleleft$

$\nwarrow + \Sigma F_y = 0: \quad E - \frac{3}{5}(90 \text{ lb}) = 0 \qquad \mathbf{E} = 54 \text{ lb} \nwarrow \triangleleft$

Free body: Frame

$+\circlearrowleft \Sigma M_B = 0: \quad -A_y(18.75 \text{ in.}) + (72 \text{ lb})(2.5 \text{ in.}) + (54 \text{ lb})(8.75 \text{ in.}) = 0$

$A_y = +34.8 \text{ lb} \qquad \mathbf{A}_y = 34.8 \text{ lb} \uparrow \triangleleft$



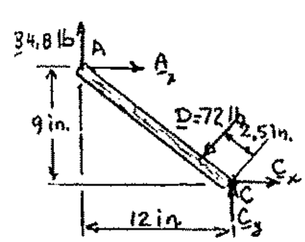
$+\uparrow \Sigma F_y = 0: \quad B_y + 34.8 \text{ lb} - \frac{4}{5}(72 \text{ lb}) - \frac{3}{5}(54 \text{ lb}) = 0$

$B_y = +55.2 \text{ lb} \qquad \mathbf{B}_y = 55.2 \text{ lb} \uparrow \triangleleft$

$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + B_x - \frac{3}{5}(72 \text{ lb}) + \frac{4}{5}(54 \text{ lb}) = 0$

$A_x + B_x = 0 \qquad (1)$

Free body: Member *AC*



$+\circlearrowleft \Sigma M_C = 0: \quad (72 \text{ lb})(2.5 \text{ in.}) - (34.8 \text{ lb})(12 \text{ in.}) - A_x(9 \text{ in.}) = 0$

$A_x = -26.4 \text{ lb} \qquad \mathbf{A}_x = 26.4 \text{ lb} \longleftarrow \triangleleft$

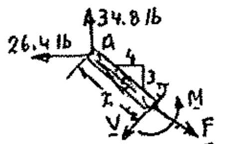
From (1): $\quad B_x = -A_x = +26.4 \text{ lb}$

$\mathbf{B}_x = 26.4 \text{ lb} \longrightarrow \triangleleft$

PROBLEM 7.19 (Continued)

Free body: Portion AJ

For $x \le 12.5$ in. $(AJ \le AD)$:

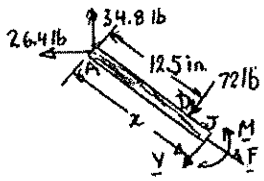


$$+\circlearrowleft \Sigma M_J = 0: \quad (26.4 \text{ lb})\frac{3}{5}x - (34.8 \text{ lb})\frac{4}{5}x + M = 0$$

$$M = 12x$$

$$M_{max} = 150 \text{ lb} \cdot \text{in. for } x = 12.5 \text{ in.}$$

$$M_{max} = 150.0 \text{ lb} \cdot \text{in. at } D \triangleleft$$



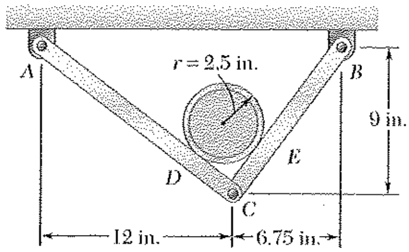
For $x > 12.5$ in.$(AJ > AD)$:

$$+\circlearrowleft \Sigma M_J = 0: \quad (26.4 \text{ lb})\frac{3}{5}x - (34.8 \text{ lb})\frac{4}{5}x + (72 \text{ lb})(x - 12.5) + M = 0$$

$$M = 900 - 60x$$

$$M_{max} = 150 \text{ lb} \cdot \text{in. for } x = 12.5 \text{ in.}$$

Thus: $$M_{max} = 150.0 \text{ lb} \cdot \text{in. at } D \blacktriangleleft$$

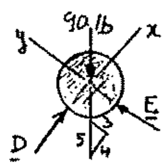


PROBLEM 7.20

For the frame of Problem 7.19, determine the magnitude and location of the maximum bending moment in member *BC*.

PROBLEM 7.19 A 5-in.-diameter pipe is supported every 9 ft by a small frame consisting of two members as shown. Knowing that the combined weight of the pipe and its contents is 10 lb/ft and neglecting the effect of friction, determine the magnitude and location of the maximum bending moment in member *AC*.

SOLUTION

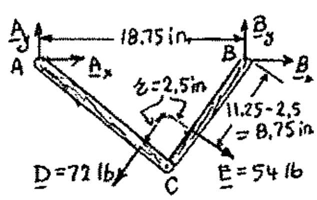


Free body: 10-ft section of pipe

$$+\nearrow \Sigma F_x = 0: \quad D - \frac{4}{5}(90 \text{ lb}) = 0 \qquad \mathbf{D} = 72 \text{ lb} \nearrow \triangleleft$$

$$\nwarrow + \Sigma F_y = 0: \quad E - \frac{3}{5}(90 \text{ lb}) = 0 \qquad \mathbf{E} = 54 \text{ lb} \nwarrow \triangleleft$$

Free body: Frame



$$+\circlearrowleft \Sigma M_B = 0: \quad -A_y(18.75 \text{ in.}) + (72 \text{ lb})(2.5 \text{ in.}) + (54 \text{ lb})(8.75 \text{ in.}) = 0$$

$$A_y = +34.8 \text{ lb} \qquad \mathbf{A}_y = 34.8 \text{ lb} \uparrow \triangleleft$$

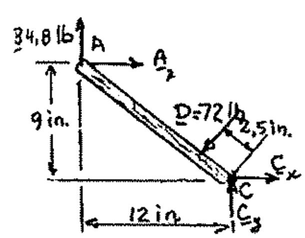
$$+\uparrow \Sigma F_y = 0: \quad B_y + 34.8 \text{ lb} - \frac{4}{5}(72 \text{ lb}) - \frac{3}{5}(54 \text{ lb}) = 0$$

$$B_y = +55.2 \text{ lb} \qquad \mathbf{B}_y = 55.2 \text{ lb} \uparrow \triangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + B_x - \frac{3}{5}(72 \text{ lb}) + \frac{4}{5}(54 \text{ lb}) = 0$$

$$A_x + B_x = 0 \qquad (1)$$

Free body: Member *AC*



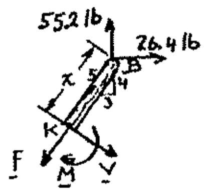
$$+\circlearrowleft \Sigma M_C = 0: \quad (72 \text{ lb})(2.5 \text{ in.}) - (34.8 \text{ lb})(12 \text{ in.}) - A_x(9 \text{ in.}) = 0$$

$$A_x = -26.4 \text{ lb} \qquad \mathbf{A}_x = 26.4 \text{ lb} \longleftarrow \triangleleft$$

From (1): $\quad B_x = -A_x = +26.4 \text{ lb}$

$$\mathbf{B}_x = 26.4 \text{ lb} \longrightarrow \triangleleft$$

PROBLEM 7.20 (Continued)



Free body: Portion BK

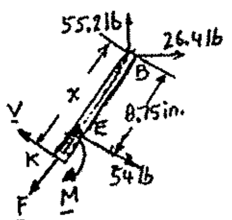
For $x \leq 8.75$ in. $(BK \leq BE)$:

$$+\circlearrowleft \Sigma M_K = 0: \quad (55.2 \text{ lb})\frac{3}{5}x - (26.4 \text{ lb})\frac{4}{5}x - M = 0$$

$$M = 12x$$

$$M_{\max} = 105.0 \text{ lb} \cdot \text{in.} \quad \text{for} \quad x = 8.75 \text{ in.}$$

$$M_{\max} = 105.0 \text{ lb} \cdot \text{in. at } E \ \triangleleft$$



For $x > 8.75$ in. $(BK > BE)$:

$$+\circlearrowleft \Sigma M_K = 0: \quad (55.2 \text{ lb})\frac{3}{5}x - (26.4 \text{ lb})\frac{4}{5}x - (54 \text{ lb})(x - 8.75 \text{ in.}) - M = 0$$

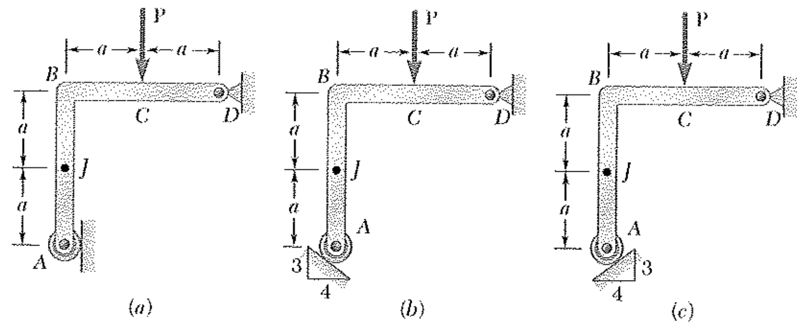
$$M = 472.5 - 42x$$

$$M_{\max} = 105.0 \text{ lb} \cdot \text{in.} \quad \text{for} \quad x = 8.75 \text{ in.}$$

Thus $$M_{\max} = 105.0 \text{ lb} \cdot \text{in. at } E \ \blacktriangleleft$$

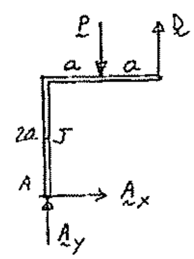
PROBLEM 7.21

A force **P** is applied to a bent rod that is supported by a roller and a pin and bracket. For each of the three cases shown, determine the internal forces at Point *J*.



SOLUTION

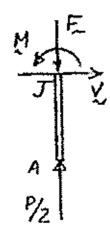
(*a*) **FBD Rod:**



$\rightarrow \Sigma F_x = 0: \quad A_x = 0$

$\circlearrowleft \Sigma M_D = 0: \quad aP - 2aA_y = 0 \qquad A_y = \dfrac{P}{2}$

FBD AJ:

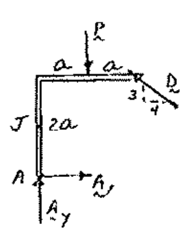


$\rightarrow \Sigma F_x = 0: \quad \mathbf{V} = 0$ ◀

$\uparrow \Sigma F_y = 0: \quad \dfrac{P}{2} - F = 0 \qquad \mathbf{F} = \dfrac{P}{2} \downarrow$ ◀

$\circlearrowleft \Sigma M_J = 0: \quad \mathbf{M} = 0$ ◀

(*b*) **FBD Rod:**

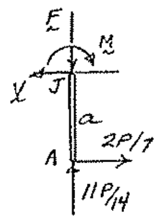


$\circlearrowleft \Sigma M_A = 0: \quad 2a\left(\dfrac{4}{5}D\right) + 2a\left(\dfrac{3}{5}D\right) - aP = 0 \qquad D = \dfrac{5P}{14}$

$\rightarrow \Sigma F_x = 0: \quad A_x - \dfrac{4}{5}\dfrac{5}{14}P = 0 \qquad A_x = \dfrac{2P}{7}$

$\uparrow \Sigma F_y = 0: \quad A_y - P + \dfrac{3}{5}\dfrac{5}{14}P = 0 \qquad A_y = \dfrac{11P}{14}$

FBD AJ:

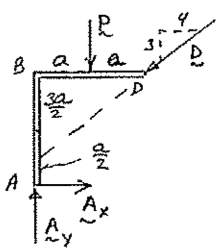


$\xrightarrow{+} \Sigma F_x = 0$: $\quad \frac{2}{7}P - V = 0 \qquad \mathbf{V} = \frac{2P}{7} \leftarrow$ ◀

$+\!\uparrow \Sigma F_y = 0$: $\quad \frac{11P}{14} - F = 0 \qquad \mathbf{F} = \frac{11P}{14} \downarrow$ ◀

$\circlearrowleft\!+ \Sigma M_J = 0$: $\quad a\frac{2P}{7} - M = 0 \qquad \mathbf{M} = \frac{2}{7}aP \circlearrowright$ ◀

(*c*) **FBD Rod:**

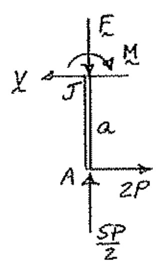


$\circlearrowleft\!+ \Sigma M_A = 0$: $\quad \frac{a}{2}\left(\frac{4D}{5}\right) - aP = 0 \qquad D = \frac{5P}{2}$

$\xrightarrow{+} \Sigma F_x = 0$: $\quad A_x - \frac{4}{5}\frac{5P}{2} = 0 \qquad A_x = 2P$

$+\!\uparrow \Sigma F_y = 0$: $\quad A_y - P - \frac{3}{5}\frac{5P}{2} = 0 \qquad A_y = \frac{5P}{2}$

FBD AJ:



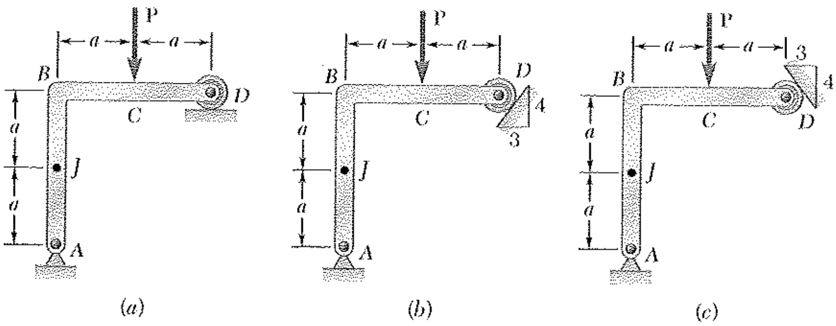
$\xrightarrow{+} \Sigma F_x = 0$: $\quad 2P - V = 0 \qquad \mathbf{V} = 2P \leftarrow$ ◀

$+\!\uparrow \Sigma F_y = 0$: $\quad \frac{5P}{2} - F = 0 \qquad \mathbf{F} = \frac{5P}{2} \downarrow$ ◀

$\circlearrowleft\!+ \Sigma M_J = 0$: $\quad a(2P) - M = 0 \qquad \mathbf{M} = 2aP \circlearrowright$ ◀

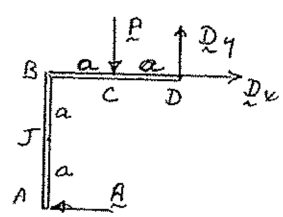
PROBLEM 7.22

A force **P** is applied to a bent rod that is supported by a roller and a pin and bracket. For each of the three cases shown, determine the internal forces at Point J.



SOLUTION

(*a*) **FBD Rod:**

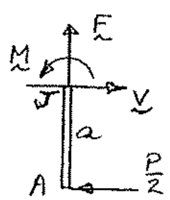


$$\circlearrowleft \Sigma M_D = 0: \quad aP - 2aA = 0$$

$$\mathbf{A} = \frac{P}{2} \longleftarrow$$

$$\longrightarrow \Sigma F_x = 0: \quad V - \frac{P}{2} = 0 \qquad \mathbf{V} = \frac{P}{2} \longrightarrow \blacktriangleleft$$

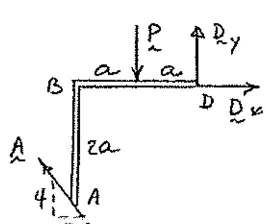
FBD AJ:



$$\uparrow \Sigma F_y = 0: \qquad \mathbf{F} = 0 \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: \quad M - a\frac{P}{2} = 0 \qquad \mathbf{M} = \frac{aP}{2} \circlearrowright \blacktriangleleft$$

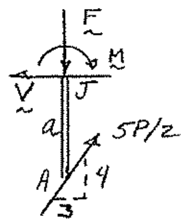
(*b*) **FBD Rod:**



$$\circlearrowleft \Sigma M_D = 0: \quad aP - \frac{a}{2}\left(\frac{4}{5}A\right) = 0$$

$$\mathbf{A} = \frac{5P}{2} \nearrow$$

FBD AJ:

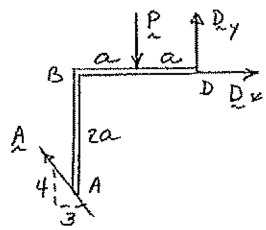


$$\rightarrow \Sigma F_x = 0: \quad \frac{3}{5}\frac{5P}{2} - V = 0 \qquad \mathbf{V} = \frac{3P}{2} \leftarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \quad \frac{4}{5}\frac{5P}{2} - F = 0 \qquad \mathbf{F} = 2P \downarrow \blacktriangleleft$$

$$\mathbf{M} = \frac{3}{2}aP \circlearrowright \blacktriangleleft$$

(*c*) **FBD Rod:**



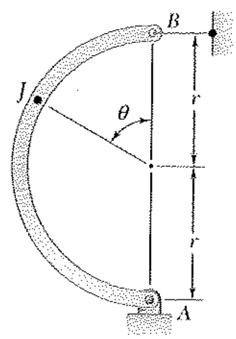
$$\circlearrowleft \Sigma M_D = 0: \quad aP - 2a\left(\frac{3}{5}A\right) - 2a\left(\frac{4}{5}A\right) = 0$$

$$A = \frac{5P}{14}$$

$$\rightarrow \Sigma F_x = 0: \quad V - \left(\frac{3}{5}\frac{5P}{14}\right) = 0 \qquad \mathbf{V} = \frac{3P}{14} \rightarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \quad \frac{4}{5}\frac{5P}{14} - F = 0 \qquad \mathbf{F} = \frac{2P}{7} \downarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: \quad M - a\left(\frac{3}{5}\frac{5P}{14}\right) = 0 \qquad \mathbf{M} = \frac{3}{14}aP \circlearrowleft \blacktriangleleft$$

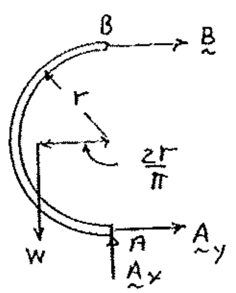


PROBLEM 7.23

A semicircular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at Point J when $\theta = 60°$.

SOLUTION

FBD Rod:



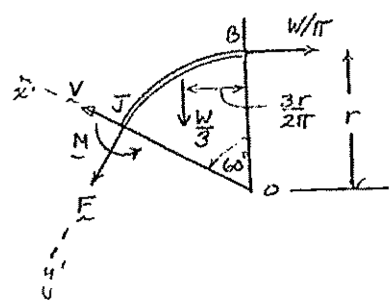
$$\circlearrowleft \Sigma M_A = 0: \quad \frac{2r}{\pi}W - 2rB = 0$$

$$\mathbf{B} = \frac{W}{\pi} \longrightarrow$$

$$\swarrow \Sigma F_{y'} = 0: \quad F + \frac{W}{3}\sin 60° - \frac{W}{\pi}\cos 60° = 0$$

$$F = -0.12952W$$

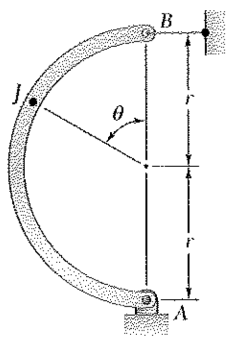
FBD BJ:



$$\circlearrowleft \Sigma M_0 = 0: \quad r\left(F - \frac{W}{\pi}\right) + \frac{3r}{2\pi}\left(\frac{W}{3}\right) + M = 0$$

$$M = Wr\left(0.12952 + \frac{1}{\pi} - \frac{1}{2\pi}\right) = 0.28868Wr$$

On BJ $\quad \mathbf{M}_J = 0.289Wr \circlearrowright$ ◀

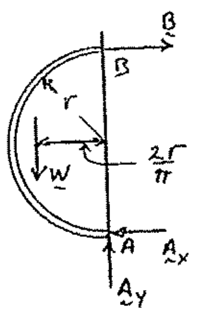


PROBLEM 7.24

A semicircular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at Point J when $\theta = 150°$.

SOLUTION

FBD Rod:

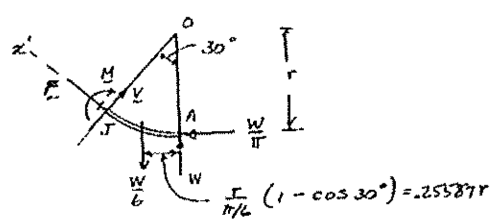


$$\uparrow \Sigma F_y = 0: \quad A_y - W = 0 \qquad \mathbf{A}_y = W \uparrow$$

$$\Sigma M_B = 0: \quad \frac{2r}{\pi} W - 2rA_x = 0$$

$$\mathbf{A}_x = \frac{W}{\pi} \longleftarrow$$

FBD AJ:



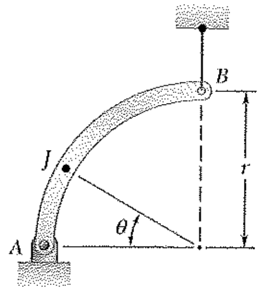
$$\searrow \Sigma F_{x'} = 0: \quad \frac{W}{\pi}\cos 30° + \frac{5W}{6}\sin 30° - F = 0 \quad F = 0.69233W \searrow$$

$$\circlearrowleft \Sigma M_0 = 0: \quad 0.25587r\left(\frac{W}{6}\right) + r\left(F - \frac{W}{\pi}\right) - M = 0$$

$$M = Wr\left[\frac{0.25587}{6} + 0.69233 - \frac{1}{\pi}\right]$$

$$M = Wr(0.4166)$$

On AJ $\quad \mathbf{M} = 0.417Wr \circlearrowright$ ◀

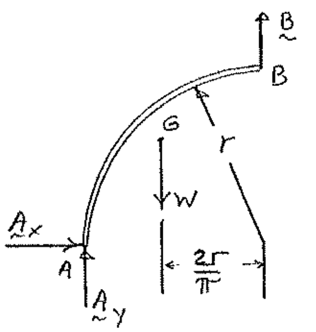


PROBLEM 7.25

A quarter-circular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at Point J when $\theta = 30°$.

SOLUTION

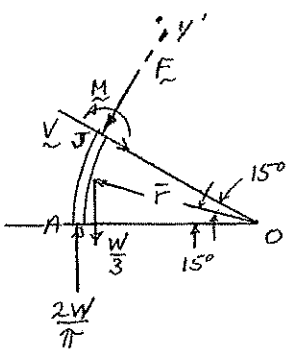
FBD Rod:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \mathbf{A}_x = 0$$

$$\circlearrowleft \Sigma M_B = 0: \quad \frac{2r}{\pi}W - rA_y = 0 \qquad \mathbf{A}_y = \frac{2W}{\pi}\uparrow$$

FBD AJ:



$$\alpha = 15°, \text{ weight of segment} = W\frac{30°}{90°} = \frac{W}{3}$$

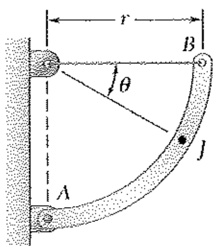
$$\bar{r} = \frac{r}{\alpha}\sin\alpha = \frac{r}{\frac{\pi}{12}}\sin 15° = 0.9886r$$

$$\nearrow \Sigma F_{y'} = 0: \quad \frac{2W}{\pi}\cos 30° - \frac{W}{3}\cos 30° - F = 0$$

$$\mathbf{F} = \frac{W\sqrt{3}}{2}\left(\frac{2}{\pi} - \frac{1}{3}\right) \swarrow$$

$$\circlearrowleft \Sigma M_0 = M + r\left(F - \frac{2W}{\pi}\right) + \bar{r}\cos 15°\frac{W}{3} = 0 \qquad \mathbf{M} = 0.0557Wr \circlearrowright \blacktriangleleft$$

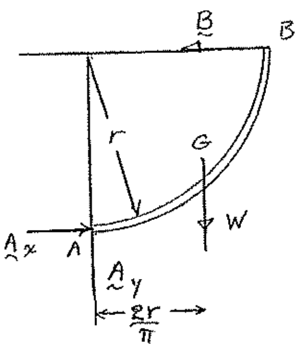
PROBLEM 7.26



A quarter-circular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at Point J when $\theta = 30°$.

SOLUTION

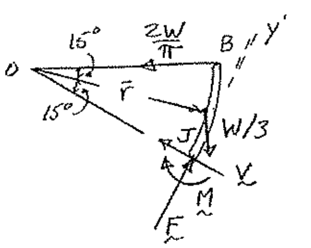
FBD Rod:



$$\circlearrowleft \Sigma M_A = 0: \quad rB - \frac{2r}{\pi}W = 0$$

$$\mathbf{B} = \frac{2W}{\pi} \longleftarrow$$

FBD BJ:



$$\alpha = 15° = \frac{\pi}{12}$$

$$\bar{r} = \frac{r}{\frac{\pi}{12}}\sin 15° = 0.98862r$$

$$\text{Weight of segment } = W\frac{30°}{90°} = \frac{W}{3}$$

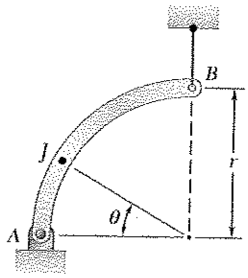
$$\nearrow \Sigma F_{y'} = 0: \quad F - \frac{W}{3}\cos 30° - \frac{2W}{\pi}\sin 30° = 0$$

$$\mathbf{F} = \left(\frac{\sqrt{3}}{6} + \frac{1}{\pi}\right)W \nearrow$$

$$\circlearrowleft \Sigma M_0 = 0: \quad rF - (\bar{r}\cos 15°)\frac{W}{3} - M = 0$$

$$M = rW\left(\frac{\sqrt{3}}{6} + \frac{1}{\pi}\right) - \left(0.98862\frac{\cos 15°}{3}\right)Wr \qquad \mathbf{M} = 0.289Wr \circlearrowright \blacktriangleleft$$

PROBLEM 7.27

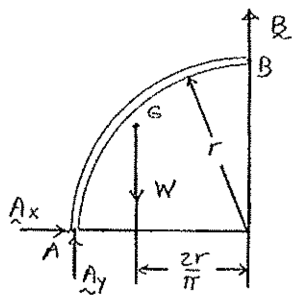


For the rod of Problem 7.25, determine the magnitude and location of the maximum bending moment.

PROBLEM 7.25 A quarter-circular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at Point J when $\theta = 30°$.

SOLUTION

FBD Rod:



$$\rightarrow \Sigma F_x = 0: \quad A_x = 0$$

$$\circlearrowleft \Sigma M_B = 0: \quad \frac{2r}{\pi}W - rA_y = 0 \qquad A_y = \frac{2W}{\pi}$$

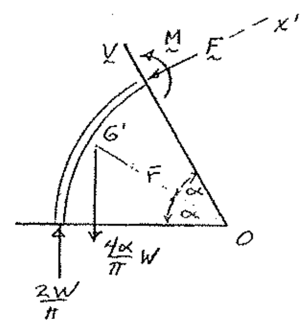
$$\alpha = \frac{\theta}{2}, \qquad \bar{r} = \frac{r}{\alpha}\sin\alpha$$

$$\text{Weight of segment} = W\frac{2\alpha}{\frac{\pi}{2}} = \frac{4\alpha}{\pi}W$$

$$\nearrow \Sigma F_{x'} = 0: \quad -F - \frac{4\alpha}{\pi}W\cos 2\alpha + \frac{2W}{\pi}\cos 2\alpha = 0$$

$$F = \frac{2W}{\pi}(1-2\alpha)\cos 2\alpha = \frac{2W}{\pi}(1-\theta)\cos\theta$$

FBD AJ: $\circlearrowleft \Sigma M_0 = 0: \quad M + \left(F - \frac{2W}{\pi}\right)r + (\bar{r}\cos\alpha)\frac{4\alpha}{\pi}W = 0$



$$M = \frac{2W}{\pi}(1 + \theta\cos\theta - \cos\theta)r - \frac{4\alpha W}{\pi}\frac{r}{\alpha}\sin\alpha\cos\alpha$$

But, $\quad \sin\alpha\cos\alpha = \frac{1}{2}\sin 2\alpha = \frac{1}{2}\sin\theta$

so $\quad M = \frac{2r}{\pi}W(1 - \cos\theta + \theta\cos\theta - \sin\theta)$

$$\frac{dM}{d\theta} = \frac{2rW}{\pi}(\sin\theta - \theta\sin\theta + \cos\theta - \cos\theta) = 0$$

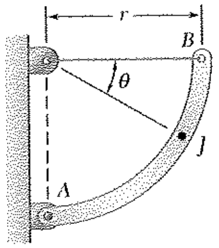
for $\quad (1-\theta)\sin\theta = 0$

$$\frac{dM}{d\theta} = 0 \quad \text{for} \quad \theta = 0, 1, n\pi\,(n = 1, 2, \cdots)$$

Only 0 and 1 in valid range

At $\quad \theta = 0 \quad M = 0, \quad \text{at} \quad \theta = 1\text{ rad}$

at $\quad \theta = 57.3°$ $\qquad M = M_{max} = 0.1009Wr$ ◀



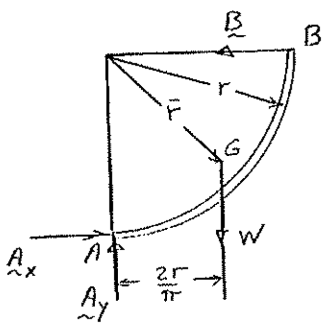
PROBLEM 7.28

For the rod of Problem 7.26, determine the magnitude and location of the maximum bending moment.

PROBLEM 7.26 A quarter-circular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at Point J when $\theta = 30°$.

SOLUTION

FBD Bar:



$$\circlearrowleft \Sigma M_A = 0: \quad rB - \frac{2r}{\pi}W = 0 \qquad \mathbf{B} = \frac{2W}{\pi} \longleftarrow$$

$$\alpha = \frac{\theta}{2} \qquad \text{so} \qquad 0 \le \alpha \le \frac{\pi}{4}$$

$$\bar{r} = \frac{r}{\alpha}\sin\alpha$$

$$\text{Weight of segment } = W\frac{2\alpha}{\frac{\pi}{2}}$$

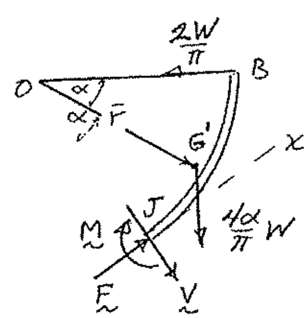
$$= \frac{4\alpha}{\pi}W$$

$$\nearrow \Sigma F_{x'} = 0: \quad F - \frac{4\alpha}{\pi}W\cos 2\alpha - \frac{2W}{\pi}\sin 2\alpha = 0$$

$$F = \frac{2W}{\pi}(\sin 2\alpha + 2\alpha\cos 2\alpha)$$

$$= \frac{2W}{\pi}(\sin\theta + \theta\cos\theta)$$

FBD BJ:



$$\circlearrowleft \Sigma M_0 = 0: \quad rF - (\bar{r}\cos\alpha)\frac{4\alpha}{\pi}W - M = 0$$

$$M = \frac{2}{\pi}Wr(\sin\theta + \theta\cos\theta) - \left(\frac{r}{\alpha}\sin\alpha\cos\alpha\right)\frac{4\alpha}{\pi}W$$

But, $\quad \sin\alpha\cos\alpha = \frac{1}{2}\sin 2\alpha = \frac{1}{2}\sin\theta$

PROBLEM 7.28 (Continued)

so $$M = \frac{2Wr}{\pi}(\sin\theta + \theta\cos\theta - \sin\theta)$$

or $$M = \frac{2}{\pi}Wr\theta\cos\theta$$

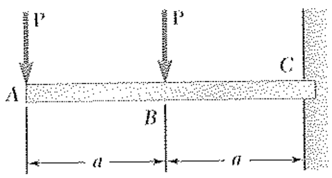
$$\frac{dM}{d\theta} = \frac{2}{\pi}Wr(\cos\theta - \theta\sin\theta) = 0 \quad \text{at } \theta\tan\theta = 1$$

Solving numerically $\theta = 0.8603$ rad

and $\mathbf{M} = 0.357Wr$ ◀

at $\theta = 49.3°$ ◀

(Since $M = 0$ at both limits, this is the maximum)

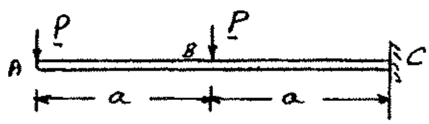


PROBLEM 7.29

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(*a*)

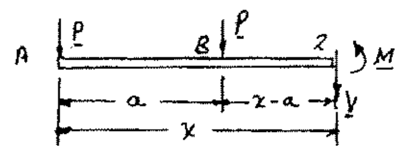


From *A* to *B*:

$+\uparrow \Sigma F_y = 0: \quad V = -P$

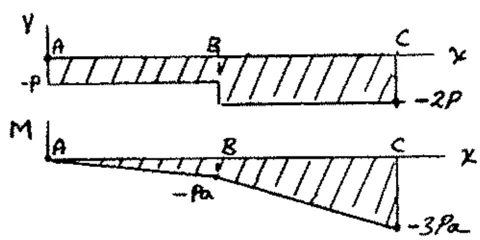
$+\circlearrowleft \Sigma M_1 = 0: \quad M = -P_x$

From *B* to *C*:

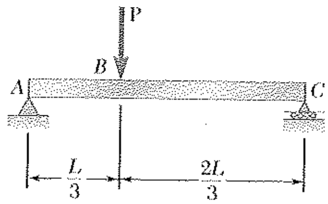


$+\uparrow \Sigma F_y = 0: \quad -P - P - V = 0 \qquad V = -2P$

$+\circlearrowleft \Sigma M_2 = 0: \quad P_x + P(x-a) + M = 0 \qquad M = -2P_x + P_a$



(*b*) $|V|_{max} = 2P; \quad |M|_{max} = 3P_a$ ◀



PROBLEM 7.30

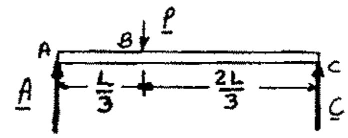
For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

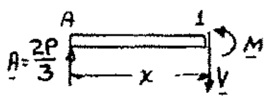
(*a*) Reactions:

$$\mathbf{A} = \frac{2P}{3}\uparrow$$

$$\mathbf{C} = \frac{P}{3}\uparrow$$



From *A* to *B*:



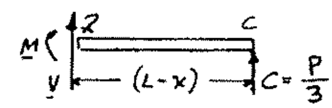
$$+\uparrow \Sigma F_y = 0: \quad V = +\frac{2P}{3}$$

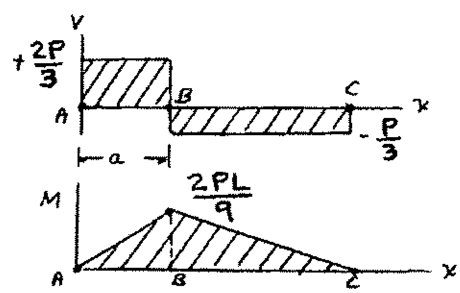
$$+\circlearrowleft \Sigma M_1 = 0: \quad M = +\frac{2P}{3}x$$

From *B* to *C*:

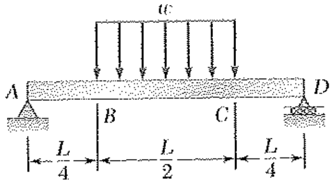
$$+\uparrow \Sigma F_y = 0: \quad V = -\frac{P}{3}$$

$$+\circlearrowleft \Sigma M_2 = 0: \quad M = +\frac{P}{3}(L-x)$$





(*b*) $|V|_{\max} = \dfrac{2P}{3}; \quad |M|_{\max} = \dfrac{2PL}{9}$ ◀



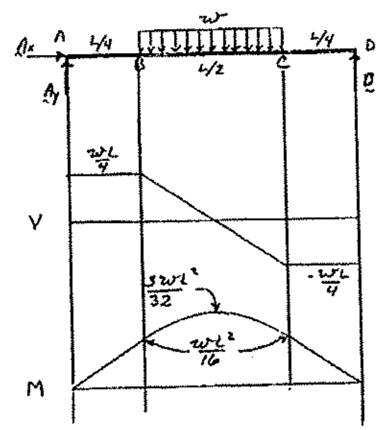
PROBLEM 7.31

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

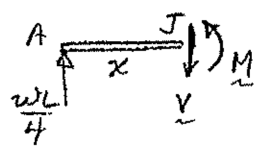
SOLUTION

FBD beam:

(*a*) By symmetry: $A_y = D = \frac{1}{2}(w)\frac{L}{2} \qquad \mathbf{A}_y = \mathbf{D} = \frac{wL}{4}\uparrow$



Along AB:

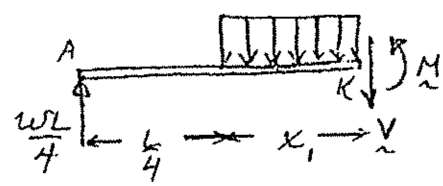


$$\uparrow \Sigma F_y = 0: \quad \frac{wL}{4} - V = 0 \qquad V = \frac{wL}{4}$$

$$\circlearrowleft \Sigma M_J = 0: \quad M - x\frac{wL}{4} = 0$$

$$M = \frac{wL}{4}x \text{ (straight)}$$

Along BC:



$$\uparrow \Sigma F_y = 0: \quad \frac{wL}{4} - wx_1 - V = 0$$

$$V = \frac{wL}{4} - wx_1$$

Straight with $\qquad V = 0 \text{ at } x_1 = \frac{L}{4}$

$$\circlearrowleft \Sigma M_k = 0: \quad M + \frac{x_1}{2}wx_1 - \left(\frac{L}{4} + x_1\right)\frac{wL}{4} = 0$$

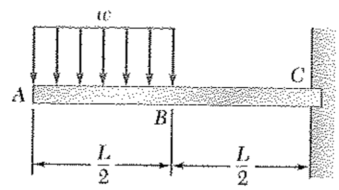
$$M = \frac{w}{2}\left(\frac{L^2}{8} + \frac{L}{2}x_1 - x_1^2\right)$$

Parabola with $\qquad M = \frac{3}{32}wL^2 \text{ at } x_1 = \frac{L}{4}$

Section *CD* by symmetry

(*b*) From diagrams: $\qquad |V|_{max} = \frac{wL}{4}$ on *AB* and *CD* ◀

$$|M|_{max} = \frac{3wL^2}{32} \text{ at center} ◀$$

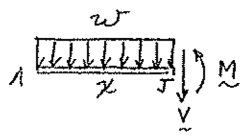


PROBLEM 7.32

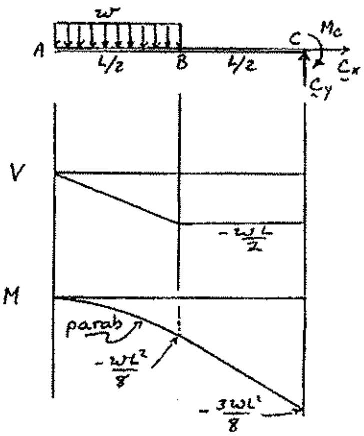
For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(*a*) **Along AB:**



$$\uparrow \Sigma F_y = 0: \quad -wx - V = 0 \qquad V = -wx$$

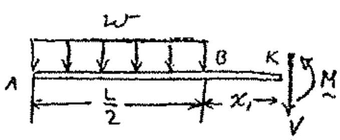


Straight with $V = -\dfrac{wL}{2}$ at $x = \dfrac{L}{2}$

$$\circlearrowleft \Sigma M_J = 0: \quad M + \frac{x}{2}wx = 0 \quad M = -\frac{1}{2}wx^2$$

Parabola with $M = -\dfrac{wL^2}{8}$ at $x = \dfrac{L}{2}$

Along BC:



$$\uparrow \Sigma F_y = 0: \quad -w\frac{L}{2} - V = 0 \qquad V = -\frac{1}{2}wL$$

$$\circlearrowleft \Sigma M_k = 0: \quad M + \left(x_1 + \frac{L}{4}\right)w\frac{L}{2} = 0$$

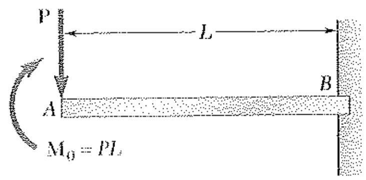
$$M = -\frac{wL}{2}\left(\frac{L}{4} + x_1\right)$$

Straight with $M = -\dfrac{3}{8}wL^2$ at $x_1 = \dfrac{L}{2}$

(*b*) From diagrams:

$$|V|_{\max} = \frac{wL}{2} \text{ on } BC \blacktriangleleft$$

$$|M|_{\max} = \frac{3wL^2}{8} \text{ at } C \blacktriangleleft$$

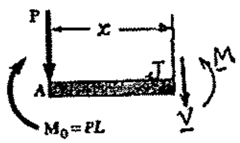


PROBLEM 7.33

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

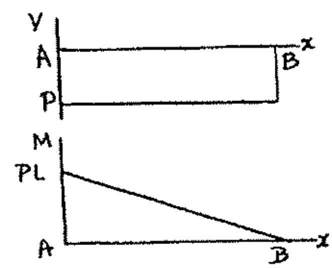
Free body: Portion *AJ*



$+\uparrow \Sigma F_y = 0: \quad -P - V = 0$ $\qquad V = -P$ ◁

$+\circlearrowleft \Sigma M_J = 0: \quad M + P_x - PL = 0$ $\qquad M = P(L - x)$ ◁

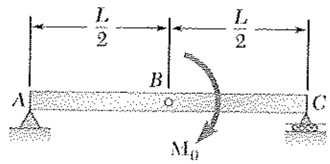
(*a*) The *V* and *M* diagrams are obtained by plotting the functions *V* and *M*.



(*b*)

$|V|_{max} = P$ ◀

$|M|_{max} = PL$ ◀



PROBLEM 7.34

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

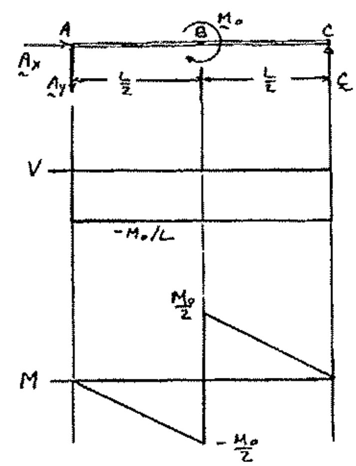
(*a*) **FBD Beam:**

$$\circlearrowleft \Sigma M_C = 0: \quad LA_y - M_0 = 0$$

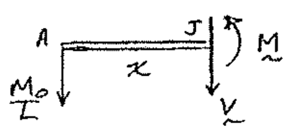
$$\mathbf{A}_y = \frac{M_0}{L} \downarrow$$

$$\uparrow \Sigma F_y = 0: \quad -A_y + C = 0$$

$$\mathbf{C} = \frac{M_0}{L} \uparrow$$



Along AB:



$$\uparrow \Sigma F_y = 0: \quad -\frac{M_0}{L} - V = 0$$

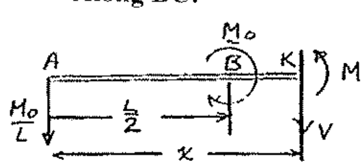
$$V = -\frac{M_0}{L}$$

$$\circlearrowleft \Sigma M_J = 0: \quad x\frac{M_0}{L} + M = 0$$

$$M = -\frac{M_0}{L}x$$

Straight with $\quad M = -\frac{M_0}{2}$ at B

Along BC:



$$\uparrow \Sigma F_y = 0: \quad -\frac{M_0}{L} - V = 0 \qquad V = -\frac{M_0}{L}$$

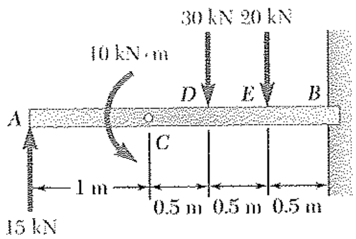
$$\circlearrowleft \Sigma M_K = 0: \quad M + x\frac{M_0}{L} - M_0 = 0 \qquad M = M_0\left(1 - \frac{x}{L}\right)$$

Straight with $\quad M = \frac{M_0}{2}$ at $B \qquad M = 0$ at C

(*b*) From diagrams:

$|V|_{max} = P$ everywhere ◀

$|M|_{max} = \frac{M_0}{2}$ at B ◀



PROBLEM 7.35

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(*a*) Just to the right of *A*:

$$+\uparrow \Sigma F_y = 0 \quad V_1 = +15\text{ kN} \quad M_1 = 0$$

Just to the left of *C*:

$$V_2 = +15\text{ kN} \quad M_2 = +15\text{ kN}\cdot\text{m}$$

Just to the right of *C*:

$$V_3 = +15\text{ kN} \quad M_3 = +5\text{ kN}\cdot\text{m}$$

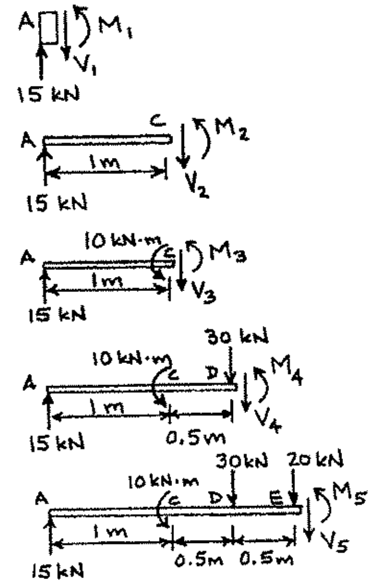
Just to the right of *D*:

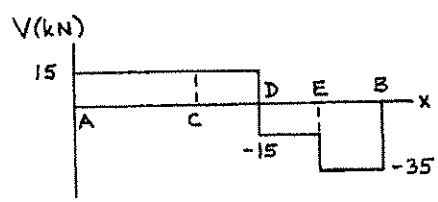
$$V_4 = -15\text{ kN} \quad M_4 = +12.5\text{ kN}\cdot\text{m}$$

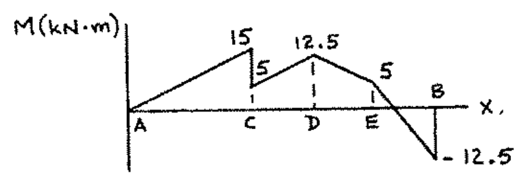
Just to the right of *E*:

$$V_5 = -35\text{ kN} \quad M_5 = +5\text{ kN}\cdot\text{m}$$

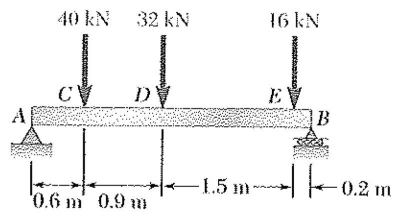
At *B*: $M_B = -12.5\text{ kN}\cdot\text{m}$







(*b*) $|V|_{max} = 35.0\text{ kN}$ $|M|_{max} = 12.50\text{ kN}\cdot\text{m}$ ◀

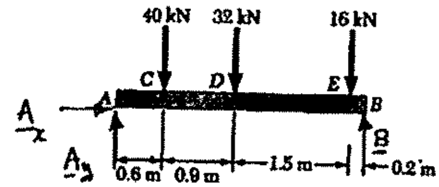


PROBLEM 7.36

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



$+\circlearrowleft \Sigma M_A = 0$: $B(3.2\text{ m}) - (40\text{ kN})(0.6\text{ m}) - (32\text{ kN})(1.5\text{ m}) - (16\text{ kN})(3\text{ m}) = 0$

$B = +37.5\text{ kN}$ $\qquad$ $\mathbf{B} = 37.5\text{ kN}\uparrow$ ◁

$\Sigma F_x = 0$: $A_x = 0$

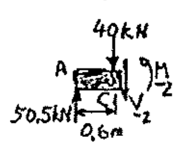
$+\uparrow \Sigma F_y = 0$: $A_y + 37.5\text{ kN} - 40\text{ kN} - 32\text{ kN} - 16\text{ kN} = 0$

$A_y = +50.5\text{ kN}$ $\qquad$ $\mathbf{A} = 50.5\text{ kN}\uparrow$ ◁

(*a*) Shear and bending moment.

Just to the right of *A*: $V_1 = 50.5\text{ kN}$ $\qquad$ $M_1 = 0$ ◁

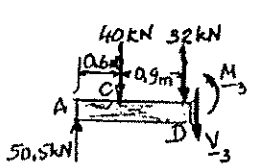
Just to the right of *C*:



$+\uparrow \Sigma F_y = 0$: $50.5\text{ kN} - 40\text{ kN} - V_2 = 0$ $\qquad$ $V_2 = +10.5\text{ kN}$ ◁

$+\circlearrowleft \Sigma M_2 = 0$: $M_2 - (50.5\text{ kN})(0.6\text{ m}) = 0$ $\qquad$ $M_2 = +30.3\text{ kN}\cdot\text{m}$ ◁

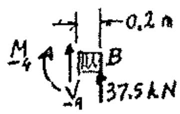
Just to the right of *D*:



$+\uparrow \Sigma F_y = 0$: $50.5 - 40 - 32 - V_3 = 0$ $\qquad$ $V_3 = -21.5\text{ kN}$ ◁

$+\circlearrowleft \Sigma M_3 = 0$: $M_3 - (50.5)(1.5) + (40)(0.9) = 0$ $\qquad$ $M_3 = +39.8\text{ kN}\cdot\text{m}$ ◁

Just to the right of E:

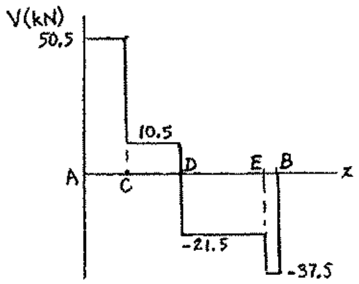


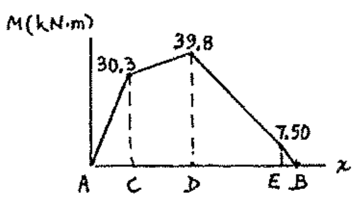
$+\uparrow \Sigma F_y = 0\colon \quad V_4 + 37.5 = 0$ $\qquad V_4 = -37.5 \text{ kN}$ ◁

$+\circlearrowleft \Sigma M_4 = 0\colon \quad -M_4 + (37.5)(0.2) = 0$ $\qquad M_4 = +7.50 \text{ kN} \cdot \text{m}$ ◁

At B: $\qquad V_B = M_B = 0$ ◁

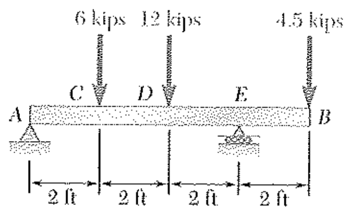
(*b*) $\qquad |V|_{\max} = 50.5 \text{ kN}$ ◀





$|M|_{\max} = 39.8 \text{ kN} \cdot \text{m}$ ◀

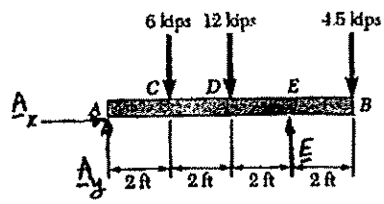
PROBLEM 7.37



For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



$+\circlearrowleft \Sigma M_A = 0$: $E(6 \text{ ft}) - (6 \text{ kips})(2 \text{ ft}) - (12 \text{ kips})(4 \text{ ft}) - (4.5 \text{ kips})(8 \text{ ft}) = 0$

$E = +16 \text{ kips}$ $\quad\quad$ $\mathbf{E} = 16 \text{ kips} \uparrow$ ◁

$\xrightarrow{+} \Sigma F_x = 0$: $A_x = 0$

$+\uparrow \Sigma F_y = 0$: $A_y + 16 \text{ kips} - 6 \text{ kips} - 12 \text{ kips} - 4.5 \text{ kips} = 0$

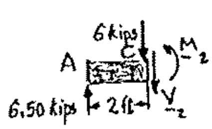
$A_y = +6.50 \text{ kips}$ $\quad\quad$ $\mathbf{A} = 6.50 \text{ kips} \uparrow$ ◁

(*a*) Shear and bending moment

Just to the right of *A*:

$V_1 = +6.50 \text{ kips} \quad M_1 = 0$ ◁

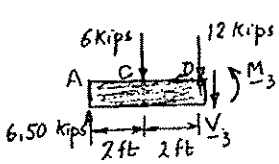
Just to the right of *C*:



$+\uparrow \Sigma F_y = 0$: $6.50 \text{ kips} - 6 \text{ kips} - V_2 = 0$ $\quad\quad$ $V_2 = +0.50 \text{ kips}$ ◁

$+\circlearrowleft \Sigma M_2 = 0$: $M_2 - (6.50 \text{ kips})(2 \text{ ft}) = 0$ $\quad\quad$ $M_2 = +13 \text{ kip}\cdot\text{ft}$ ◁

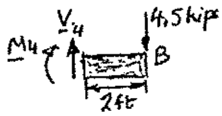
Just to the right of *D*:



$+\uparrow \Sigma F_y = 0$: $6.50 - 6 - 12 - V_3 = 0$ $\quad\quad$ $V_3 = +11.5 \text{ kips}$ ◁

$+\circlearrowleft \Sigma M_3 = 0$: $M_3 - (6.50)(4) - (6)(2) = 0$ $\quad\quad$ $M_3 = +14 \text{ kip}\cdot\text{ft}$ ◁

Just to the right of E:

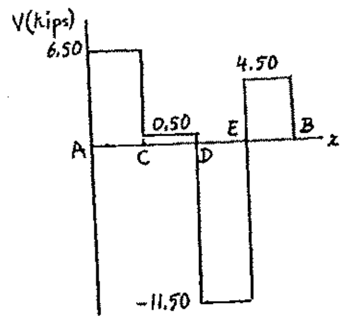


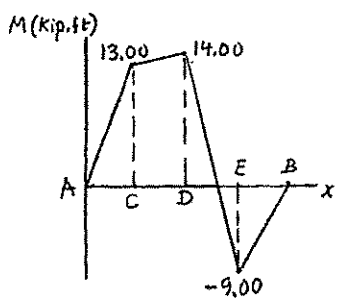
$+\uparrow \Sigma F_y = 0$: $V_4 - 4.5 = 0$ $\qquad V_4 = +4.5 \text{ kips}$ ◁

$+\circlearrowleft \Sigma M_4 = 0$: $-M_4 - (4.5)2 = 0$ $\qquad M_4 = -9 \text{ kip} \cdot \text{ft}$ ◁

At B: $\qquad V_B = M_B = 0$ ◁

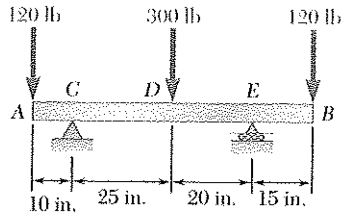
(*b*) $\qquad |V|_{max} = 11.50 \text{ kips}$ ◀





$|M|_{max} = 14.00 \text{ kip} \cdot \text{ft}$ ◀

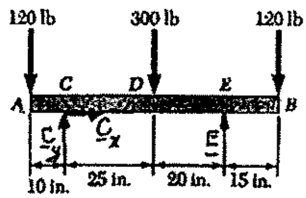
PROBLEM 7.38



For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



$$+\circlearrowleft \Sigma M_C = 0: \quad (120 \text{ lb})(10 \text{ in.}) - (300 \text{ lb})(25 \text{ in.}) + E(45 \text{ in.}) - (120 \text{ lb})(60 \text{ in.}) = 0$$

$E = +300$ lb $\qquad$ $\mathbf{E} = 300 \text{ lb}\uparrow$ ◁

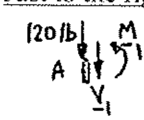
$\Sigma F_x = 0: \quad C_x = 0$

$+\uparrow \Sigma F_y = 0: \quad C_y + 300 \text{ lb} - 120 \text{ lb} - 300 \text{ lb} - 120 \text{ lb} = 0$

$C_y = +240$ lb $\qquad$ $\mathbf{C} = 240 \text{ lb}\uparrow$ ◁

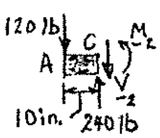
(*a*) Shear and bending moment

Just to the right of *A*:



$+\uparrow \Sigma F_y = 0: \quad -120 \text{ lb} - V_1 = 0$ $\qquad$ $V_1 = -120 \text{ lb}, \quad M_1 = 0$ ◁

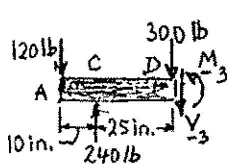
Just to the right of *C*:



$+\uparrow \Sigma F_y = 0: \quad 240 \text{ lb} - 120 \text{ lb} - V_2 = 0$ $\qquad$ $V_2 = +120 \text{ lb}$ ◁

$+\circlearrowleft \Sigma M_C = 0: \quad M_2 + (120 \text{ lb})(10 \text{ in.}) = 0$ $\qquad$ $M_2 = -1200 \text{ lb} \cdot \text{in.}$ ◁

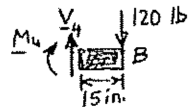
Just to the right of *D*:



$+\uparrow \Sigma F_y = 0: \quad 240 - 120 - 300 - V_3 = 0$ $\qquad$ $V_3 = -180 \text{ lb}$ ◁

$+\circlearrowleft \Sigma M_3 = 0: \quad M_3 + (120)(35) - (240)(25) = 0,$ $\qquad$ $M_3 = +1800 \text{ lb} \cdot \text{in.}$ ◁

Just to the right of E:

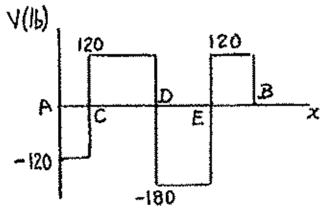


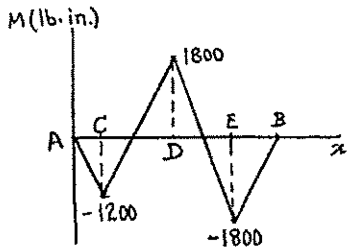
$+\Sigma F_y = 0$: $V_4 - 120 \text{ lb} = 0$ $\qquad V_4 = +120 \text{ lb}$ ◁

$+\circlearrowleft \Sigma M_4 = 0$: $-M_4 - (120 \text{ lb})(15 \text{ in.}) = 0$ $\qquad M_4 = -1800 \text{ lb}\cdot\text{in.}$ ◁

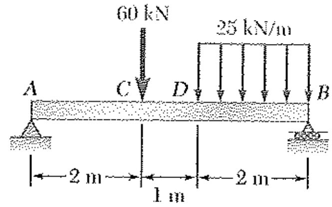
At B: $\qquad V_B = M_B = 0$ ◁

(*b*) $\qquad |V|_{max} = 180.0 \text{ lb}$ ◀





$|M|_{max} = 1800 \text{ lb}\cdot\text{in.}$ ◀

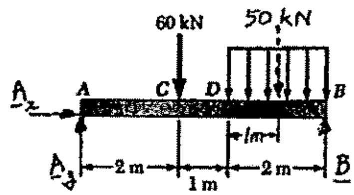


PROBLEM 7.39

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



$+\circlearrowleft \Sigma M_A = 0: \quad B(5\text{ m}) - (60\text{ kN})(2\text{ m}) - (50\text{ kN})(4\text{ m}) = 0$

$B = +64.0\text{ kN}$ $\qquad$ $\mathbf{B} = 64.0\text{ kN}\uparrow$ ◁

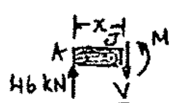
$\Sigma F_x = 0: \quad A_x = 0$

$+\uparrow \Sigma F_y = 0: \quad A_y + 64.0\text{ kN} - 6.0\text{ kN} - 50\text{ kN} = 0$

$A_y = +46.0\text{ kN}$ $\qquad$ $\mathbf{A} = 46.0\text{ kN}\uparrow$ ◁

(*a*) Shear and bending-moment diagrams.

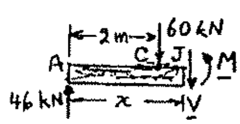
From *A* to *C*:



$+\uparrow \Sigma F_y = 0: \quad 46 - V = 0$ $\qquad$ $V = +46\text{ kN}$ ◁

$+\circlearrowleft \Sigma M_y = 0: \quad M - 46x = 0$ $\qquad$ $M = (46x)\text{kN}\cdot\text{m}$ ◁

From *C* to *D*:



$+\uparrow \Sigma F_y = 0: \quad 46 - 60 - V = 0$ $\qquad$ $V = -14\text{ kN}$ ◁

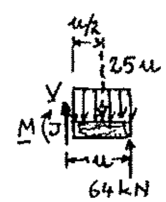
$+\circlearrowleft \Sigma M_j = 0: \quad M - 46x + 60(x-2) = 0$

$M = (120 - 14x)\text{kN}\cdot\text{m}$

For $\quad x = 2\text{ m}: \quad M_C = +92.0\text{ kN}\cdot\text{m}$ ◁

For $\quad x = 3\text{ m}: \quad M_D = +78.0\text{ kN}\cdot\text{m}$ ◁

From D to B:



$$+\uparrow \Sigma F_y = 0: \quad V + 64 - 25\mu = 0$$

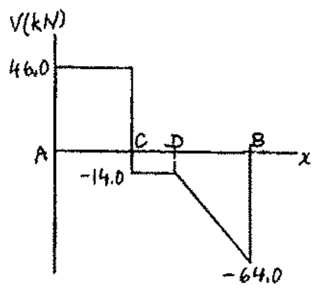
$$V = (25\mu - 64)\text{kN}$$

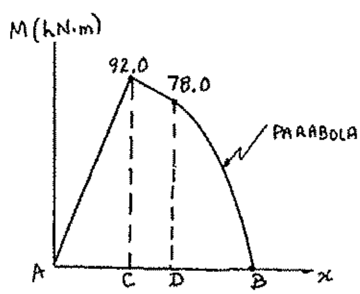
$$+\circlearrowleft \Sigma M_j = 0: \quad 64\mu - (25\mu)\left(\frac{\mu}{2}\right) - M = 0$$

$$M = (64\mu - 12.5\mu^2)\text{kN} \cdot \text{m}$$

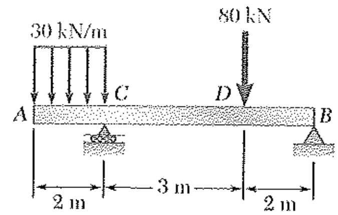
For $\mu = 0$: $V_B = -64$ kN $\qquad M_B = 0$ ◁

(*b*) $|V|_{max} = 64.0$ kN ◀





$|M|_{max} = 92.0$ kN · m ◀

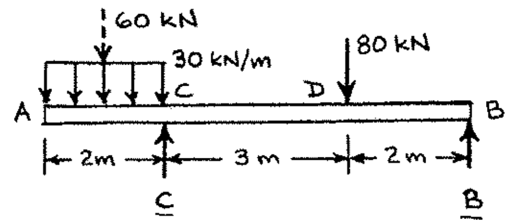


PROBLEM 7.40

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



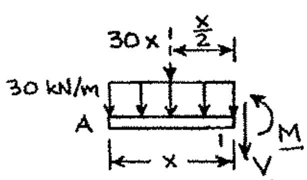
$$+\circlearrowleft \Sigma M_B = 0: \quad (60 \text{ kN})(6 \text{ m}) - C(5 \text{ m}) + (80 \text{ kN})(2 \text{ m}) = 0$$

$C = +104 \text{ kN}$ $\qquad$ $\mathbf{C} = 104 \text{ kN} \uparrow$

$+\uparrow \Sigma F_y = 0: \quad 104 - 60 - 80 + B = 0$ $\qquad$ $\mathbf{B} = 36 \text{ kN} \uparrow$

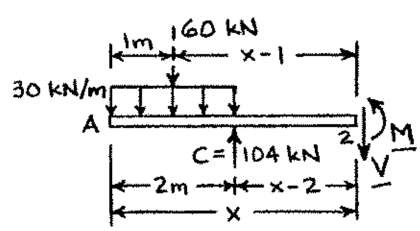
From A to C:

$+\uparrow \Sigma F_y = 0: \quad -30x - V = 0$ $\qquad$ $V = -30x$



$$+\circlearrowleft \Sigma M_1 = 0: \quad (30x)\left(\frac{x}{2}\right) + M = 0$$ $\qquad$ $M = -15x^2$

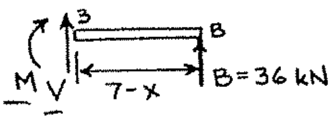
From C to D:



$+\uparrow \Sigma F_y = 0: \quad 104 - 60 - V = 0$ $\qquad$ $V = +44 \text{ kN}$

$+\circlearrowleft \Sigma M_2 = 0: \quad (60)(x-1) - (104)(x-2) + M = 0$ $\qquad$ $M = 44x - 148$

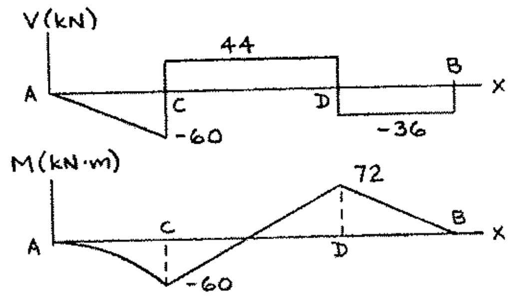
From D to B:



$+\uparrow \Sigma F_y = 0$: $V = -36$ kN

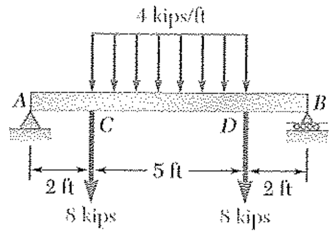
$+\circlearrowleft \Sigma M_3 = 0$: $(36)(7-x) - M = 0$

$$M = -36x + 252$$



(*b*) $|V|_{max} = 60.0$ kN $|M|_{max} = 72.0$ kN · m. ◄

PROBLEM 7.41

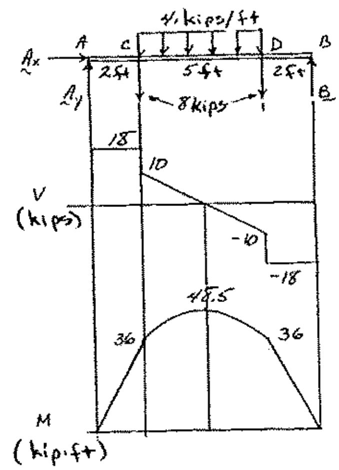


For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

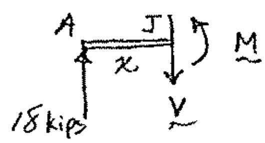
SOLUTION

(*a*) By symmetry:

$$A_y = B = 8 \text{ kips} + \frac{1}{2}(4 \text{ kips})(5 \text{ ft}) \quad \mathbf{A}_y = \mathbf{B} = 18 \text{ kips}\uparrow$$



Along AC:

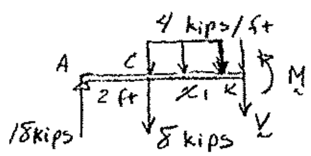


$$+\uparrow\Sigma F_y = 0: \quad 18 \text{ kips} - V = 0 \quad V = 18 \text{ kips}$$

$$\circlearrowleft \Sigma M_J = 0: \quad M - x(18 \text{ kips}) \quad M = (18 \text{ kips})x$$

$$M = 36 \text{ kip}\cdot\text{ft at } C(x = 2 \text{ ft})$$

Along CD:



$$+\uparrow\Sigma F_y = 0: \quad 18 \text{ kips} - 8 \text{ kips} - (4 \text{ kips/ft})x_1 - V = 0$$

$$V = 10 \text{ kips} - (4 \text{ kips/ft})x_1$$

$$V = 0 \text{ at } x_1 = 2.5 \text{ ft (at center)}$$

$$\circlearrowleft \Sigma M_K = 0: \quad M + \frac{x_1}{2}(4 \text{ kips/ft})x_1 + (8 \text{ kips})x_1 - (2 \text{ ft} + x_1)(18 \text{ kips}) = 0$$

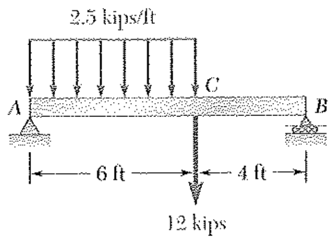
$$M = 36 \text{ kip}\cdot\text{ft} + (10 \text{ kips/ft})x_1 - (2 \text{ kips/ft})x_1^2$$

$$M = 48.5 \text{ kip}\cdot\text{ft} \text{ at } x_1 = 2.5 \text{ ft}$$

Complete diagram by symmetry

(*b*) From diagrams: $|V|_{max} = 18.00$ kips on *AC* and *DB* ◀

$|M|_{max} = 48.5$ kip · ft at center ◀

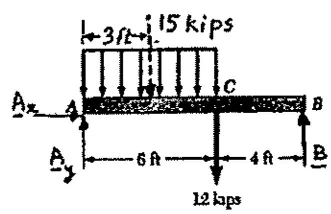


PROBLEM 7.42

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



$$+\circlearrowleft \Sigma M_A = 0: \quad B(10\text{ ft}) - (15\text{ kips})(3\text{ ft}) - (12\text{ kips})(6\text{ ft}) = 0$$

$$B = +11.70\text{ kips} \qquad \mathbf{B} = 11.70\text{ kips}\uparrow \triangleleft$$

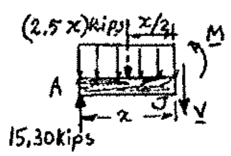
$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 15 - 12 + 11.70 = 0$$

$$A_y = +15.30\text{ kips} \qquad \mathbf{A} = 15.30\text{ kips}\uparrow \triangleleft$$

(*a*) Shear and bending-moment diagrams

From *A* to *C*:



$$+\uparrow \Sigma F_y = 0: \quad 15.30 - 2.5x - V = 0$$

$$V = (15.30 - 2.5x)\text{ kips}$$

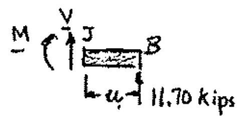
$$+\circlearrowleft \Sigma M_J = 0: \quad M + (2.5x)\left(\frac{x}{2}\right) - 15.30x = 0$$

$$M = 15.30x - 1.25x^2$$

For $x = 0$: $V_A = +15.30$ kips $\qquad M_A = 0 \triangleleft$

For $x = 6$ ft: $V_C = +0.300$ kip $\qquad M_C = +46.8$ kip·ft $\triangleleft$

From C to B:



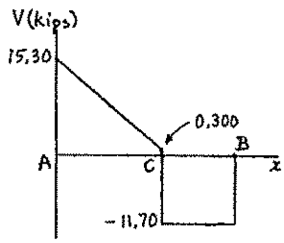
$+\uparrow \Sigma F_y = 0$: $V + 11.70 = 0$ $\quad V = -11.70$ kips ◁

$+\circlearrowleft \Sigma M_J = 0$: $11.70\mu - M = 0$

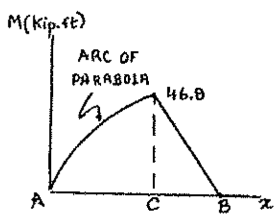
$M = (11.70\mu)$ kip · ft

For $\mu = 4$ ft: $\quad M_C = +46.8$ kip · ft ◁

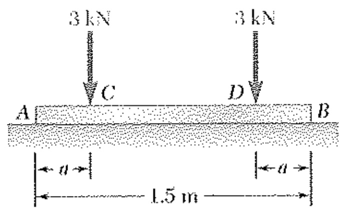
For $\mu = 0$: $\quad M_B = 0$ ◁



(*b*) $\quad |V|_{max} = 15.30$ kips ◀



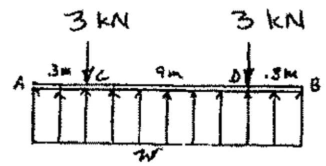
$|M|_{max} = 46.8$ kip · ft ◀



PROBLEM 7.43

Assuming the upward reaction of the ground on beam AB to be uniformly distributed and knowing that $a = 0.3$ m, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

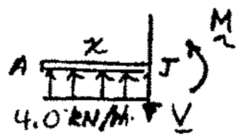
SOLUTION



(a) **FBD Beam:** $+\uparrow \Sigma F_y = 0$: $w(1.5 \text{ m}) - 2(3.0 \text{ kN}) = 0$

$$w = 4.0 \text{ kN/m}$$

Along AC:

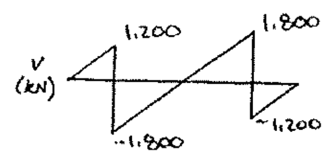


$+\uparrow \Sigma F_y = 0$: $(4.0 \text{ kN/m})x - V = 0$

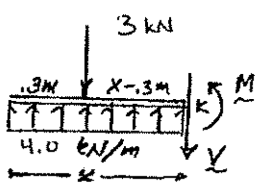
$$V = (4.0 \text{ kN/m})x$$

$\circlearrowleft \Sigma M_J = 0$: $M - \frac{x}{2}(4.0 \text{ kN/m})x = 0$

$$M = (2.0 \text{ kN/m})x^2$$

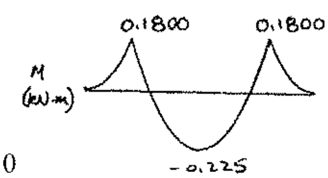


Along CD:



$+\uparrow \Sigma F_y = 0$: $(4.0 \text{ kN/m})x - 3.0 \text{ kN} - V = 0$

$$V = (4.0 \text{ kN/m})x - 3.0 \text{ kN}$$



$\circlearrowleft \Sigma M_K = 0$: $M + (x - 0.3 \text{ m})(3.0 \text{ kN}) - \frac{x}{2}(4.0 \text{ kN/m})x = 0$

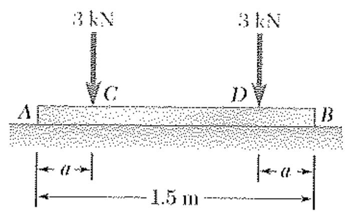
$$M = 0.9 \text{ kN} \cdot \text{m} - (3.0 \text{ kN})x + (2.0 \text{ kN/m})x^2$$

Note: $V = 0$ at $x = 0.75$ m, where $M = -0.225 \text{ kN} \cdot \text{m}$

Complete diagrams using symmetry.

(b) $|V|_{max} = 1.800$ kN at C and D ◀

$|M|_{max} = 0.225 \text{ kN} \cdot \text{m}$ at center ◀



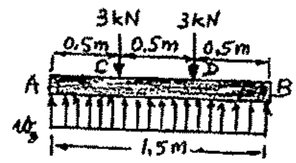
PROBLEM 7.44

Solve Problem 7.43 knowing that $a = 0.5$ m.

PROBLEM 7.43 Assuming the upward reaction of the ground on beam AB to be uniformly distributed and knowing that $a = 0.3$ m, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

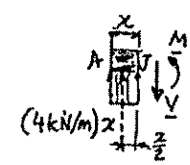
Free body: Entire beam



$$+\uparrow \Sigma F_y = 0: \quad w_g(1.5\text{ m}) - 3\text{ kN} - 3\text{ kN} = 0 \qquad w_g = 4\text{ kN/m} \;\triangleleft$$

(a) Shear and bending moment

From A to C:



$$+\uparrow \Sigma F_y = 0: \quad 4x - V = 0 \quad V = (4x)\text{ kN}$$

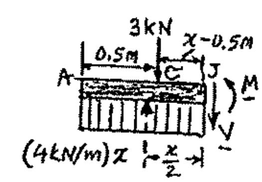
$$+\circlearrowleft \Sigma M_J = 0: \quad M - (4x)\frac{x}{2} = 0, \quad M = (2x^2)\text{ kN}\cdot\text{m}$$

For $x = 0$: $\qquad V_A = M_A = 0 \;\triangleleft$

For $x = 0.5$ m: $\qquad V_C = 2$ kN, $\qquad M_C = 0.500\text{ kN}\cdot\text{m} \;\triangleleft$

From C to D: $\qquad +\uparrow \Sigma F_y = 0: \quad 4x - 3\text{ kN} - V = 0$

$$V = (4x - 3)\text{ kN}$$



$$+\circlearrowleft \Sigma M_J = 0: \quad M + (3\text{ kN})(x - 0.5) - (4x)\frac{x}{2} = 0$$

$$M = (2x^2 - 3x + 1.5)\text{ kN}\cdot\text{m}$$

For $x = 0.5$ m: $\qquad V_C = -1.00$ kN, $\qquad M_C = 0.500\text{ kN}\cdot\text{m} \;\triangleleft$

For $x = 0.75$ m: $\qquad V_C = 0$, $\qquad M_C = 0.375\text{ kN}\cdot\text{m} \;\triangleleft$

For $x = 1.0$ m: $\qquad V_C = 1.00$ kN, $\qquad M_C = 0.500\text{ kN}\cdot\text{m} \;\triangleleft$

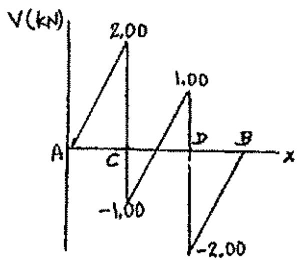
<u>From D to B:</u>

$$+\uparrow \Sigma F_y = 0: \quad V + 4\mu = 0 \quad V = -(4\mu)\text{ kN}$$

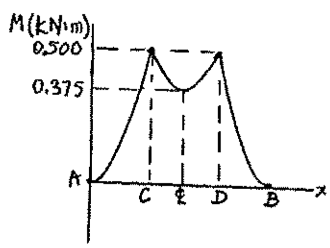
$$+\circlearrowleft \Sigma M_J = 0: \quad (4\mu)\frac{\mu}{2} - M = 0, \quad M = 2\mu^2$$

For $\mu = 0$: $V_B = M_B = 0$ ◁

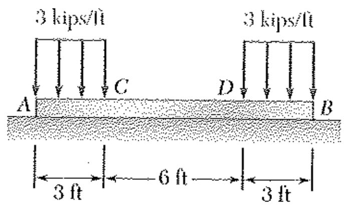
For $\mu = 0.5$ m: $V_D = -2$ kN, $M_D = 0.500$ kN·m ◁



(*b*) $|V|_{max} = 2.00$ kN ◀



$|M|_{max} = 0.500$ kN·m ◀

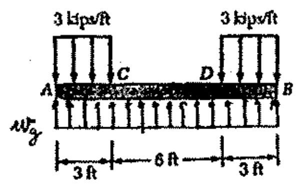


PROBLEM 7.45

Assuming the upward reaction of the ground on beam AB to be uniformly distributed, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

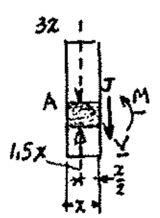
Free body: Entire beam



$+\uparrow \Sigma F_y = 0$: $w_g(12\text{ ft}) - (3\text{ kips/ft})(6\text{ ft}) = 0$ $\qquad w_g = 1.5\text{ kips/ft} \triangleleft$

(a) Shear and bending-moment diagrams.

From A to C:



$+\uparrow \Sigma F_y = 0$: $1.5x - 3x - V = 0$

$V = (-1.5x)$ kips

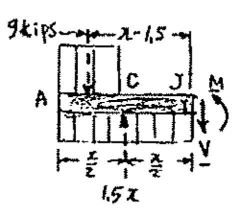
$+\circlearrowleft \Sigma M_J = 0$: $M + (3x)\dfrac{x}{2} - (1.5x)\dfrac{x}{2} = 0$

$M = (-0.75x^2)$ kip · ft

For $x = 0$: $\qquad V_A = M_A = 0 \triangleleft$

For $x = 3$ ft: $\qquad V_C = -4.5$ kips $\quad M_C = -6.75$ kip · ft $\triangleleft$

From C to D:



$+\uparrow \Sigma F_y = 0$: $1.5x = 9 - V = 0$, $\quad V = (1.5x - 9)$ kips

$+\circlearrowleft \Sigma M_J = 0$: $M + 9(x - 1.5) - (1.5x)\dfrac{x}{2} = 0$

$M = 0.75x^2 - 9x + 13.5$

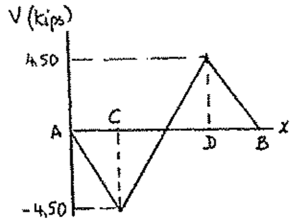
For $x = 3$ ft: $\qquad V_C = -4.5$ kips, $\qquad M_C = -6.75$ kip · ft $\triangleleft$

For $x = 6$ ft: $\qquad V_C = 0$, $\qquad M_C = -13.50$ kip · ft $\triangleleft$

PROBLEM 7.45 (Continued)

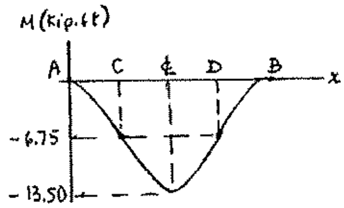
For $x = 9$ ft: $V_D = 4.5$ kips, $M_D = -6.75$ kip · ft ◁

At B: $V_B = M_B = 0$ ◁



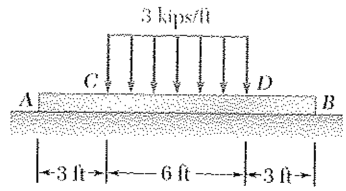
(*b*) $|V|_{\max} = 4.50$ kips ◀

Bending moment diagram consists of three distinct arcs of parabola.



$|M|_{\max} = 13.50$ kip · ft ◀

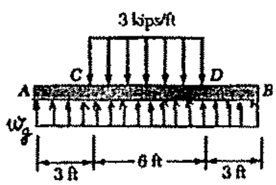
Since entire diagram is below the x axis: $M \leq 0$ everywhere ◀



PROBLEM 7.46

Assuming the upward reaction of the ground on beam AB to be uniformly distributed, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

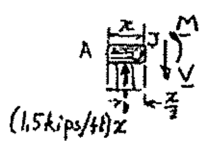
SOLUTION



Free body: Entire beam

$$+\uparrow \Sigma F_y = 0: \quad w_g(12\text{ ft}) - (3\text{ kips/ft})(6\text{ ft}) = 0$$

$$w_g = 1.5\text{ kips/ft} \quad \triangleleft$$



(*a*) Shear and bending-moment diagrams from A to C:

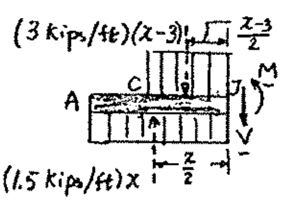
$$+\uparrow \Sigma F_y = 0: \quad 1.5x - V = 0 \qquad V = (1.5x)\text{kips}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M - (1.5x)\frac{x}{2} \qquad M = (0.75x^2)\text{kip}\cdot\text{ft}$$

For $x = 0$: $\quad V_A = M_A = 0 \quad \triangleleft$

For $x = 3$ ft: $V_C = 4.5$ kips, $\quad M_C = 6.75\text{ kip}\cdot\text{ft} \quad \triangleleft$

From C to D:



$$+\uparrow \Sigma F_y = 0: \quad 1.5x - 3(x-3) - V = 0$$

$$V = (9 - 1.5x)\text{kips}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M + 3(x-3)\frac{x-3}{2} - (1.5x)\frac{x}{2} = 0$$

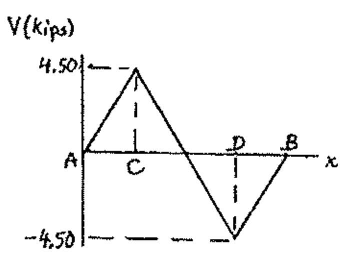
$$M = [0.75x^2 - 1.5(x-3)^2]\text{kip}\cdot\text{ft}$$

For $x = 3$ ft: $V_C = 4.5$ kips, $\quad M_C = 6.75\text{ kip}\cdot\text{ft} \quad \triangleleft$

For $x = 6$ ft: $V_{℄} = 0$, $\quad M_{℄} = 13.50\text{ kip}\cdot\text{ft} \quad \triangleleft$

For $x = 9$ ft: $V_D = -4.5$ kips, $\quad M_D = 6.75\text{ kip}\cdot\text{ft} \quad \triangleleft$

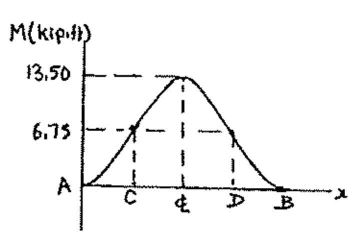
At B: $\quad V_B = M_B = 0 \quad \triangleleft$



(*b*)

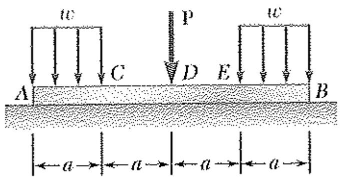
$$|V|_{max} = 4.50\text{ kips} \quad \blacktriangleleft$$

$$|M|_{max} = 13.50\text{ kip}\cdot\text{ft} \quad \blacktriangleleft$$



Bending-moment diagram consists or three distinct arcs of parabola, all located above the x axis.

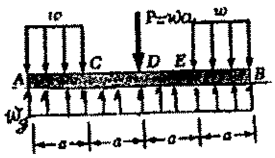
Thus: $M \geq 0$ everywhere $\blacktriangleleft$



PROBLEM 7.47

Assuming the upward reaction of the ground on beam AB to be uniformly distributed and knowing that $P = wa$, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION



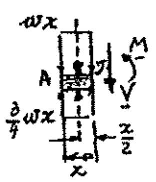
Free body: Entire beam

$$+\uparrow \Sigma F_y = 0: \quad w_g(4a) - 2wa - wa = 0$$

$$w_g = \frac{3}{4}w \ \triangleleft$$

(a) Shear and bending-moment diagrams

From A to C:



$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{4}wx - wx - V = 0$$

$$V = -\frac{1}{4}wx$$

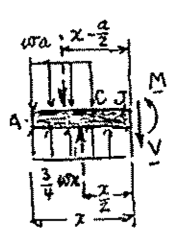
$$+\circlearrowleft \Sigma M_J = 0: \quad M + (wx)\frac{x}{2} - \left(\frac{3}{4}wx\right)\frac{x}{2} = 0$$

$$M = -\frac{1}{8}wx^2$$

For $x = 0$: $\qquad V_A = M_A = 0 \ \triangleleft$

For $x = a$: $\qquad V_C = -\frac{1}{4}wa \qquad M_C = -\frac{1}{8}wa^2 \ \triangleleft$

From C to D:



$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{4}wx - wa - V = 0$$

$$V = \left(\frac{3}{4}x - a\right)w$$

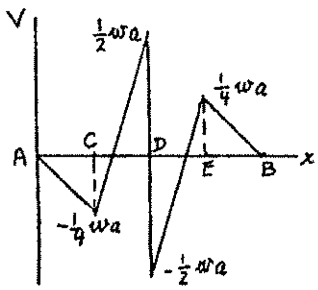
$$+\circlearrowleft \Sigma M_J = 0: \quad M + wa\left(x - \frac{a}{2}\right) - \frac{3}{4}wx\left(\frac{x}{2}\right) = 0$$

$$M = \frac{3}{8}wx^2 - wa\left(x - \frac{a}{2}\right) \qquad (1)$$

For $x = a$: $V_C = -\frac{1}{4}wa$ $M_C = -\frac{1}{8}wa^2$ ◁

For $x = 2a$: $V_D = +\frac{1}{2}wa$ $M_D = 0$ ◁

Because of the symmetry of the loading, we can deduce the values of V and M for the right-hand half of the beam from the values obtained for its left-hand half.

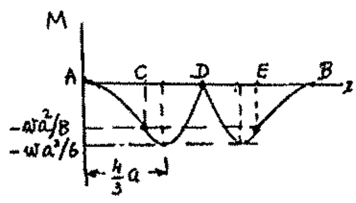


(*b*) $|V|_{\max} = \frac{1}{2}wa$ ◀

To find $|M|_{\max}$, we differentiate Eq. (1) and set $\frac{dM}{dx} = 0$:

$$\frac{dM}{dx} = \frac{3}{4}wx - wa = 0, \quad x = \frac{4}{3}a$$

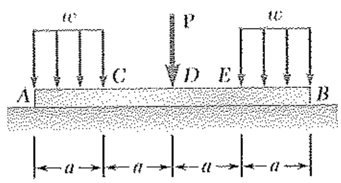
$$M = \frac{3}{8}w\left(\frac{4}{3}a\right)^2 - wa^2\left(\frac{4}{3} - \frac{1}{2}\right) = -\frac{wa^2}{6}$$



$|M|_{\max} = \frac{1}{6}wa^2$ ◀

Bending-moment diagram consists of four distinct arcs of parabola.

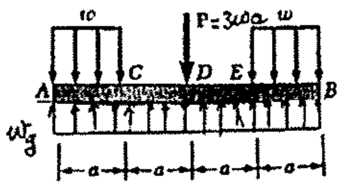
PROBLEM 7.48



Solve Problem 7.47 knowing that $P = 3wa$.

PROBLEM 7.47 Assuming the upward reaction of the ground on beam AB to be uniformly distributed and knowing that $P = wa$, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION



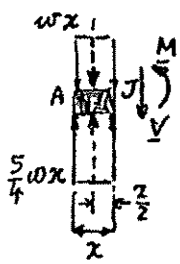
Free body: Entire beam

$$+\uparrow \Sigma F_y = 0: \quad w_g(4a) - 2wa - 3wa = 0$$

$$w_g = \frac{5}{4}w \quad \triangleleft$$

(a) Shear and bending-moment diagrams

From A to C:



$$+\uparrow \Sigma F_y = 0: \quad \frac{5}{4}wx - wx - V = 0$$

$$V = +\frac{1}{4}wx$$

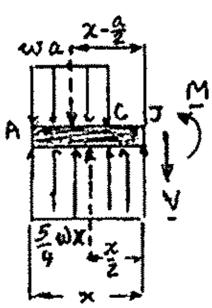
$$+\circlearrowleft \Sigma M_J = 0: \quad M + (wx)\frac{x}{2} - \left(\frac{5}{4}wx\right)\frac{x}{2} = 0$$

$$M = +\frac{1}{8}wx^2$$

For $x = 0$: $\quad V_A = M_A = 0 \quad \triangleleft$

For $x = a$: $\quad V_C = +\frac{1}{4}wa \qquad M_C = +\frac{1}{8}wa^2 \quad \triangleleft$

From C to D:



$$+\uparrow \Sigma F_y = 0: \quad \frac{5}{4}wx - wa - V = 0$$

$$V = \left(\frac{5}{4}x - a\right)w$$

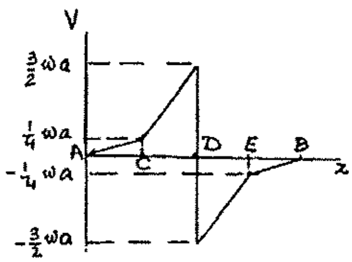
$$+\circlearrowleft \Sigma M_J = 0: \quad M + wa\left(x - \frac{a}{2}\right) - \frac{5}{4}wx\left(\frac{x}{2}\right) = 0$$

$$M = \frac{5}{8}wx^2 - wa\left(x - \frac{a}{2}\right) \qquad (1)$$

For $x = a$: $V_C = +\frac{1}{4}wa,\ \ M_C = +\frac{1}{8}wa^2$ ◁

For $x = 2a$: $V_D = +\frac{3}{2}wa,\ \ M_D = +wa^2$ ◁

Because of the symmetry of the loading, we can deduce the values of V and M for the right-hand half of the beam from the values obtained for its left-hand half.



(*b*) $|V|_{max} = \frac{3}{2}wa$ ◀

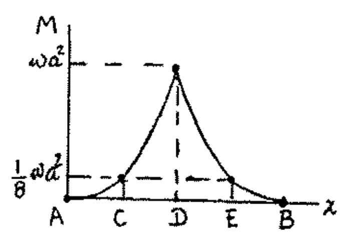
To find $|M|_{max}$, we differentiate Eq. (1) and set $\frac{dM}{dx} = 0$:

$$\frac{dM}{dx} = \frac{5}{4}wx - wa = 0$$

$$x = \frac{4}{5}a < a \quad \text{(outside portion } CD\text{)}$$

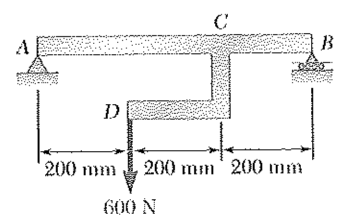
The maximum value of $|M|$ occurs at D:

$$|M|_{max} = wa^2 \ ◀$$



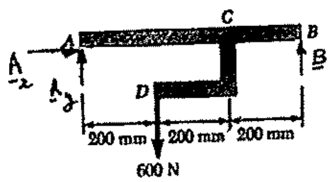
Bending-moment diagram consists of four distinct arcs of parabola.

PROBLEM 7.49



Draw the shear and bending-moment diagrams for the beam AB, and determine the shear and bending moment (a) just to the left of C, (b) just to the right of C.

SOLUTION



Free body: Entire beam

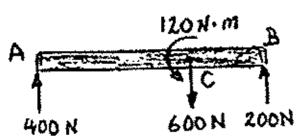
$+\circlearrowleft \Sigma M_A = 0{:}\quad B(0.6\text{ m}) - (600\text{ N})(0.2\text{ m}) = 0$

$B = +200\text{ N}$ $\qquad$ $\mathbf{B} = 200\text{ N}\uparrow$ ◁

$\Sigma F_x = 0{:}\quad A_x = 0$

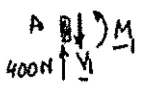
$+\uparrow \Sigma F_y = 0{:}\quad A_y - 600\text{ N} + 200\text{ N} = 0$

$A_y = +400\text{ N}$ $\qquad$ $\mathbf{A} = 400\text{ N}\uparrow$ ◁



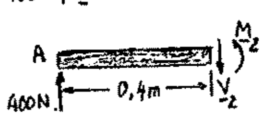
We replace the 600-N load by an equivalent force-couple system at C

Just to the right of A:



$V_1 = +400\text{ N},\quad M_1 = 0$ ◁

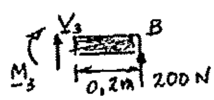
(a) Just to the left of C:



$V_2 = +400\text{ N}$ ◀

$M_2 = (400\text{ N})(0.4\text{ m})$ $\qquad$ $M_2 = +160.0\text{ N}\cdot\text{m}$ ◀

(b) Just to the right of C:

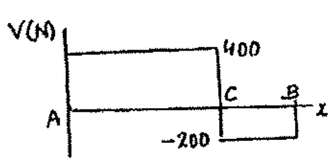


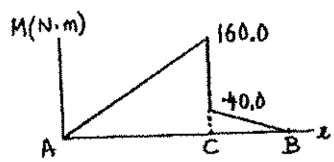
$V_3 = -200\text{ N}$ ◀

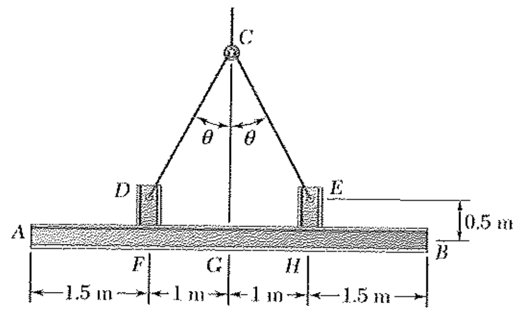
$M_3 = (200\text{ N})(0.2\text{ m})$ $\qquad$ $M_3 = +40.0\text{ N}\cdot\text{m}$ ◀

Just to the left of B:

$V_4 = -200\text{ N},\quad M_4 = 0$ ◁





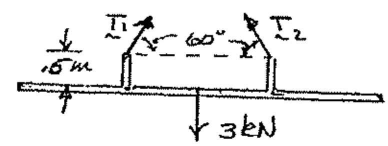


PROBLEM 7.50

Two small channel sections DF and EH have been welded to the uniform beam AB of weight $W = 3$ kN to form the rigid structural member shown. This member is being lifted by two cables attached at D and E. Knowing that $\theta = 30°$ and neglecting the weight of the channel sections, (a) draw the shear and bending-moment diagrams for beam AB, (b) determine the maximum absolute values of the shear and bending moment in the beam.

SOLUTION

FBD Beam + channels:



(a) By symmetry: $T_1 = T_2 = T$

$$+\uparrow \Sigma F_y = 0: \quad 2T \sin 60° - 3 \text{ kN} = 0$$

$$T = \frac{3}{\sqrt{3}} \text{ kN}$$

$$T_{1x} = \frac{3}{2\sqrt{3}}$$

$$T_{1y} = \frac{3}{2} \text{ kN}$$

FBD Beam:

$$M = (0.5 \text{ m})\frac{3}{2\sqrt{3}} \text{ kN}$$

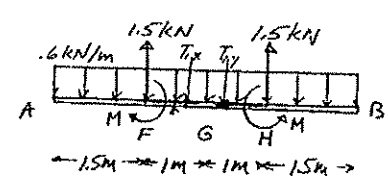
$$= 0.433 \text{ kN} \cdot \text{m}$$

With cable force replaced by equivalent force-couple system at F and G

Shear Diagram: V is piecewise linear

$$\left(\frac{dV}{dx} = -0.6 \text{ kN/m}\right) \text{with } 1.5 \text{ kN}$$

discontinuities at F and H.



$$V_{F^-} = -(0.6 \text{ kN/m})(1.5 \text{ m}) = 0.9 \text{ kN}$$

V increases by 1.5 kN to $+0.6$ kN at F^+

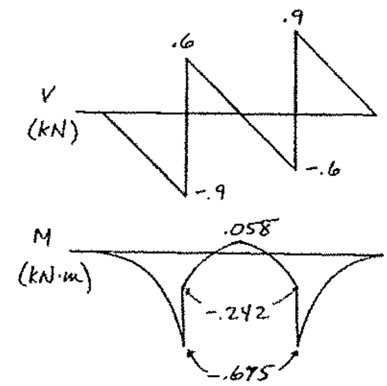
$$V_G = 0.6 \text{ kN} - (0.6 \text{ kN/m})(1 \text{ m}) = 0$$

Finish by invoking symmetry

Moment diagram: M is piecewise parabolic

$$\left(\frac{dM}{dx} \text{ decreasing with } V\right)$$

with discontinuities of .433 kN at F and H.



$$M_{F^-} = -\frac{1}{2}(0.9\text{ kN})(1.5\text{ m})$$
$$= -0.675\text{ kN}\cdot\text{m}$$

M increases by 0.433 kN m to -0.242 kN · m at F^+

$$M_G = -0.242\text{ kN}\cdot\text{m} + \frac{1}{2}(0.6\text{ kN})(1\text{ m})$$
$$= 0.058\text{ kN}\cdot\text{m}$$

Finish by invoking symmetry

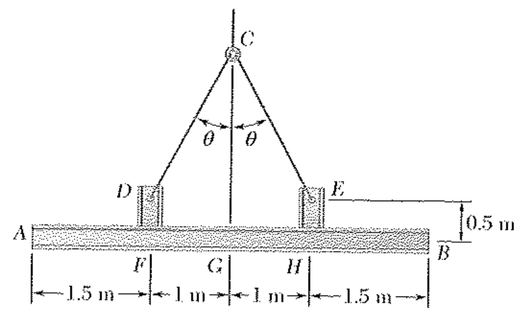
(*b*)

$|V|_{max} = 900\text{ N}$ ◀

at F^- and G^+

$|M|_{max} = 675\text{ N}\cdot\text{m}$ ◀

at F and G



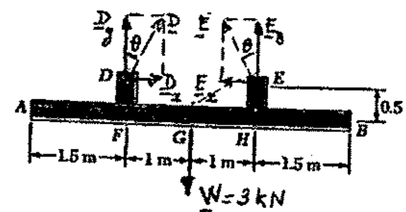
PROBLEM 7.51

Solve Problem 7.50 when $\theta = 60°$.

PROBLEM 7.50 Two small channel sections DF and EH have been welded to the uniform beam AB of weight $W = 3$ kN to form the rigid structural member shown. This member is being lifted by two cables attached at D and E. Knowing that $\theta = 30°$ and neglecting the weight of the channel sections, (a) draw the shear and bending-moment diagrams for beam AB, (b) determine the maximum absolute values of the shear and bending moment in the beam.

SOLUTION

Free body: Beam and channels



From symmetry:

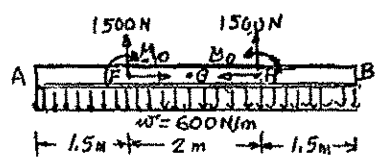
$$E_y = D_y$$

Thus: $$E_x = D_x = D_y \tan\theta \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad D_y + E_y - 3\text{ kN} = 0 \qquad \mathbf{D}_y = \mathbf{E}_y = 1.5\text{ kN}\uparrow \triangleleft$$

From (1): $$\mathbf{D}_x = (1.5\text{ kN})\tan\theta \longrightarrow \qquad \mathbf{E} = (1.5\text{ kN})\tan\theta \longleftarrow \triangleleft$$

We replace the forces at D and E by equivalent force-couple systems at F and H, where



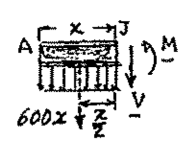
$$M_0 = (1.5\text{ kN}\tan\theta)(0.5\text{ m}) = (750\text{ N}\cdot\text{m})\tan\theta \qquad (2)$$

We note that the weight of the beam per unit length is

$$w = \frac{W}{L} = \frac{3\text{ kN}}{5\text{ m}} = 0.6\text{ kN/m} = 600\text{ N/m}$$

(a) Shear and bending moment diagrams

From A to F:



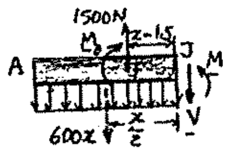
$$+\uparrow \Sigma F_y = 0: \quad -V - 600x = 0 \quad V = (-600x)\text{N}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M + (600x)\frac{x}{2} = 0, \quad M = (-300x^2)\text{N}\cdot\text{m}$$

For $x = 0$: $$V_A = M_A = 0 \triangleleft$$

For $x = 1.5$ m: $$V_F = -900\text{ N}, \quad M_F = -675\text{ N}\cdot\text{m} \triangleleft$$

From F to H:



$$+\uparrow \Sigma F_y = 0: \quad 1500 - 600x - V = 0$$

$$V = (1500 - 600x)\text{N}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M + (600x)\frac{x}{2} - 1500(x - 1.5) - M_0 = 0$$

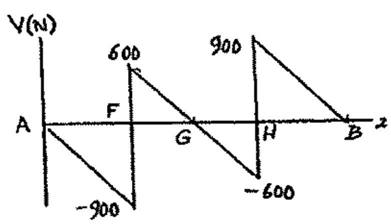
$$M = M_0 - 300x^2 + 1500(x - 1.5)\text{N}\cdot\text{m}$$

For $x = 1.5$ m: $V_F = +600$ N, $M_F = (M_0 - 675)$ N·m ◁

For $x = 2.5$ m: $V_G = 0$, $M_G = (M_0 - 375)$ N·m ◁

From G To B, The V and M diagrams will be obtained by symmetry,

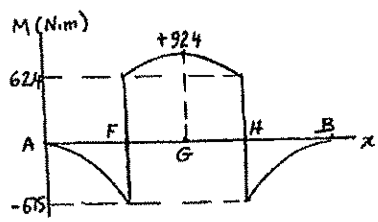
(*b*) $|V|_{max} = 900$ N ◀



Making $\theta = 60°$ in Eq. (2): $M_0 = 750 \tan 60° = 1299$ N·m

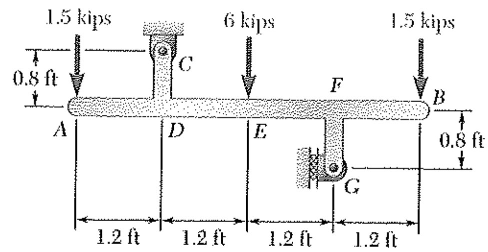
Thus, just to the right of F: $M = 1299 - 675 = 624$ N·m ◁

and $M_G = 1299 - 375 = 924$ N·m ◁



(*b*) $|V|_{max} = 900$ N ◀

$|M|_{max} = 924$ N·m ◀

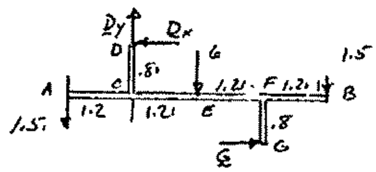


PROBLEM 7.52

Draw the shear and bending-moment diagrams for the beam *AB*, and determine the maximum absolute values of the shear and bending moment.

SOLUTION

FBD whole:



$$\circlearrowleft \Sigma M_D = 0: \quad (1.2 \text{ ft})(1.5 \text{ kips}) - (1.2 \text{ ft})(6 \text{ kips}) - (3.6 \text{ ft})(1.5 \text{ kips}) + (1.6 \text{ ft})G = 0$$

$$\mathbf{G} = 6.75 \text{ kips} \longrightarrow$$

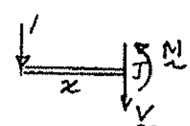
(Dimensions in ft., loads in kips, moments in kips · ft)

$$\longrightarrow \Sigma F_x = 0: \quad -D_x + G = 0 \qquad \mathbf{D}_x = 6.75 \text{ kips} \longleftarrow$$

$$\uparrow \Sigma F_y = 0: \quad D_y - 1.5 \text{ kips} - 6 \text{ kips} - 1.5 \text{ kips} = 0 \qquad \mathbf{D}_y = 9 \text{ kips} \uparrow$$

Beam AB, with forces **D** and **G** replaced by equivalent force/couples at *C* and *F*

Along AD:

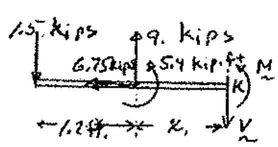


$$\uparrow \Sigma F_y = 0: \quad -1.5 \text{ kips} - V = 0 \quad V = -1.5 \text{ kips}$$

$$\circlearrowleft \Sigma M_J = 0: \quad x(1.5 \text{ kips}) + M = 0 \quad M = -(1.5 \text{ kips})x$$

$$M = -1.8 \text{ kips at } x = 1.2 \text{ ft}$$

Along DE:



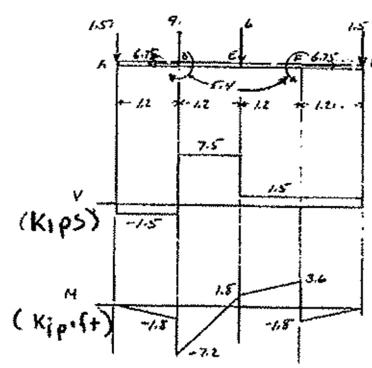
$$\uparrow \Sigma F_y = 0: \quad -1.5 \text{ kips} + 9 \text{ kips} - V = 0$$

$$V = 7.5 \text{ kips}$$

$$\circlearrowleft \Sigma M_K = 0: \quad M + 5.4 \text{ kip}\cdot\text{ft} - x_1(9 \text{ kips}) + (1.2 \text{ ft} + x_1)(1.5 \text{ kips}) = 0$$

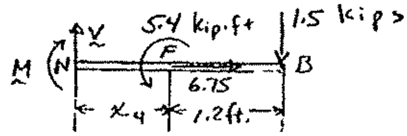
$$M = 7.2 \text{ kip}\cdot\text{ft} + (7.5 \text{ kips})x_1$$

$$M = 1.8 \text{ kip}\cdot\text{ft} \text{ at } x_1 = 1.2 \text{ ft}$$



PROBLEM 7.52 (Continued)

Along EF:

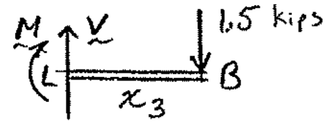


$+\uparrow \Sigma F_y = 0$: $V - 1.5 \text{ kips} = 0 \quad V = 1.5 \text{ kips}$

$+\circlearrowleft \Sigma M_N = 0$: $-M + 5.4 \text{ kip}\cdot\text{ft} - (x_4 + 1.2 \text{ ft})(1.5 \text{ kips})$

$$M = 3.6 \text{ kip}\cdot\text{ft} - (1.5 \text{ kips})x_4$$
$$M = 1.8 \text{ kip}\cdot\text{ft} \text{ at } x_4 = 1.2 \text{ ft}$$
$$M = 3.6 \text{ kip}\cdot\text{ft} \text{ at } x_4 = 0$$

Along FB:



$+\uparrow \Sigma F_y = 0$: $V - 1.5 \text{ kips} = 0$

$$V = 1.5 \text{ kips}$$

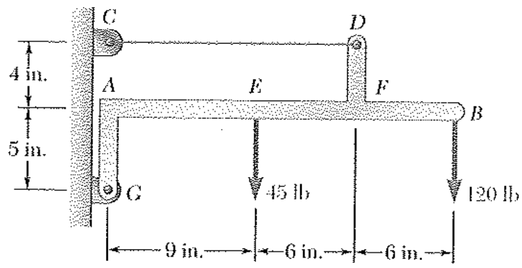
$+\circlearrowleft \Sigma M_L = 0$: $-M - x_3(1.5 \text{ kips}) = 0$

$$M = (-1.5 \text{ kips})x_3$$
$$M = -1.8 \text{ kip}\cdot\text{ft at } x_3 = 1.2 \text{ ft}$$

From diagrams:

$|V|_{max} = 7.50 \text{ kips on } DE$ ◀

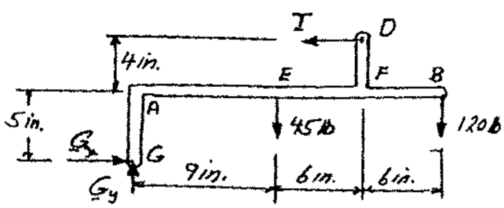
$|M|_{max} = 7.20 \text{ kip}\cdot\text{ft at } D^+$ ◀



PROBLEM 7.53

Draw the shear and bending-moment diagrams for the beam AB, and determine the maximum absolute values of the shear and bending moment.

SOLUTION

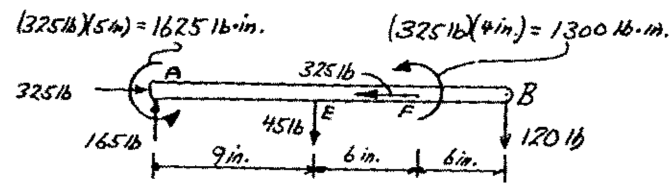


$+\circlearrowleft \Sigma F_G = 0$: $T(9 \text{ in.}) - (45 \text{ lb})(9 \text{ in.}) - (120 \text{ lb})(21 \text{ in.}) = 0$ $\qquad T = 325 \text{ lb}$

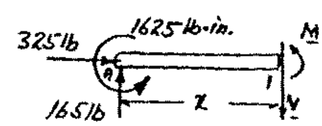
$\overset{+}{\rightarrow} \Sigma F_x = 0$: $-325 \text{ lb} + G_x = 0$ $\qquad \mathbf{G}_x = 325 \text{ lb} \longrightarrow$

$+\uparrow \Sigma F_y = 0$: $G_y - 45 \text{ lb} - 120 \text{ lb} = 0$ $\qquad \mathbf{G}_y = 165 \text{ lb} \uparrow$

Equivalent loading on straight part of beam AB



From A to E:

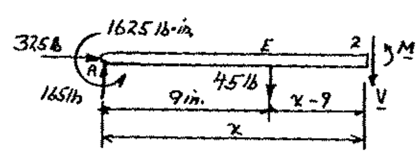


$\Sigma F_y = 0$: $V = +165 \text{ lb}$

$+\circlearrowleft \Sigma M_1 = 0$: $+1625 \text{ lb} \cdot \text{in.} - (165 \text{ lb})x + M = 0$

$$M = -1625 + 165x$$

From E to F:



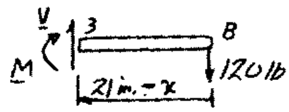
$+\uparrow \Sigma F_y = 0$: $165 - 45 - V = 0$

$$V = +120 \text{ lb}$$

$+\circlearrowleft \Sigma M_2 = 0$: $+1625 \text{ lb} \cdot \text{in.} - (165 \text{ lb})x + (45 \text{ lb})(x - 9) + M = 0$

$$M = -1220 + 120x$$

PROBLEM 7.53 (Continued)

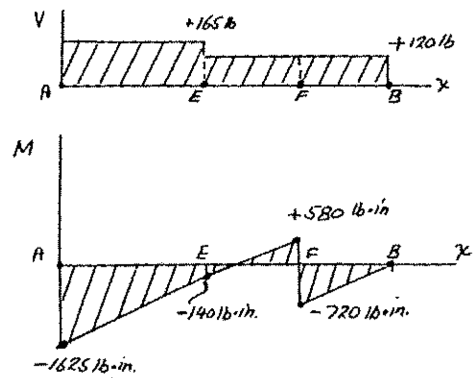


From F to B:

$$\Sigma F_y = 0 \quad V = +120 \text{ lb}$$

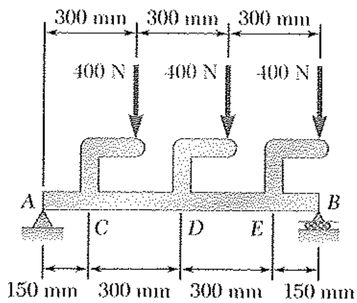
$$+\circlearrowleft \Sigma M_3 = 0: \quad -(120)(21 - x) - M = 0$$

$$M = -2520 + 120x$$



$$|V|_{\max} = 165.0 \text{ lb} \blacktriangleleft$$

$$|M|_{\max} = 1625 \text{ lb} \cdot \text{in.} \blacktriangleleft$$

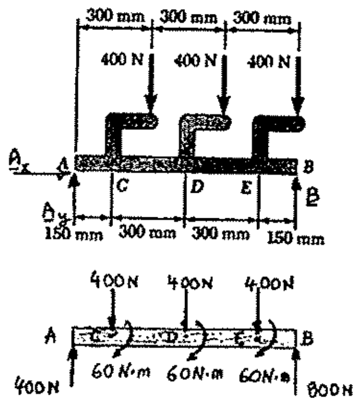


PROBLEM 7.54

Draw the shear and bending-moment diagrams for the beam AB, and determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



$+\circlearrowleft \Sigma M_A = 0$: $B(0.9\text{ m}) - (400\text{ N})(0.3\text{ m}) - (400\text{ N})(0.6\text{ m}) - (400\text{ N})(0.9\text{ m}) = 0$

$B = +800\text{ N}$ $\qquad$ $\mathbf{B} = 800\text{ N}\uparrow$ ◁

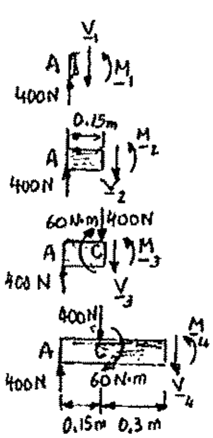
$\Sigma F_x = 0$: $A_x = 0$

$+\uparrow \Sigma F_y = 0$: $A_y + 800\text{ N} - 3(400\text{ N}) = 0$

$A_y = +400\text{ N}$ $\qquad$ $\mathbf{A} = 400\text{ N}\uparrow$ ◁

We replace the loads by equivalent force-couple systems at C, D, and E.

We consider successively the following F-B diagrams.



$V_1 = +400\text{ N}$ $\qquad$ $M_1 = 0$

$V_2 = +400\text{ N}$ $\qquad$ $M_2 = +60\text{ N}\cdot\text{m}$

$V_3 = 0$ $\qquad$ $M_3 = +120\text{ N}\cdot\text{m}$

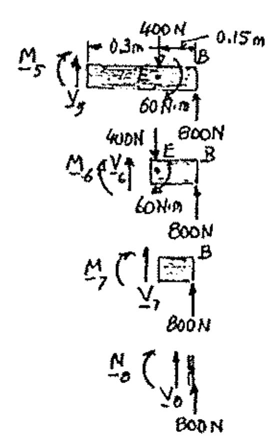
$V_4 = 0$ $\qquad$ $M_4 = +120\text{ N}\cdot\text{m}$

$V_5 = -400\text{ N}$ $\qquad$ $M_5 = +180\text{ N}\cdot\text{m}$

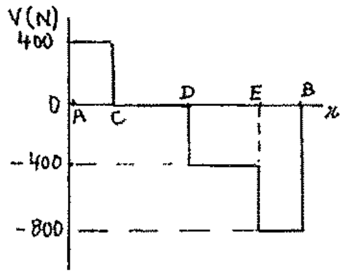
$V_6 = -400\text{ N}$ $\qquad$ $M_6 = +60\text{ N}\cdot\text{m}$

$V_7 = -800\text{ N}$ $\qquad$ $M_7 = +120\text{ N}\cdot\text{m}$

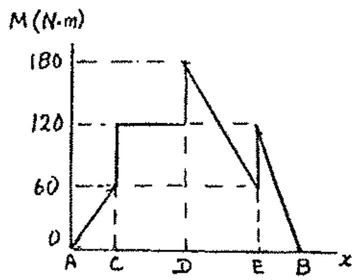
$V_8 = -800\text{ N}$ $\qquad$ $M_8 = 0$



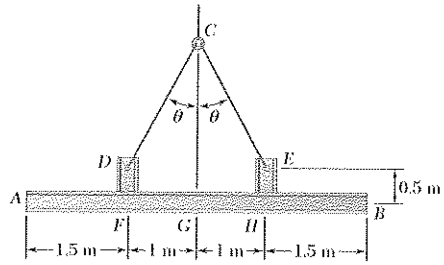
PROBLEM 7.54 (Continued)



(*b*) $|V|_{max} = 800 \text{ N}$ ◀



$|M|_{max} = 180.0 \text{ N} \cdot \text{m}$ ◀



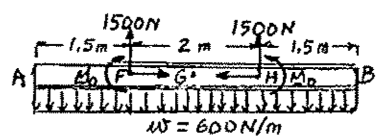
PROBLEM 7.55

For the structural member of Problem 7.50, determine (*a*) the angle θ for which the maximum absolute value of the bending moment in beam AB is as small as possible, (*b*) the corresponding value of $|M|_{max}$. (*Hint:* Draw the bending-moment diagram and then equate the absolute values of the largest positive and negative bending moments obtained.)

PROBLEM 7.50 Two small channel sections DF and EH have been welded to the uniform beam AB of weight $W = 3$ kN to form the rigid structural member shown. This member is being lifted by two cables attached at D and E. Knowing that $\theta = 30°$ and neglecting the weight of the channel sections, (*a*) draw the shear and bending-moment diagrams for beam AB, (*b*) determine the maximum absolute values of the shear and bending moment in the beam.

SOLUTION

See solution of Problem 7.50 for reduction of loading or beam AB to the following:



where $M_0 = (750 \text{ N}\cdot\text{m})\tan\theta$ ◁

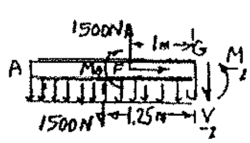
[Equation (2)]

The largest negative bending moment occurs Just to the left of F:

$$+\circlearrowleft \Sigma M_1 = 0: \quad M_1 + (900 \text{ N})\left(\frac{1.5 \text{ m}}{2}\right) = 0 \qquad M_1 = -675 \text{ N}\cdot\text{m} \ \triangleleft$$

The largest positive bending moment occurs

At G:



$$+\circlearrowleft \Sigma M_2 = 0: \quad M_2 - M_0 + (1500 \text{ N})(1.25 \text{ m} - 1 \text{ m}) = 0$$

$$M_2 = M_0 - 375 \text{ N}\cdot\text{m} \ \triangleleft$$

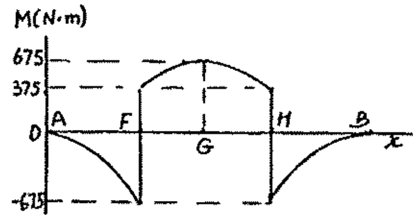
Equating M_2 and $-M_1$:

$$M_0 - 375 = +675$$

$$M_0 = 1050 \text{ N}\cdot\text{m}$$

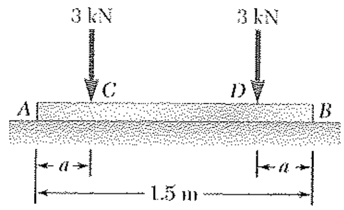
PROBLEM 7.55 (Continued)

(*a*) From Equation (2): $\tan\theta = \dfrac{1050}{750} = 1.400$ $\theta = 54.5°$ ◀



(*b*) $|M|_{max} = 675\ \text{N}\cdot\text{m}$ ◀

PROBLEM 7.56

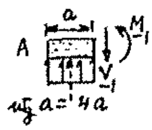


For the beam of Problem 7.43, determine (*a*) the distance a for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding value of $|M|_{max}$. (See hint for Problem 7.55.)

PROBLEM 7.43 Assuming the upward reaction of the ground on beam AB to be uniformly distributed and knowing that $a = 0.3$ m, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Force per unit length exerted by ground:



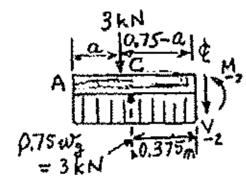
$$w_g = \frac{6\text{ kN}}{1.5\text{ m}} = 4\text{ kN/m}$$

The largest positive bending moment occurs Just to the left of C:

$$+\circlearrowleft\Sigma M_1 = 0: \quad M_1 = (4a)\frac{a}{2}$$

$M_1 = 2a^2$ ◁

The largest negative bending moment occurs



At the center line:

$$+\circlearrowleft\Sigma M_2 = 0: \quad M_2 + 3(0.75 - a) - 3(0.375) = 0$$

$M_2 = -(1.125 - 3a)$ ◁

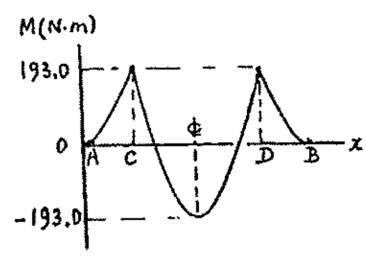
Equating M_1 and $-M_2$:

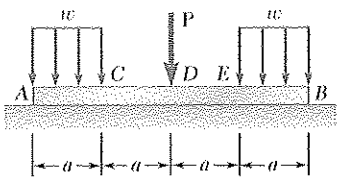
$$2a^2 = 1.125 - 3a$$

$$a^2 + 1.5a - 0.5625 = 0$$

(*a*) Solving the quadratic equation: $a = 0.31066,$ $a = 0.311\text{ m}$ ◀

(*b*) Substituting: $|M|_{max} = M_1 = 2(0.31066)^2$ $|M|_{max} = 193.0\text{ N}\cdot\text{m}$ ◀



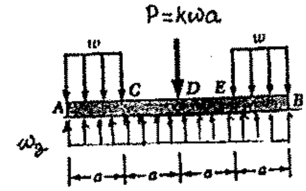


PROBLEM 7.57

For the beam of Problem 7.47, determine (*a*) the ratio $k = P/wa$ for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding value of $|M|_{max}$. (See hint for Problem 7.55.)

PROBLEM 7.47 Assuming the upward reaction of the ground on beam AB to be uniformly distributed and knowing that $P = wa$, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



Free body: Entire beam

$$+\uparrow \Sigma F_y = 0: \quad w_g(4a) - 2wa - kwa = 0$$

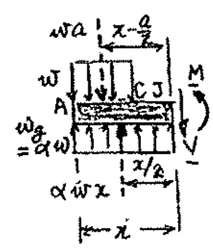
$$w_g = \frac{w}{4}(2+k)$$

Setting $$\frac{w_g}{w} = \alpha \qquad (1)$$

We have $$k = 4\alpha - 2 \qquad (2)$$

Minimum value of B.M. For M to have negative values, we must have $w_g < w$. We verify that M will then be negative and keep decreasing in the portion AC of the beam. Therefore, M_{min} will occur between C and D.

From C to D:



$$+\circlearrowleft \Sigma M_J = 0: \quad M + wa\left(x - \frac{a}{2}\right) - \alpha wx\left(\frac{x}{2}\right) = 0$$

$$M = \frac{1}{2}w(\alpha x^2 - 2ax + a^2) \qquad (3)$$

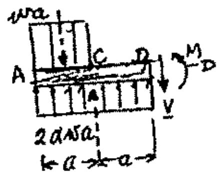
We differentiate and set $\frac{dM}{dx} = 0$: $$\alpha x - a = 0 \qquad x_{min} = \frac{a}{\alpha} \qquad (4)$$

Substituting in (3):

$$M_{min} = \frac{1}{2}wa^2\left(\frac{1}{\alpha} - \frac{2}{\alpha} + 1\right)$$

$$M_{min} = -wa^2\frac{1-\alpha}{2\alpha} \qquad (5)$$

Maximum value of bending moment occurs at D



$$+\circlearrowleft \Sigma M_D = 0: \quad M_D + wa\left(\frac{3a}{2}\right) - (2\alpha wa)a = 0$$

$$M_{max} = M_D = wa^2\left(2\alpha - \frac{3}{2}\right) \qquad (6)$$

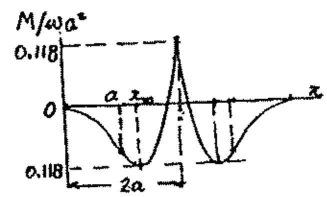
Equating $-M_{min}$ and M_{max}:

$$wa^2\frac{1-\alpha}{2\alpha} = wa^2\left(2\alpha - \frac{3}{2}\right)$$

$$4\alpha^2 - 2\alpha - 1 = 0$$

$$\alpha = \frac{2+\sqrt{20}}{8} \qquad \alpha = \frac{1+\sqrt{5}}{4} = 0.809$$

(*a*) Substitute in (2): $k = 4(0.809) - 2$ $\quad k = 1.236$ ◀

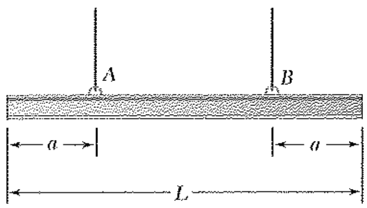


(*b*) Substitute for α in (5):

$$|M|_{max} = -M_{min} = -wa^2\frac{1-0.809}{2(0.809)} \qquad |M|_{max} = 0.1180wa^2 \blacktriangleleft$$

Substitute for α in (4): $\quad x_{min} = \dfrac{a}{0.809}1.236a$ ◁

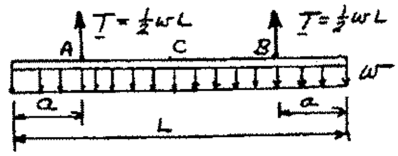
PROBLEM 7.58



A uniform beam is to be picked up by crane cables attached at A and B. Determine the distance a from the ends of the beam to the points where the cables should be attached if the maximum absolute value of the bending moment in the beam is to be as small as possible. (*Hint:* Draw the bending-moment diagram in terms of a, L, and the weight w per unit length, and then equate the absolute values of the largest positive and negative bending moments obtained.)

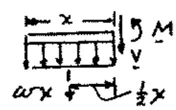
SOLUTION

w = weight per unit length



To the left of A:

$$+\circlearrowleft \Sigma M_1 = 0: \quad M + wx\left(\frac{x}{2}\right) = 0$$

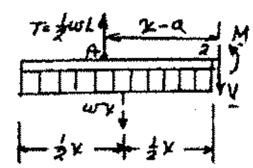


$$M = -\frac{1}{2}wx^2$$

$$M_A = -\frac{1}{2}wa^2$$

Between A and B:

$$+\circlearrowleft \Sigma M_2 = 0: \quad M - \left(\frac{1}{2}wL\right)(x-a) + (wx)\left(\frac{1}{2}x\right) = 0$$

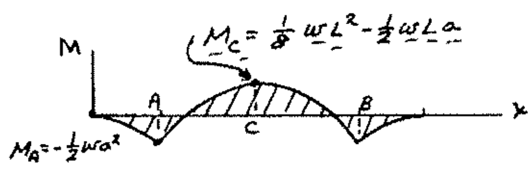


$$M = -\frac{1}{2}wx^2 + \frac{1}{2}wLx - \frac{1}{2}wLa$$

At center C:

$$x = \frac{L}{2}$$

$$M_C = -\frac{1}{2}w\left(\frac{L}{2}\right)^2 + \frac{1}{2}wL\left(\frac{L}{2}\right) - \frac{1}{2}wLa$$



PROBLEM 7.58 (Continued)

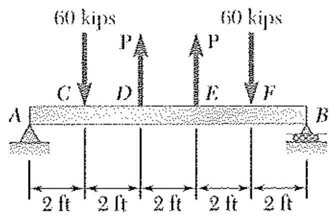
We set $|M_A| = |M_C|$: $\left|-\frac{1}{2}wa^2\right| = \left|\frac{1}{8}wL^2 - \frac{1}{2}wLa\right| + \frac{1}{2}wa^2 = \frac{1}{8}wL^2 - \frac{1}{2}wLa$

$$a^2 + La - 0.25L^2 = 0$$

$$a = \frac{1}{2}(L \pm \sqrt{L^2 + L^2}) = \frac{1}{2}(\sqrt{2} - 1)L$$

$$M_{\max} = \frac{1}{2}w(0.207L)^2 = 0.0214wL^2$$

$a = 0.207L$ ◀

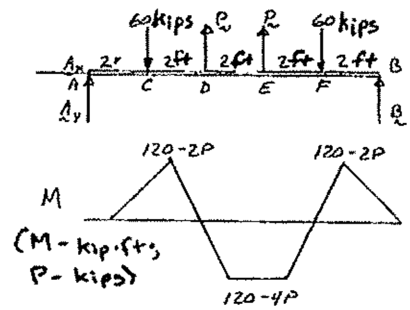


PROBLEM 7.59

For the beam shown, determine (*a*) the magnitude P of the two upward forces for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding value of $|M|_{max}$. (See hint for Problem 7.55.)

SOLUTION

By symmetry: $A_y = B = 60 \text{ kips} - P$

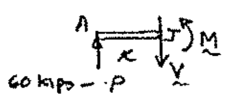


Along AC:

$+\circlearrowleft \Sigma M_J = 0$: $M - x(60 \text{ kips} - P) = 0$

$M = (60 \text{ kips} - P)x$

$M = 120 \text{ kips} \cdot \text{ft} - (2 \text{ ft})P$ at $x = 2$ ft

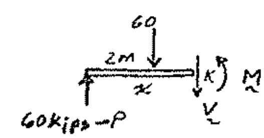


Along CD:

$+\circlearrowleft \Sigma M_K = 0$: $M + (x - 2 \text{ ft})(60 \text{ kips}) - x(60 \text{ kips} - P) = 0$

$M = 120 \text{ kip} \cdot \text{ft} - Px$

$M = 120 \text{ kip} \cdot \text{ft} - (4 \text{ ft})P$ at $x = 4$ ft

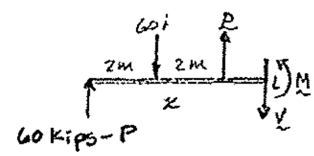


Along DE:

$+\circlearrowleft \Sigma M_L = 0$: $M - (x - 4 \text{ ft})P + (x - 2 \text{ ft})(60 \text{ kips})$

$-x(60 \text{ kips} - P) = 0$

$M = 120 \text{ kip} \cdot \text{ft} - (4 \text{ ft})P$ (const)



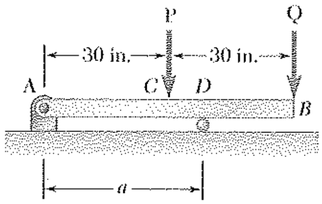
Complete diagram by symmetry

For minimum $|M|_{max}$, set $M_{max} = -M_{min}$

$$120 \text{ kip} \cdot \text{ft} - (2 \text{ ft})P = -[120 \text{ kip} \cdot \text{ft} - (4 \text{ ft})P]$$

(*a*) $P = 40.0 \text{ kips}$ ◀

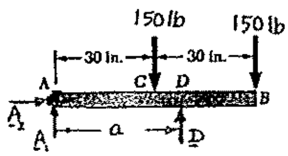
$M_{min} = 120 \text{ kip} \cdot \text{ft} - (4 \text{ ft})P$ (*b*) $|M|_{max} = 40.0 \text{ kip} \cdot \text{ft}$ ◀



PROBLEM 7.60

Knowing that $P = Q = 150$ lb, determine (*a*) the distance a for which the maximum absolute value of the bending moment in beam AB is as small as possible, (*b*) the corresponding value of $|M|_{max}$. (See hint for Problem 7.55.)

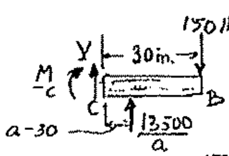
SOLUTION



Free body: Entire beam

$$+\circlearrowleft \Sigma M_A = 0: \quad Da - (150)(30) - (150)(60) = 0$$

$$D = \frac{13,500}{a} \quad \triangleleft$$



Free body: CB

$$+\circlearrowleft \Sigma M_C = 0: \quad -M_C - (150)(30) + \frac{13,500}{a}(a - 30) = 0$$

$$M_C = 9000\left(1 - \frac{45}{a}\right) \quad \triangleleft$$

Free body: DB

$$+\circlearrowleft \Sigma M_D = 0: \quad -M_D - (150)(60 - a) = 0$$

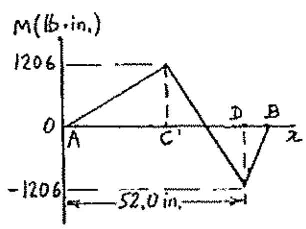
$$M_D = -150(60 - a) \quad \triangleleft$$

(*a*) We set

$$M_{max} = |M_{min}| \quad \text{or} \quad M_C = -M_D: \quad 9000\left(1 - \frac{45}{a}\right) = 150(60 - a)$$

$$60 - \frac{2700}{a} = 60 - a$$

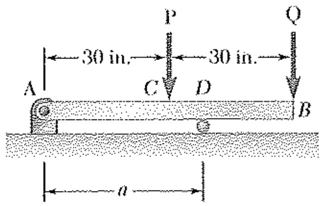
$$a^2 = 2700 \quad a = 51.96 \text{ in.} \qquad a = 52.0 \text{ in.} \blacktriangleleft$$



(*b*) $|M|_{max} = -M_D = 150(60 - 51.96)$

$$|M|_{max} = 1206 \text{ lb} \cdot \text{in.} \blacktriangleleft$$

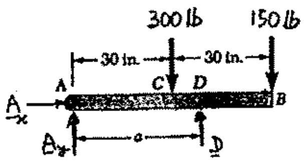
PROBLEM 7.61



Solve Problem 7.60 assuming that $P = 300$ lb and $Q = 150$ lb.

PROBLEM 7.60 Knowing that $P = Q = 150$ lb, determine (*a*) the distance a for which the maximum absolute value of the bending moment in beam AB is as small as possible, (*b*) the corresponding value of $|M|_{max}$. (See hint for Problem 7.55.)

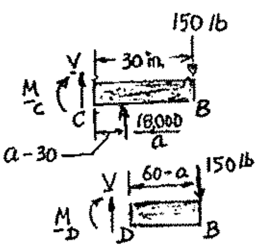
SOLUTION



Free body: Entire beam

$$+\circlearrowleft \Sigma M_A = 0: \quad Da - (300)(30) - (150)(60) = 0$$

$$D = \frac{18{,}000}{a} \quad \triangleleft$$



Free body: CB

$$+\circlearrowleft \Sigma M_C = 0: \quad -M_C - (150)(30) + \frac{18{,}000}{a}(a-30) = 0$$

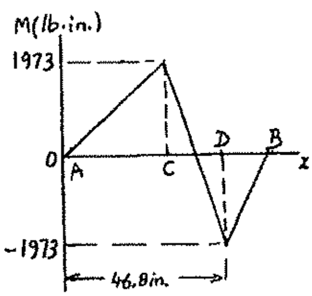
$$M_C = 13{,}500\left(1 - \frac{40}{a}\right) \quad \triangleleft$$

Free body: DB

$$+\circlearrowleft \Sigma M_D = 0: \quad -M_D - (150)(60-a) = 0$$

$$M_D = -150(60-a) \quad \triangleleft$$

(*a*) We set



$$M_{max} = |M_{min}| \quad \text{or} \quad M_C = -M_D: \quad 13{,}500\left(1 - \frac{40}{a}\right) = 150(60-a)$$

$$90 - \frac{3600}{a} = 60 - a$$

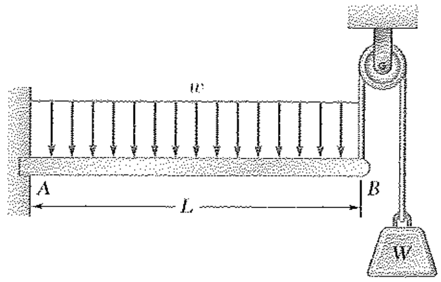
$$a^2 + 30a - 3600 = 0$$

$$a = \frac{-30 + \sqrt{15.300}}{2} = 46.847$$

$$a = 46.8 \text{ in.} \blacktriangleleft$$

(*b*) $|M|_{max} = -M_D = 150(60 - 46.847)$

$$|M|_{max} = 1973 \text{ lb} \cdot \text{in.} \blacktriangleleft$$

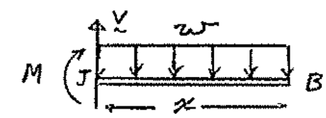


PROBLEM 7.62*

In order to reduce the bending moment in the cantilever beam AB, a cable and counterweight are permanently attached at end B. Determine the magnitude of the counterweight for which the maximum absolute value of the bending moment in the beam is as small as possible and the corresponding value of $|M|_{max}$. Consider (a) the case when the distributed load is permanently applied to the beam, (b) the more general case when the distributed load may either be applied or removed.

SOLUTION

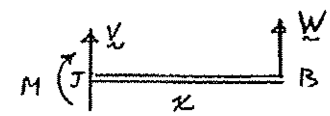
M due to distributed load:



$$\circlearrowleft \Sigma M_J = 0: \quad -M - \frac{x}{2}wx = 0$$

$$M = -\frac{1}{2}wx^2$$

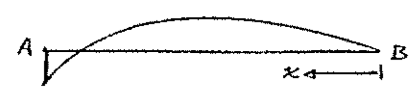
M due to counter weight:



$$\circlearrowleft \Sigma M_J = 0: \quad -M + xw = 0$$

$$M = wx$$

(a) **Both applied:**



$$M = W_x - \frac{w}{2}x^2$$

$$\frac{dM}{dx} = W - wx = 0 \text{ at } x = \frac{W}{w}$$

And here $M = \dfrac{W^2}{2w} > 0$ so M_{max}; M_{min} must be at $x = L$

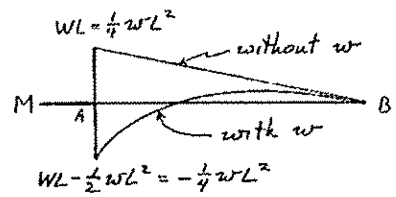
So $M_{min} = WL - \dfrac{1}{2}wL^2$. For minimum $|M|_{max}$ set $M_{max} = -M_{min}$,

so $$\frac{W^2}{2w} = -WL + \frac{1}{2}wL^2 \quad \text{or} \quad W^2 + 2wLW - w^2L^2 = 0$$

$$W = -wL \pm \sqrt{2w^2L^2} \text{ (need+)} \qquad W = (\sqrt{2} - 1)wL = 0.414\, wL \blacktriangleleft$$

(*b*) **w may be removed**

$$M_{\max} = \frac{W^2}{2w} = \frac{(\sqrt{2}-1)^2}{2} wL^2$$ $M_{\max} = 0.858\, wL^2$ ◀



Without w, $M = Wx$

$$M_{\max} = WL \text{ at } A$$

With w (see Part *a*) $M = Wx - \frac{w}{2}x^2$

$$M_{\max} = \frac{W^2}{2w} \text{ at } x = \frac{W}{w}$$

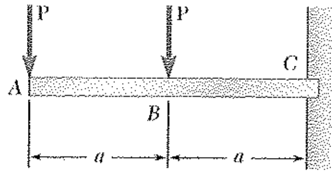
$$M_{\min} = WL - \frac{1}{2}wL^2 \text{ at } x = L$$

For minimum $M_{\max}$, set $M_{\max}(\text{no } w) = -M_{\min}(\text{with } w)$

$$WL = -WL + \frac{1}{2}wL^2 \rightarrow W = \frac{1}{4}wL \rightarrow$$ $M_{\max} = \frac{1}{4}wL^2$ ◀

With $W = \frac{1}{4}wL$ ◀

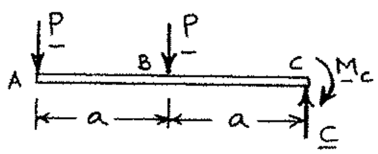
PROBLEM 7.63



Using the method of Section 7.6, solve Problem 7.29.

PROBLEM 7.29 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



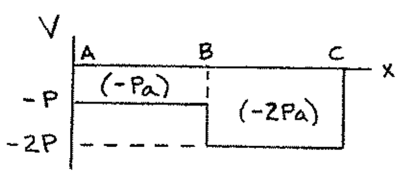
Free body: Entire beam

$$+\uparrow \Sigma F_y = 0: \quad C - P - P = 0$$

$$\mathbf{C} = 2P \uparrow$$

$$+\circlearrowleft \Sigma M_C = 0: \quad P(2a) + P(a) - M_C = 0$$

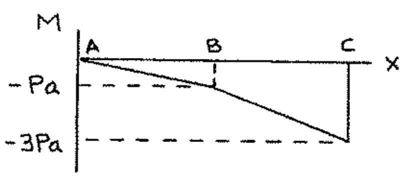
$$\mathbf{M}_C = 3Pa \circlearrowleft$$



Shear diagram

At A: $\qquad V_A = -P$

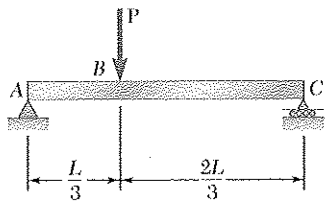
$$|V|_{max} = 2P \blacktriangleleft$$



Bending-moment diagram

At A: $\qquad M_A = 0$

$$|M|_{max} = 3Pa \blacktriangleleft$$

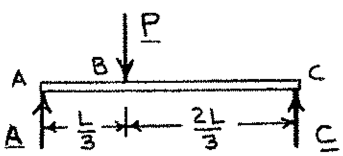


PROBLEM 7.64

Using the method of Section 7.6, solve Problem 7.30.

PROBLEM 7.30 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



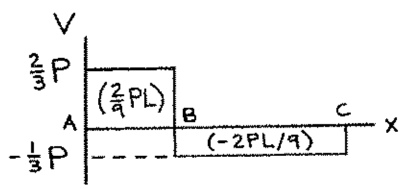
Free body: Entire beam

$$+\circlearrowleft \Sigma M_C = 0: \quad P\left(\frac{2L}{3}\right) - A(L) = 0$$

$$\mathbf{A} = \frac{2}{3}P \uparrow$$

$$+\uparrow \Sigma F_Y = 0: \quad \frac{2}{3}P - P + C = 0$$

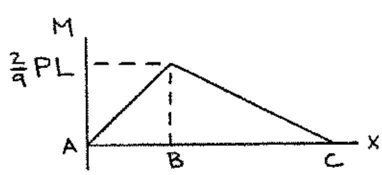
$$\mathbf{C} = \frac{1}{3}P \uparrow$$



Shear diagram

At A: $\quad V_A = \frac{2}{3}P$

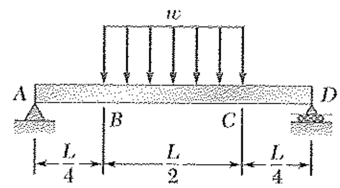
$$|V|_{max} = \frac{2}{3}P \blacktriangleleft$$



Bending-moment diagram

At A: $\quad M_A = 0$

$$|M|_{max} = \frac{2}{9}PL \blacktriangleleft$$

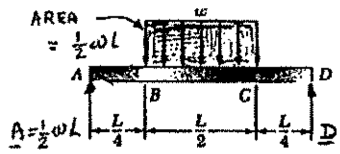


PROBLEM 7.65

Using the method of Section 7.6, solve Problem 7.31.

PROBLEM 7.31 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

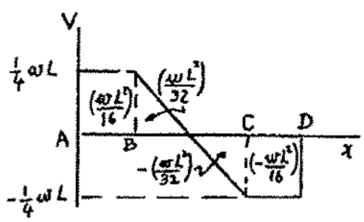


Reactions at A and D

Because of the symmetry of the supports and loading.

$$A = D = \frac{1}{2}\left(w\frac{L}{2}\right) = \frac{1}{4}wL$$

$$\mathbf{A} = \mathbf{D} = \frac{1}{4}wL\uparrow \triangleleft$$

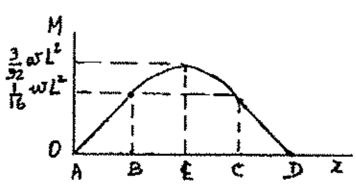


Shear diagram

At A: $V_A = +\frac{1}{4}wL$

From B to C:

Oblique straight line $|V|_{max} = \frac{1}{4}wL$ ◀

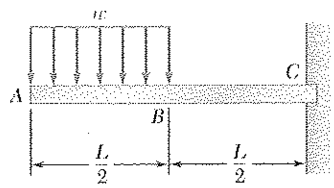


Bending-moment diagram

At A: $M_A = 0$

From B to C: *ARC* of parabola $|M|_{max} = \frac{3}{32}wL^2$ ◀

Since V has no discontinuity at B nor C, the slope of the parabola at these points is the same as the slope of the adjoining straight line segment.

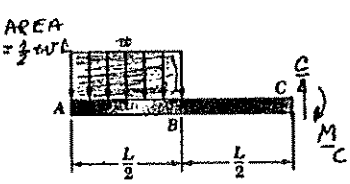


PROBLEM 7.66

Using the method of Section 7.6, solve Problem 7.32.

PROBLEM 7.32 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



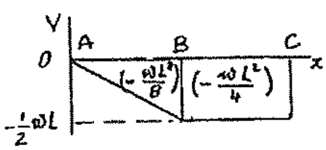
Free body: Entire beam

$$+\uparrow \Sigma F_y = 0: \quad C - w\frac{L}{2} = 0$$

$$\mathbf{C} = \frac{1}{2}wL \uparrow$$

$$+\circlearrowleft \Sigma M_C = 0: \quad \left(\frac{1}{2}wL\right)\left(\frac{3L}{4}\right) = M_C = 0$$

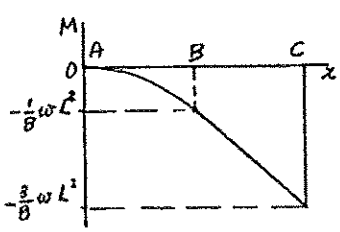
$$\mathbf{M}_C = \frac{3}{8}wL^2 \circlearrowleft$$



Shear diagram

At A: $\quad V_A = 0$

$$|V|_{max} = \frac{1}{2}wL \blacktriangleleft$$



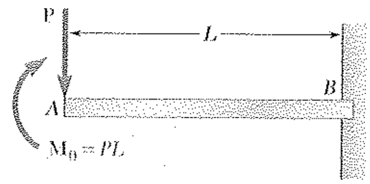
Bending-moment diagram

At A: $\quad M = 0 \frac{dM}{dx} = V = 0$

$$|M|_{max} = \frac{3}{8}wL^2 \blacktriangleleft$$

From A to B: ARC of parabola

Since V has no discontinuity at B, the slope of the parabola at B is equal to the slope of the straightline segment.

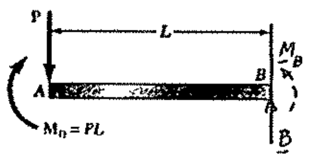


PROBLEM 7.67

Using the method of Section 7.6, solve Problem 7.33.

PROBLEM 7.33 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



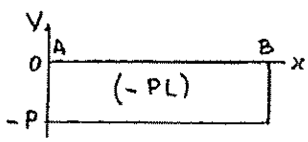
Free body: Entire beam

$$+\uparrow \Sigma F_y = 0: \quad B - P = 0$$

$$\mathbf{B} = P \uparrow$$

$$+\circlearrowleft \Sigma M_B = 0: \quad M_B - M_0 + PL = 0$$

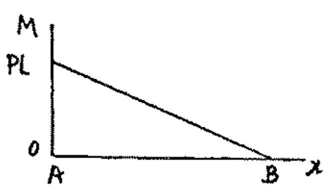
$$\mathbf{M}_B = 0$$



Shear diagram

At A: $\quad V_A = -P$

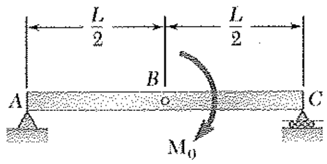
$$|V|_{\max} = P \blacktriangleleft$$



Bending-moment diagram

At A: $\quad M_A = M_0 = PL$

$$|M|_{\max} = PL \blacktriangleleft$$

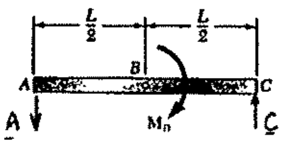


PROBLEM 7.68

Using the method of Section 7.6, solve Problem 7.34.

PROBLEM 7.34 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

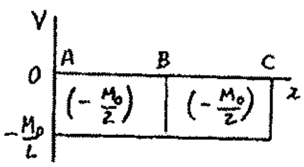


Free body: Entire beam

$$\Sigma F_y = 0: \quad A = C$$

$$+\circlearrowleft \Sigma M_C = 0: \quad Al - M_0 = 0$$

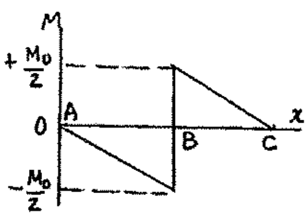
$$A = C = \frac{M_0}{L}$$



Shear diagram

At A: $$V_A = -\frac{M_0}{L}$$

$$|V|_{\max} = \frac{M_0}{L} \blacktriangleleft$$

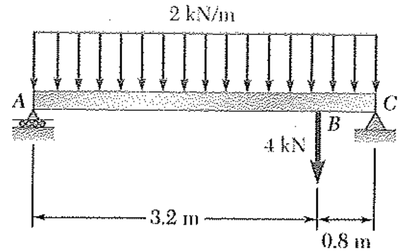


Bending-moment diagram

At A: $$M_A = 0$$

At B, M increases by M_0 on account of applied couple.

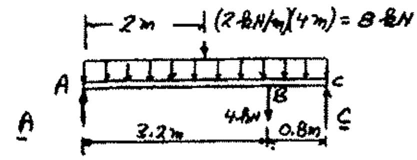
$$|M|_{\max} = M_0/2 \blacktriangleleft$$



PROBLEM 7.69

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

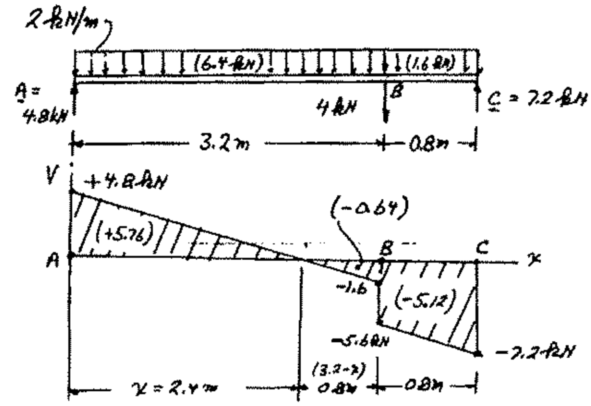


$+\circlearrowleft \Sigma M_A = 0$: $(8)(2) + (4)(3.2) - 4C = 0$

$\mathbf{C} = 7.2 \text{ kN} \uparrow$

$\Sigma F_y = 0$: $\mathbf{A} = 4.8 \text{ kN} \uparrow$

(*a*) Shear diagram

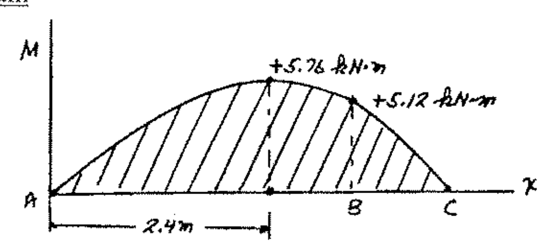


Similar Triangles: $\dfrac{x}{4.8} = \dfrac{3.2 - x}{1.6} = \dfrac{3.2}{6.4}$; $x = 2.4 \text{ m}$

↑

Add num. & den.

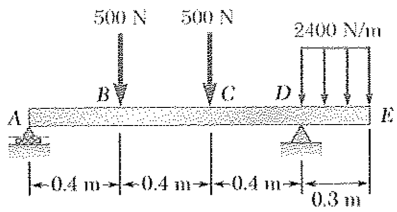
Bending-moment diagram



(*b*)

$|V|_{max} = 7.20 \text{ kN}$ ◀

$|M|_{max} = 5.76 \text{ kN} \cdot \text{m}$ ◀

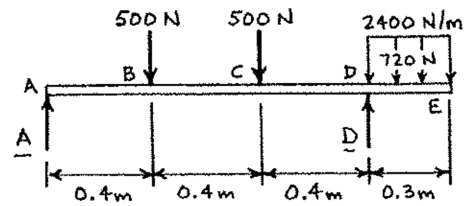


PROBLEM 7.70

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



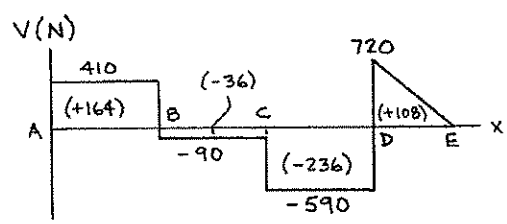
$$+\circlearrowleft \Sigma M_D = 0: \quad (500\text{ N})(0.8\text{ m}) + (500\text{ N})(0.4\text{ m}) - (2400\text{ N/m})(0.3\text{ m})(0.15\text{ m}) - A(1.2\text{ m}) = 0$$

$$\mathbf{A} = 410\text{ N}\uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad 410 - 2(500) - 2400(0.3) + D = 0$$

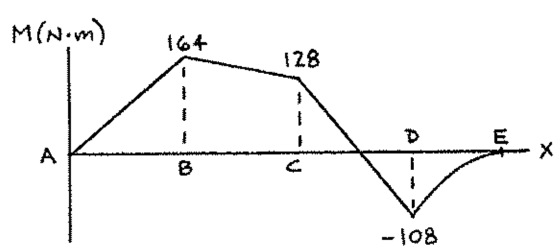
$$\mathbf{D} = 1310\text{ N}\uparrow$$

Shear diagram



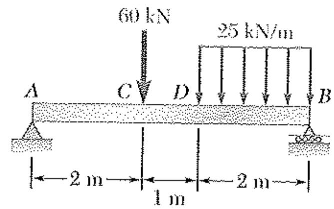
At A: $V_A = +410\text{ N}$ $|V|_{max} = 720\text{ N}$ ◀

Bending-moment diagram



At A: $M_A = 0$ $|M|_{max} = 164.0\text{ N}\cdot\text{m}$ ◀

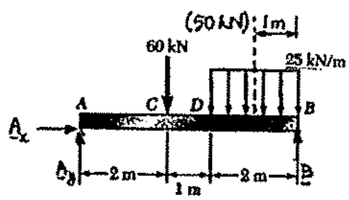
PROBLEM 7.71



Using the method of Section 7.6, solve Problem 7.39.

PROBLEM 7.39 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



Free body: Beam

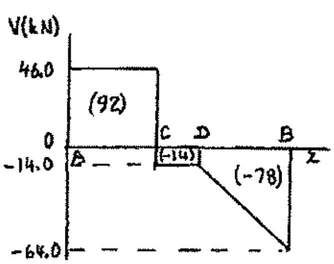
$\Sigma F_x = 0$: $A_x = 0$

$+\circlearrowleft \Sigma M_B = 0$: $(60 \text{ kN})(3 \text{ m}) + (50 \text{ kN})(1 \text{ m}) - A_y(5 \text{ m}) = 0$

$A_y = +46.0 \text{ kN}$ ◁

$+\uparrow \Sigma F_y = 0$: $B + 46.0 \text{ kN} - 60 \text{ kN} - 50 \text{ kN} = 0$

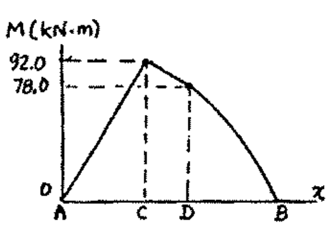
$B = +64.0 \text{ kN}$ ◁



Shear diagram

At A: $V_A = A_y = +46.0 \text{ kN}$

$|V|_{\max} = 64.0 \text{ kN}$ ◀

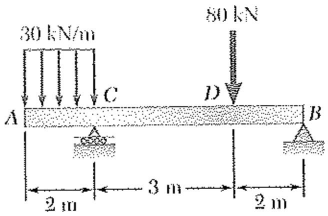


Bending-moment diagram

At A: $M_A = 0$

$|M|_{\max} = 92.0 \text{ kN} \cdot \text{m}$ ◀

Parabola from D to B. Its slope at D is same as that of straight-line segment CD since V has no discontinuity at D.

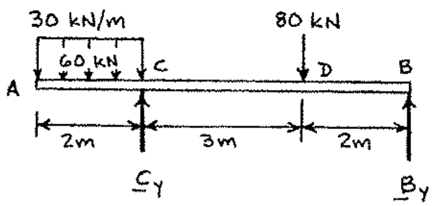


PROBLEM 7.72

Using the method of Section 7.6, solve Problem 7.40.

PROBLEM 7.40 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



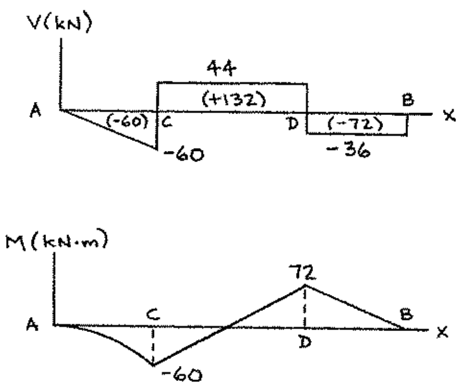
Free body: Entire beam

$$+\circlearrowleft \Sigma M_B = 0: \quad (30\text{ kN/m})(2\text{ m})(6\text{ m}) - C(5\text{ m}) + (80\text{ kN})(2\text{ m}) = 0$$

$$\mathbf{C} = 104\text{ kN}\uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad 104 - 30(2) - 80 + B = 0$$

$$\mathbf{B} = 36\text{ kN}\uparrow$$



Shear diagram

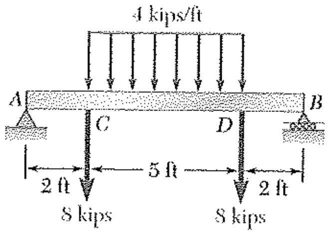
At A: $V_A = 0$

$$|V|_{max} = 60.0\text{ kN} \blacktriangleleft$$

Bending-moment diagram

At A: $M_A = 0$

$$|M|_{max} = 72.0\text{ kN}\cdot\text{m} \blacktriangleleft$$

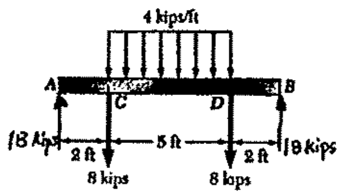


PROBLEM 7.73

Using the method of Section 7.6, solve Problem 7.41.

PROBLEM 7.41 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

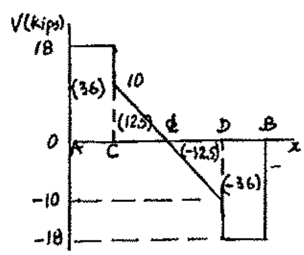


Reactions at supports.

Because of the symmetry:

$$A = B = \frac{1}{2}(8 + 8 + 4 \times 5)\,\text{kips}$$

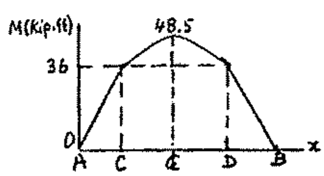
$\mathbf{A} = \mathbf{B} = 18\ \text{kips} \uparrow$ ◁



Shear diagram

At A: $V_A = +18\ \text{kips}$

$|V|_{max} = 18.00\ \text{kips}$ ◀

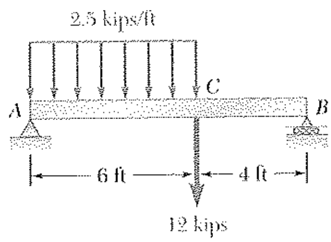


Bending-moment diagram

At A: $M_A = 0$

$|M|_{max} = 48.5\ \text{kip} \cdot \text{ft}$ ◀

Discontinuities in slope at C and D, due to the discontinuities of V.

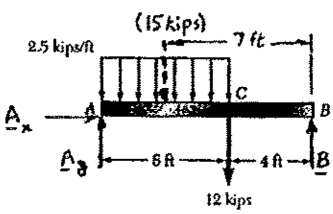


PROBLEM 7.74

Using the method of Section 7.6, solve Problem 7.42.

PROBLEM 7.42 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



Free body: Beam

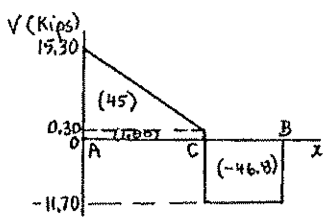
$\Sigma F_x = 0$: $A_x = 0$

$+\circlearrowleft \Sigma M_B = 0$: $(12 \text{ kips})(4 \text{ ft}) + (15 \text{ kips})(7 \text{ ft}) - A_y(10 \text{ ft}) = 0$

$A_y = +15.3 \text{ kips}$ ◁

$+\uparrow \Sigma F_y = 0$: $B + 15.3 - 15 - 12 = 0$

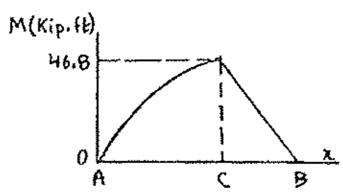
$B = +11.7 \text{ kips}$ ◁



Shear diagram

At A: $V_A = A_y = 15.3 \text{ kips}$

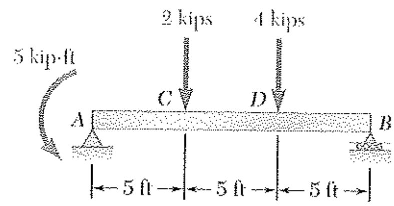
$|V|_{max} = 15.30 \text{ kips}$ ◀



Bending-moment diagram

At A: $M_A = 0$

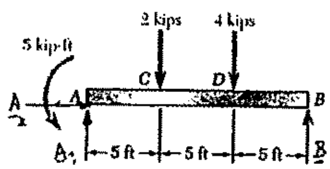
$|M|_{max} = 46.8 \text{ kip} \cdot \text{ft}$ ◀



PROBLEM 7.75

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

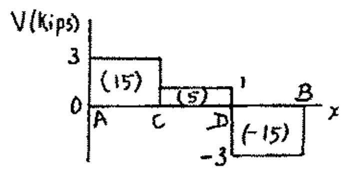


Free body: Beam

$$+\circlearrowleft \Sigma M_B = 0: \quad 5 \text{ kip}\cdot\text{ft} + (1 \text{ kips})(10 \text{ ft}) + (4 \text{ kips})(5 \text{ ft}) - A_y(15 \text{ ft}) = 0$$

$$A_y = +3.00 \text{ kips} \;\triangleleft$$

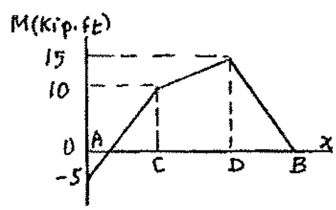
$$\Sigma F_x = 0: \quad A_x = 0$$



Shear diagram

At A: $\quad V_A = A_y = +3.00 \text{ kips}$

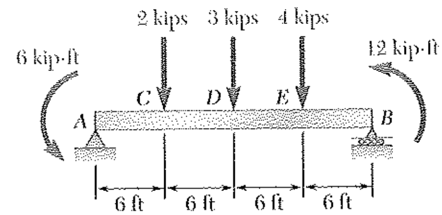
$$|V|_{max} = 3.00 \text{ kips} \;\blacktriangleleft$$



Bending-moment diagram

At A: $\quad M_A = -5 \text{ kip}\cdot\text{ft}$

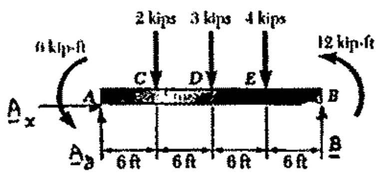
$$|M|_{max} = 15.00 \text{ kip}\cdot\text{ft} \;\blacktriangleleft$$



PROBLEM 7.76

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

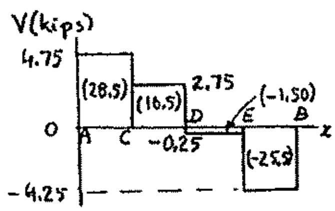


Free body: Beam

$$+\circlearrowleft \Sigma M_B = 0: \quad 6\text{ kip}\cdot\text{ft} + 12\text{ kip}\cdot\text{ft} + (2\text{ kips})(18\text{ ft}) + (3\text{ kips})(12\text{ ft}) + (4\text{ kips})(6\text{ ft}) - A_y(24\text{ ft}) = 0$$

$$A_y = +4.75\text{ kips} \triangleleft$$

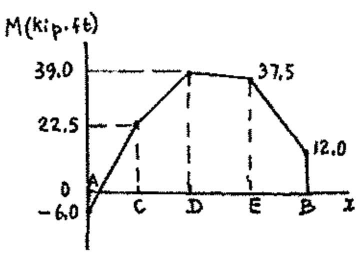
$$\Sigma F_x = 0: \quad A_x = 0$$



Shear diagram

At A: $\quad V_A = A_y = +4.75\text{ kips}$

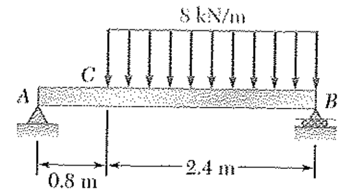
$$|V|_{max} = 4.75\text{ kips} \blacktriangleleft$$



Bending-moment diagram

At A: $\quad M_A = -6\text{ kip}\cdot\text{ft}$

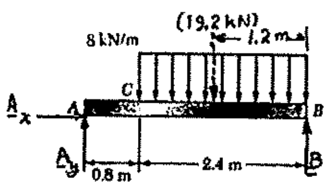
$$|M|_{max} = 39.0\text{ kip}\cdot\text{ft} \blacktriangleleft$$



PROBLEM 7.77

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

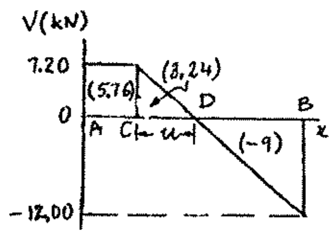


Free body: Beam

$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (19.2 \text{ kN})(1.2 \text{ m}) - A_y(3.2 \text{ m}) = 0$$

$$A_y = +7.20 \text{ kN} \; \triangleleft$$



Shear diagram

$$V_A = V_C = A_y = +7.20 \text{ kN}$$

To determine Point D where $V = 0$, we write

$$V_D - V_C = wu$$

$$0 - 7.20 \text{ kN} = -(8 \text{ kN/m})u \qquad u = 0.9 \text{ m} \; \triangleleft$$

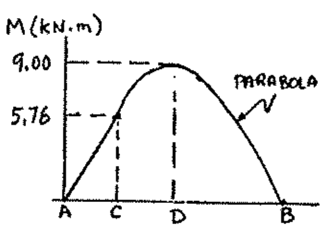
We next compute all areas

Bending-moment diagram

At A: $\quad M_A = 0$

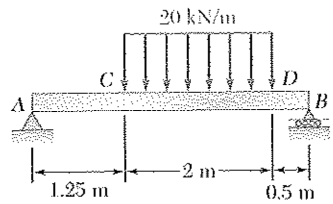
Largest value occurs at D with

$$AD = 0.8 + 0.9 = 1.700 \text{ m}$$



$$|M|_{max} = 9.00 \text{ kN} \cdot \text{m} \; \blacktriangleleft$$

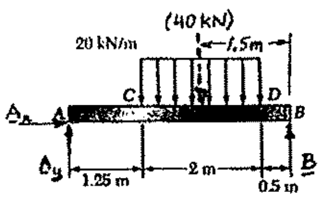
$$1.700 \text{ m from } A \; \blacktriangleleft$$



PROBLEM 7.78

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum bending moment.

SOLUTION



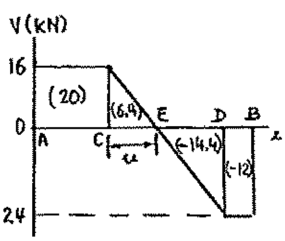
Free body: Beam

$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (40 \text{ kN})(1.5 \text{ m}) - A_y(3.75 \text{ m}) = 0$$

$$A_y = +16.00 \text{ kN} \ \triangleleft$$

Shear diagram



$$V_A = V_C = A_y = +16.00 \text{ kN}$$

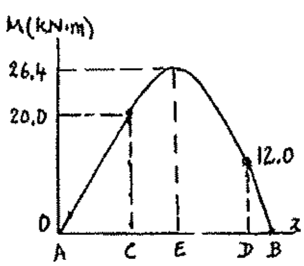
To determine Point E where $V = 0$, we write

$$V_E - V_C = -wu$$

$$0 - 16 \text{ kN} = -(20 \text{ kN/m})u \qquad u = 0.800 \text{ m} \ \triangleleft$$

We next compute all areas

Bending-moment diagram



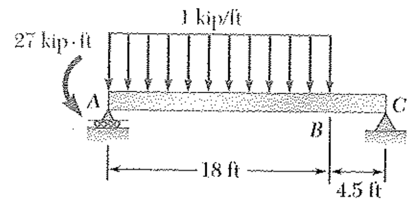
At A: $\quad M_A = 0$

Largest value occurs at E with

$$AE = 1.25 + 0.8 = 2.05 \text{ m} \qquad |M|_{max} = 26.4 \text{ kN} \cdot \text{m} \ \blacktriangleleft$$

2.05 m from A ◀

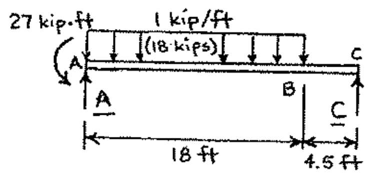
From A to C and D to B: Straight line segments. From C to D: Parabola



PROBLEM 7.79

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION



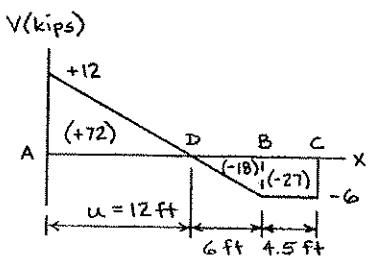
Free body: Entire beam

$$+\circlearrowleft \Sigma M_C = 0: \quad -(27 \text{ kip}\cdot\text{ft}) + A(22.5 \text{ ft}) - (1 \text{ kip/ft})(18 \text{ ft})(13.5 \text{ ft}) = 0$$

$$\mathbf{A} = 12 \text{ kips} \uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad 12 - 1(18) + C = 0$$

$$\mathbf{C} = 6 \text{ kips} \uparrow$$



Shear diagram

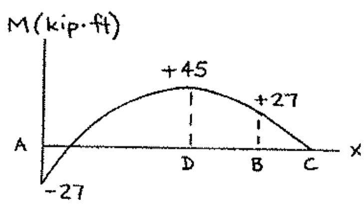
At A: $\quad V_A = +12$ kips

To locate Point D (where $V = 0$)

$$V_D - V_A = -wu$$

$$0 - 12 \text{ kips} = -(1 \text{ kip/ft})u$$

$$u = 12 \text{ ft}$$

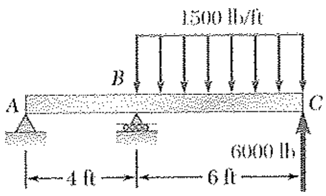


Bending-moment diagram

At A: $\quad M_A = 0$

$|M|_{max} = 45.0 \text{ kip}\cdot\text{ft}$ ◀

12.00 ft from A ◀

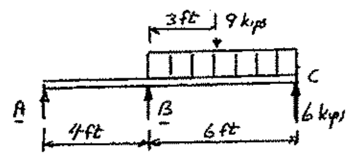


PROBLEM 7.80

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION

Free body: Entire beam



$+\circlearrowleft \Sigma M_A = 0$: $(6 \text{ kips})(10 \text{ ft}) - (9 \text{ kips})(7 \text{ ft}) + 8(4 \text{ ft}) = 0$

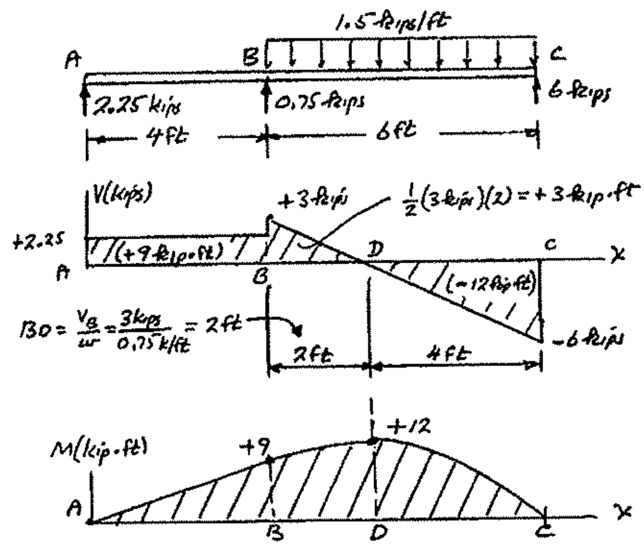
$$B = +0.75 \text{ kips}$$

$$\mathbf{B} = 0.75 \text{ kips} \uparrow$$

$+\uparrow \Sigma F_y = 0$: $A + 0.75 \text{ kips} - 9 \text{ kips} + 6 \text{ kips} = 0$

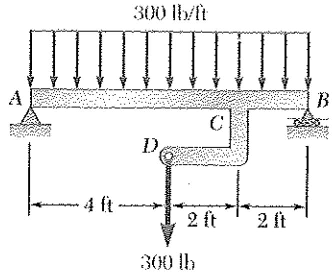
$$A = +2.25 \text{ kips}$$

$$\mathbf{A} = 2.25 \text{ kips} \uparrow$$



$M_{max} = 12.00 \text{ kip} \cdot \text{ft}$ ◀

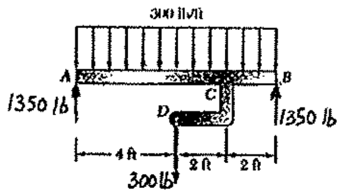
6.00 ft from A



PROBLEM 7.81

(*a*) Draw the shear and bending-moment diagrams for beam *AB*, (*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION

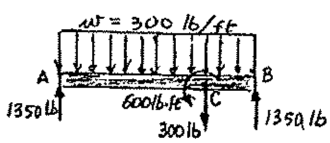


Reactions at supports

Because of symmetry of load

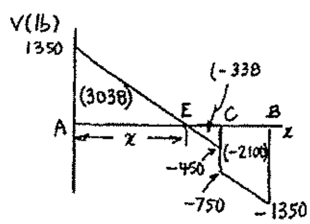
$$A = B = \frac{1}{2}(300 \times 8 + 300)$$

$\mathbf{A} = \mathbf{B} = 1350\ \text{lb}\uparrow$ ◁



Load diagram for *AB*

The 300-lb force at *D* is replaced by an equivalent force-couple system at *C*.



Shear diagram

At *A*: $V_A = A = 1350\ \text{lb}$

To determine Point *E* where $V = 0$:

$$V_E - V_A = -wx$$

$$0 - 1350\ \text{lb} = -(300\ \text{lb/ft})x$$

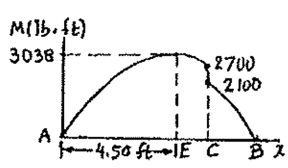
$x = 4.50\ \text{ft}$ ◁

We compute all areas

Bending-moment diagram

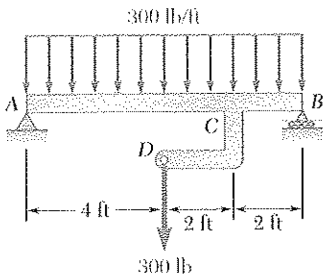
At *A*: $M_A = 0$

Note $600-\text{lb}\cdot\text{ft}$ drop at *C* due to couple



$|M|_{\max} = 3040\ \text{lb}\cdot\text{ft}$ ◀

4.50 ft from *A* ◀

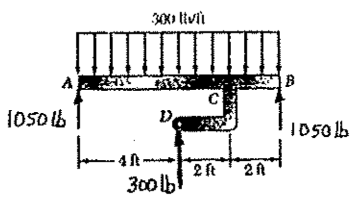


PROBLEM 7.82

Solve Problem 7.81 assuming that the 300-lb force applied at D is directed upward.

PROBLEM 7.81 (*a*) Draw the shear and bending-moment diagrams for beam AB, (*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION

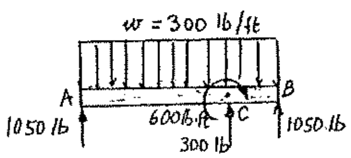


Reactions at supports

Because of symmetry of load:

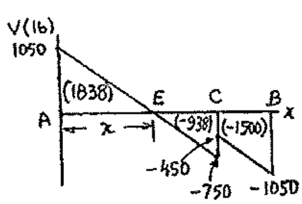
$$A = B = \frac{1}{2}(300 \times 8 - 300)$$

$\mathbf{A} = \mathbf{B} = 1050 \text{ lb}\uparrow$ ◁



Load diagram

The 300-lb force at D is replaced by an equivalent force-couple system at C.



Shear diagram

At A: $\quad V_A = A = 1050 \text{ lb}$

To determine Point E where $V = 0$:

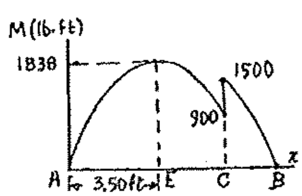
$$V_E - V_A = -wx$$
$$0 - 1050 \text{ lb} = -(300 \text{ lb/ft})x$$

$x = 3.50 \text{ ft}$ ◁

We compute all areas

Bending-moment diagram

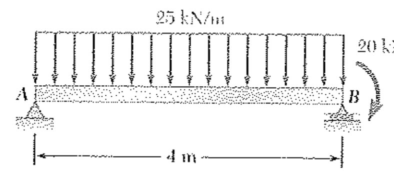
At A: $\quad M_A = 0$



Note $600 - \text{lb} \cdot \text{ft}$ increase at C due to couple

$|M|_{max} = 1838 \text{ lb} \cdot \text{ft}$ ◀

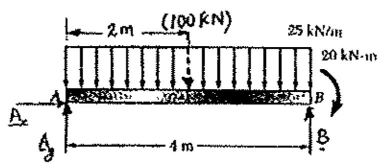
3.50 ft from A ◀



PROBLEM 7.83

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION



Free body: Beam

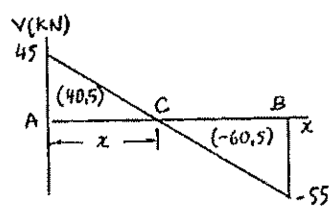
$$+\circlearrowleft \Sigma M_A = 0: \quad B(4\text{ m}) - (100\text{ kN})(2\text{ m}) - 20\text{ kN}\cdot\text{m} = 0$$

$B = +55\text{ kN}$ ◁

$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + 55 - 100 = 0$$

$A_y = +45\text{ kN}$ ◁



Shear diagram

At A: $\quad V_A = A_y = +45\text{ kN}$

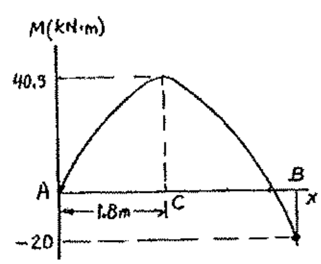
To determine Point C where $V = 0$:

$$V_C - V_A = -wx$$
$$0 - 45\text{ kN} = -(25\text{ kN}\cdot\text{m})x$$

$x = 1.8\text{ m}$ ◁

We compute all areas bending-moment

Bending-moment diagram



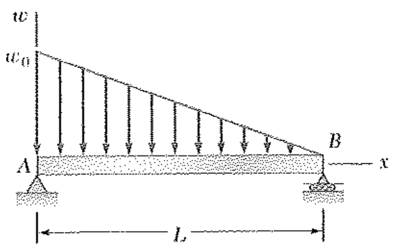
At A: $\quad M_A = 0$

At B: $\quad M_B = -20\text{ kN}\cdot\text{m}$

$|M|_{max} = 40.5\text{ kN}\cdot\text{m}$ ◀

1.800 m from A ◀

Single arc of parabola

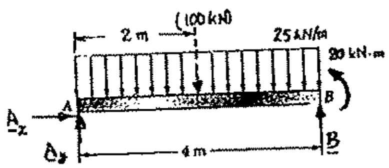


PROBLEM 7.84

Solve Problem 7.83 assuming that the 20-kN · m couple applied at B is counterclockwise.

PROBLEM 7.83 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION



Free body: Beam

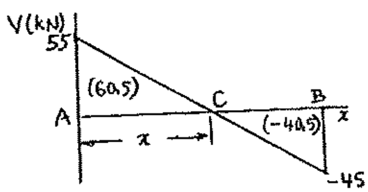
$$+\circlearrowleft \Sigma M_A = 0: \quad B(4\text{ m}) - (100\text{ kN})(2\text{ m}) - 20\text{ kN}\cdot\text{m} = 0$$

$B = +45\text{ kN}$ ◁

$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + 45 - 100 = 0$$

$A_y = +55\text{ kN}$ ◁



Shear diagram

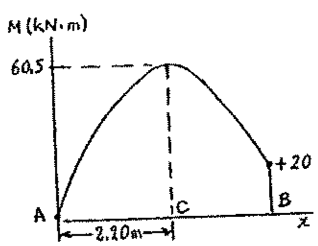
At A: $\quad V_A = A_y = +55\text{ kN}$

To determine Point C where $V = 0$:

$$V_C - V_A = -wx$$
$$0 - 55\text{ kN} = -(25\text{ kN/m})x$$

$x = 2.20\text{ m}$ ◁

We compute all areas bending-moment



Bending-moment diagram

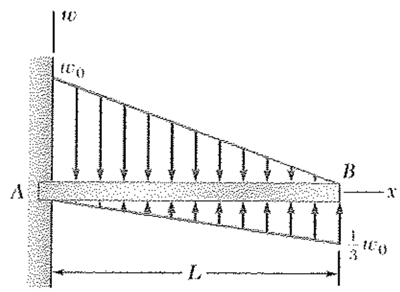
At A: $\quad M_A = 0$

At B: $\quad M_B = +20\text{ kN}\cdot\text{m}$

$|M|_{\max} = 60.5\text{ kN}\cdot\text{m}$ ◀

2.20 m from A ◀

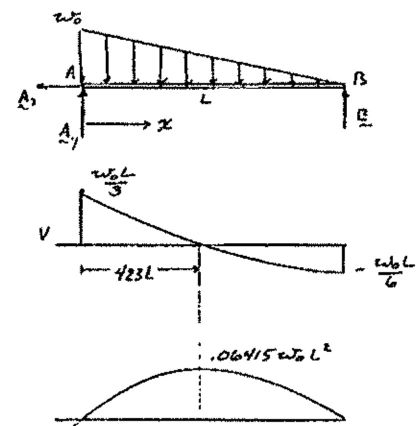
Single arc of parabola



PROBLEM 7.85

For the beam and loading shown, (*a*) write the equations of the shear and bending-moment curves, (*b*) determine the magnitude and location of the maximum bending moment.

SOLUTION



Distributed load

$$w = w_0\left(1-\frac{x}{L}\right) \quad \left(\text{Total} = \frac{1}{2}w_0L\right)$$

$$\circlearrowleft \Sigma M_A = 0: \quad \frac{L}{3}\left(\frac{1}{2}w_0L\right) - LB = 0 \qquad \mathbf{B} = \frac{w_0L}{6}\uparrow$$

$$\uparrow \Sigma F_y = 0: \quad A_y - \frac{1}{2}w_0L + \frac{w_0L}{6} = 0 \quad \mathbf{A}_y = \frac{w_0L}{3}\uparrow$$

Shear: $\quad V_A = A_y = \dfrac{w_0L}{3}$

Then $\quad \dfrac{dV}{dx} = -w \rightarrow V$

$$= V_A - \int_0^x w_0\left(1-\frac{x}{L}\right)dx$$

$$V = \left(\frac{w_0L}{3}\right) - w_0x + \frac{1}{2}\frac{w_0}{L}x^2$$

$$= w_0L\left[\frac{1}{3} - \frac{x}{L} + \frac{1}{2}\left(\frac{x}{L}\right)^2\right]$$

Note: At $\quad x = L$

$$V = -\frac{w_0L}{6}$$

$$V = 0 \text{ at} \left(\frac{x}{L}\right)^2 - 2\left(\frac{x}{L}\right) + \frac{2}{3}$$

$$= 0 \rightarrow \frac{x}{L} = 1 - \sqrt{\frac{1}{3}}$$

PROBLEM 7.85 (Continued)

Moment: $M_A = 0$

Then $$\left(\frac{dM}{dx}\right) = V \rightarrow M = \int_0^x V dx = L\int_0^{x/L} V\left(\frac{x}{L}\right)d\left(\frac{x}{L}\right)$$

$$M = w_0 L^2 \int_0^{x/L}\left[\frac{1}{3} - \frac{x}{L} + \frac{1}{2}\left(\frac{x}{L}\right)^2\right]d\left(\frac{x}{L}\right)$$

$$M = w_0 L^2\left[\frac{1}{3}\left(\frac{x}{L}\right) - \frac{1}{2}\left(\frac{x}{L}\right)^2 + \frac{1}{6}\left(\frac{x}{L}\right)^3\right]$$

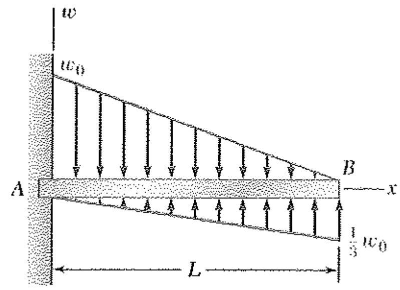
$$M_{\max}\left(\text{at } \frac{x}{L} = 1 - \sqrt{\frac{1}{3}}\right) = 0.06415 w_0 L^2$$

(*a*) $$V = w_0 L\left[\frac{1}{3} - \frac{x}{L} + \frac{1}{2}\left(\frac{x}{L}\right)^2\right] \blacktriangleleft$$

$$M = w_0 L^2\left[\frac{1}{3}\left(\frac{x}{L}\right) - \frac{1}{2}\left(\frac{x}{L}\right)^2 + \frac{1}{6}\left(\frac{x}{L}\right)^3\right] \blacktriangleleft$$

(*c*) $$M_{\max} = 0.0642\ w_0 L^2 \blacktriangleleft$$

at $x = 0.423L$ $\blacktriangleleft$



PROBLEM 7.86

For the beam and loading shown, (*a*) write the equations of the shear and bending-moment curves, (*b*) determine the magnitude and location of the maximum bending moment.

SOLUTION

(*a*) We note that at $B(x=L)$: $V_B = 0, \quad M_B = 0$ (1)

Load: $w(x) = w_0\left(1-\frac{x}{L}\right) - \frac{1}{3}w_0\left(\frac{x}{L}\right) = w_0\left(1-\frac{4x}{3L}\right)$

Shear: We use Eq. (7.2) between $C(x=x)$ and $B(x=L)$:

$$V_B - V_C = -\int_x^L w(x)dx \quad 0 - V(x) = -\int_x^L w(x)dx$$

$$V(x) = w_0\int_x^L\left(1-\frac{4x}{3L}\right)dx$$

$$= w_0\left[x-\frac{2x^2}{3L}\right]_x^L = w_0\left(L-\frac{2L}{3}-x+\frac{2x^2}{3L}\right)$$

$$V(x) = \frac{w_0}{3L}(2x^2-3Lx+L^2) \quad (2) \blacktriangleleft$$

Bending moment: We use to Eq. (7.4) between $C(x=x)$ and $B(x=L)$:

$$M_B - M_C = \int_x^L V(x)dx \quad 0 - M(x)$$

$$= \frac{w_0}{3L}\int_x^L(2x^2-3Lx+L^2)dx$$

$$M(x) = -\frac{w_0}{3L}\left[\frac{2}{3}x^3-\frac{3}{2}Lx^2+L^2x\right]^L$$

$$= -\frac{w_0}{18L}[4x^3-9Lx^2+6L^2x]_x^L$$

$$= -\frac{w_0}{18L}[(4L^3-9L^3+6L^3)-(4x^3-9Lx^2+6L^2x)]$$

$$M(x) = \frac{w_0}{18L}(4x^3-9Lx^2+6L^2x-L^3) \quad (3) \blacktriangleleft$$

PROBLEM 7.86 (Continued)

(*b*) Maximum bending moment

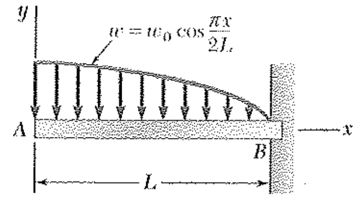
$$\frac{dM}{dx} = V = 0$$

Eq. (2): $2x^2 - 3Lx + L^2 = 0$

$$x = \frac{3 - \sqrt{9-8}}{4} L = \frac{L}{2}$$

Carrying into (3): $M_{\max} = \dfrac{w_0 L^2}{72}$, At $x = \dfrac{L}{2}$ ◀

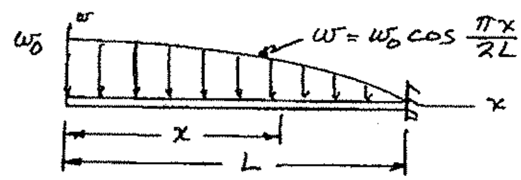
Note: $|M|_{\max} = \dfrac{w_0 L^2}{18}$ At $x = 0$



PROBLEM 7.87

For the beam and loading shown, (*a*) write the equations of the shear and bending-moment curves, (*b*) determine the magnitude and location of the maximum bending moment.

SOLUTION



$$\frac{dv}{dx} = -w = w_0 \cos\frac{\pi}{2}\frac{x}{L}$$

$$V = -\int w dx = -w_0\left(\frac{2L}{\pi}\right)\sin\frac{\pi x}{2L} + C_1 \qquad (1)$$

$$\frac{dM}{dx} = V = -w_0\left(\frac{2L}{\pi}\right)\sin\frac{\pi x}{2L} + C_1$$

$$M = \int V dx = +w_0\left(\frac{2L}{\pi}\right)^2 \cos\frac{\pi x}{2L} + C_1 x + C_2 \qquad (2)$$

Boundary conditions

At $x = 0$: $\quad V = C_1 = 0 \quad C_1 = 0$

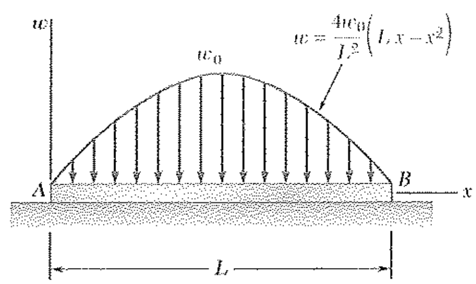
At $x = 0$: $\quad M = +w_0\left(\frac{2L}{\pi}\right)^2 \cos(0) + C_2 = 0$

$$C_2 = -w_0\left(\frac{2L}{\pi}\right)^2$$

Eq. (1) $\quad V = -w_0\left(\frac{2L}{\pi}\right)\sin\frac{\pi x}{2L}$ ◀

$$M = w_0\left(\frac{2L}{\pi}\right)^2\left(-1 + \cos\frac{\pi x}{2L}\right) \text{ ◀}$$

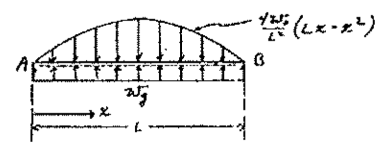
M_{max} at $x = L$: $\quad |M_{max}| = w_0\left(\frac{2L}{\pi}\right)^2 |-1+0| = \frac{4}{\pi^2} w_0 L^2$ ◀



PROBLEM 7.88

The beam AB, which lies on the ground, supports the parabolic load shown. Assuming the upward reaction of the ground to be uniformly distributed, (*a*) write the equations of the shear and bending-moment curves, (*b*) determine the maximum bending moment.

SOLUTION



(*a*)

$$+\uparrow \Sigma F_y = 0: \quad w_g L - \int_0^L \frac{4w_0}{L^2}(Lx - x^2)dx = 0$$

$$w_g L = \frac{4w_0}{L^2}\left(\frac{1}{2}LL^2 - \frac{1}{3}L^3\right) = \frac{2}{3}w_0 L$$

$$w_g = \frac{2w_0}{3}$$

Define

$$\xi = \frac{x}{L} \text{ so } d\xi = \frac{dx}{L} \longrightarrow \text{net load } w = 4w_0\left[\frac{x}{L} - \left(\frac{x}{L}\right)^2\right] - \frac{2}{3}w_0$$

or

$$w = 4w_0\left(-\frac{1}{6} + \xi - \xi^2\right)$$

$$V = V(0) - \int_0^\xi 4w_0 L\left(-\frac{1}{6} + \xi - \xi^2\right)d\xi$$

$$= 0 + 4w_0 L\left(\frac{1}{6}\xi + \frac{1}{2}\xi^2 - \frac{1}{3}\xi^3\right) \qquad V = \frac{2}{3}w_0 L(\xi - 3\xi^2 + 2\xi^3) \blacktriangleleft$$

$$M = M_0 + \int_0^x V dx = 0 + \frac{2}{3}w_0 L^2 \int_0^\xi (\xi - 3\xi^2 + 2\xi^3)d\xi$$

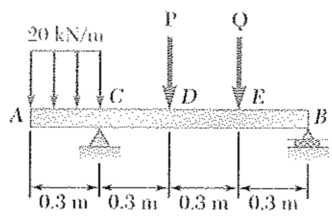
$$= \frac{2}{3}w_0 L^2\left(\frac{1}{2}\xi^2 - \xi^3 + \frac{1}{2}\xi^4\right) = \frac{1}{3}w_0 L^2(\xi^2 - 2\xi^3 + \xi^4) \blacktriangleleft$$

(*b*) Max M occurs where

$$V = 0 \longrightarrow 1 - 3\xi + 2\xi^2 = 0 \longrightarrow \xi = \frac{1}{2}$$

$$M\left(\xi = \frac{1}{2}\right) = \frac{1}{3}w_0 L^2\left(\frac{1}{4} - \frac{2}{8} + \frac{1}{16}\right) = \frac{w_0 L^2}{48}$$

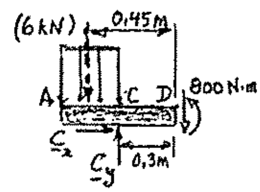
$$M_{\max} = \frac{w_0 L^2}{48} \text{ at center of beam} \blacktriangleleft$$



PROBLEM 7.89

The beam AB is subjected to the uniformly distributed load shown and to two unknown forces **P** and **Q**. Knowing that it has been experimentally determined that the bending moment is $+800\ \text{N}\cdot\text{m}$ at D and $+1300\ \text{N}\cdot\text{m}$ at E, (*a*) determine **P** and **Q**, (*b*) draw the shear and bending-moment diagrams for the beam.

SOLUTION

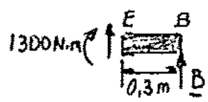


(*a*) Free body: Portion AD

$$\Sigma F_x = 0: \quad C_x = 0$$

$$+\circlearrowleft \Sigma M_D = 0: \quad -C_y(0.3\text{ m}) + 0.800\text{ kN}\cdot\text{m} + (6\text{ kN})(0.45\text{ m}) = 0$$

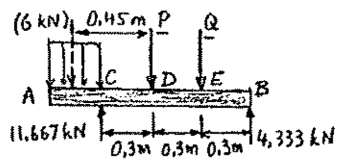
$$C_y = +11.667\text{ kN} \qquad \mathbf{C} = 11.667\text{ kN}\uparrow \triangleleft$$



Free body: Portion EB

$$+\circlearrowleft \Sigma M_E = 0: \quad B(0.3\text{ m}) - 1.300\text{ kN}\cdot\text{m} = 0$$

$$\mathbf{B} = 4.333\text{ kN}\uparrow \triangleleft$$



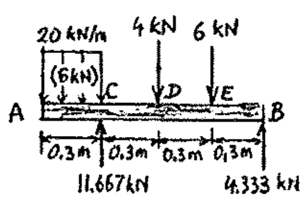
Free body: Entire beam

$$+\circlearrowleft \Sigma M_D = 0: \quad (6\text{ kN})(0.45\text{ m}) - (11.667\text{ kN})(0.3\text{ m})$$
$$-Q(0.3\text{ m}) + (4.333\text{ kN})(0.6\text{ m}) = 0$$

$$\mathbf{Q} = 6.00\text{ kN}\downarrow \blacktriangleleft$$

$$+\uparrow \Sigma M_y = 0: \quad 11.667\text{ kN} + 4.333\text{ kN}$$
$$-6\text{ kN} - P - 6\text{ kN} = 0$$

$$\mathbf{P} = 4.00\text{ kN}\downarrow \blacktriangleleft$$

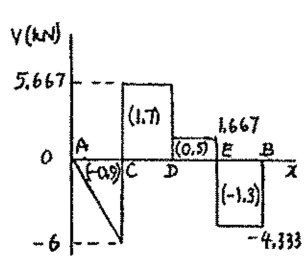


Load diagram

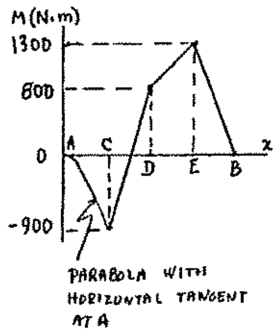
(*b*) Shear diagram

At A: $V_A = 0$

$$|V|_{max} = 6\text{ kN} \triangleleft$$



PROBLEM 7.89 (Continued)



Bending-moment diagram

At A: $M_A = 0$

$|M|_{max} = 1300\ \text{N}\cdot\text{m}$ ◁

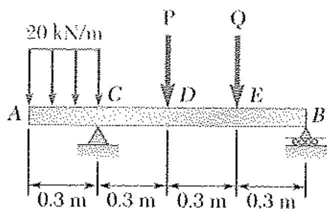
We check that

$$M_D = +800\ \text{N}\cdot\text{m} \quad \text{and} \quad M_E = +1300\ \text{N}\cdot\text{m}$$

As given:

At C: $M_C = -900\ \text{N}\cdot\text{m}$ ◀

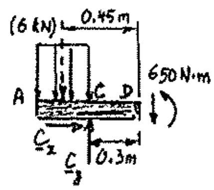
PROBLEM 7.90



Solve Problem 7.89 assuming that the bending moment was found to be +650 N·m at D and +1450 N·m at E.

PROBLEM 7.89 The beam AB is subjected to the uniformly distributed load shown and to two unknown forces **P** and **Q**. Knowing that it has been experimentally determined that the bending moment is +800 N·m at D and +1300 N·m at E, (*a*) determine **P** and **Q**, (*b*) draw the shear and bending-moment diagrams for the beam.

SOLUTION

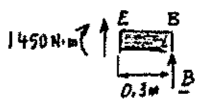


(*a*) Free body: Portion AD

$$\Sigma F_x = 0: \quad C_x = 0$$

$$+\circlearrowleft \Sigma M_D = 0: \quad -C(0.3 \text{ m}) + 0.650 \text{ kN}\cdot\text{m} + (6 \text{ kN})(0.45 \text{ m}) = 0$$

$$C_y = +11.167 \text{ kN} \qquad \mathbf{C} = 11.167 \text{ kN} \uparrow \triangleleft$$

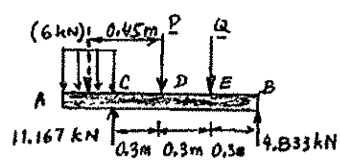


Free body: Portion EB

$$+\circlearrowleft \Sigma M_E = 0: \quad B(0.3 \text{ m}) - 1.450 \text{ kN}\cdot\text{m} = 0$$

$$\mathbf{B} = 4.833 \text{ kN} \uparrow \triangleleft$$

Free body: Entire beam

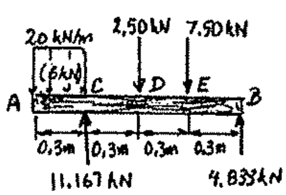


$$+\circlearrowleft \Sigma M_D = 0: \quad (6 \text{ kN})(0.45 \text{ m}) - (11.167 \text{ kN})(0.3 \text{ m}) - Q(0.3 \text{ m}) + (4.833 \text{ kN})(0.6 \text{ m}) = 0$$

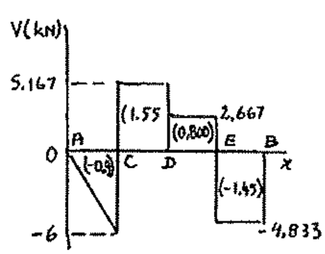
$$\mathbf{Q} = 7.50 \text{ kN} \downarrow \blacktriangleleft$$

$$+\uparrow \Sigma M_y = 0: \quad 11.167 \text{ kN} + 4.833 \text{ kN} - 6 \text{ kN} - P - 7.50 \text{ kN} = 0$$

$$\mathbf{P} = 2.50 \text{ kN} \downarrow \blacktriangleleft$$



Load diagram

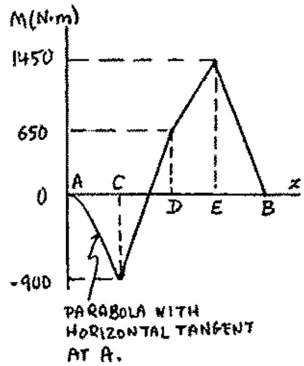


(*b*) Shear diagram

At A: $\quad V_A = 0$

$$|V|_{\max} = 6 \text{ kN} \triangleleft$$

PROBLEM 7.90 (Continued)



Bending-moment diagram

At A: $M_A = 0$

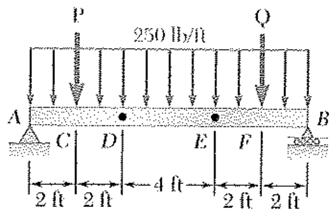
$|M|_{max} = 1450\ \text{N}\cdot\text{m}$ ◁

We check that

$$M_D = +650\ \text{N}\cdot\text{m} \quad \text{and} \quad M_E = +1450\ \text{N}\cdot\text{m}$$

As given:

At C: $M_C = -900\ \text{N}\cdot\text{m}$ ◀

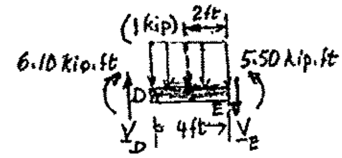


PROBLEM 7.91*

The beam AB is subjected to the uniformly distributed load shown and to two unknown forces **P** and **Q**. Knowing that it has been experimentally determined that the bending moment is +6.10 kip·ft at D and +5.50 kip·ft at E, (a) determine **P** and **Q**, (b) draw the shear and bending-moment diagrams for the beam.

SOLUTION

(a) Free body: Portion DE



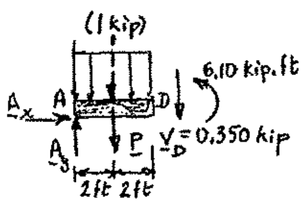
$$+\circlearrowleft \Sigma M_E = 0: \quad 5.50 \text{ kip} \cdot \text{ft} - 6.10 \text{ kip} \cdot \text{ft} + (1 \text{ kip})(2 \text{ ft}) - V_D(4 \text{ ft}) = 0$$

$$V_D = +0.350 \text{ kip}$$

$$+\uparrow \Sigma F_y = 0: \quad 0.350 \text{ kip} - 1 \text{ kip} - V_E = 0$$

$$V_E = -0.650 \text{ kip}$$

Free body: Portion AD



$$+\circlearrowleft \Sigma M_A = 0: \quad 6.10 \text{ kip} \cdot \text{ft} - P(2 \text{ ft}) - (1 \text{ kip})(2 \text{ ft}) - (0.350 \text{ kip})(4 \text{ ft}) = 0$$

P = 1.350 kips↓ ◀

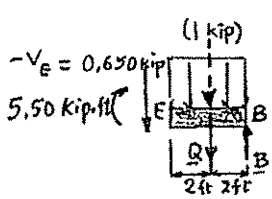
$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 1 \text{ kip} - 1.350 \text{ kip} - 0.350 \text{ kip} = 0$$

$$A_y = +2.70 \text{ kips}$$

A = 2.70 kips↑ ◀

Free body: Portion EB



$$+\circlearrowleft \Sigma M_B = 0: \quad (0.650 \text{ kip})(4 \text{ ft}) + (1 \text{ kip})(2 \text{ ft}) + Q(2 \text{ ft}) - 5.50 \text{ kip} \cdot \text{ft} = 0$$

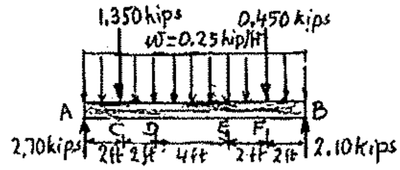
Q = 0.450 kip↓ ◀

$$+\uparrow \Sigma F_y = 0: \quad B - 0.450 - 1 - 0.650 = 0$$

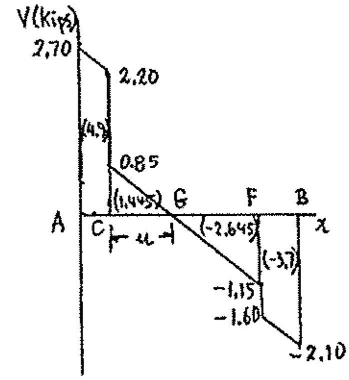
B = 2.10 kips↑ ◀

PROBLEM 7.91* (Continued)

(*b*) Load diagram



Shear diagram



At A: $V_A = A = +2.70$ kips

To determine Point G where $V = 0$, we write

$$V_G - V_C = -w\mu$$

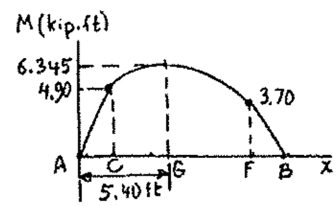
$$0 - 0.85 \text{ kips} = -(0.25 \text{ kip/ft})\mu$$

$\mu = 3.40$ ft ◀

We next compute all areas

$|V|_{max} = 2.70$ kips at A ◀

Bending-moment diagram



At A: $M_A = 0$

Largest value occurs at G with

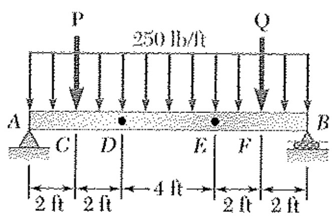
$$AG = 2 + 3.40 = 5.40 \text{ ft}$$

$|M|_{max} = 6.345$ kip·ft ◀

5.40 ft from A ◀

Bending-moment diagram consists of 3 distinct arcs of parabolas.

PROBLEM 7.92*

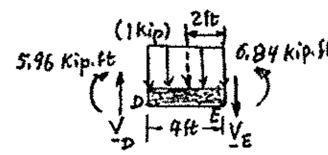


Solve Problem 7.91 assuming that the bending moment was found to be +5.96 kip·ft at D and +6.84 kip·ft at E.

PROBLEM 7.91* The beam AB is subjected to the uniformly distributed load shown and to two unknown forces **P** and **Q**. Knowing that it has been experimentally determined that the bending moment is +6.10 kip·ft at D and +5.50 kip·ft at E, (a) determine **P** and **Q**, (b) draw the shear and bending-moment diagrams for the beam.

SOLUTION

(a) Free body: Portion DE



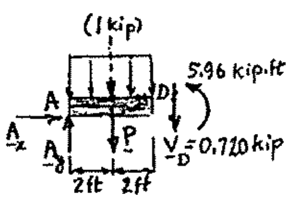
$$+\circlearrowleft \Sigma M_E = 0: \quad 6.84 \text{ kip}\cdot\text{ft} - 5.96 \text{ kip}\cdot\text{ft} + (1 \text{ kip})(2 \text{ ft}) - V_D(4 \text{ ft}) = 0$$

$$V_D = +0.720 \text{ kip}$$

$$+\uparrow \Sigma F_y = 0: \quad 0.720 \text{ kip} - 1 \text{ kip} - V_E = 0$$

$$V_E = -0.280 \text{ kip}$$

Free body: Portion AD



$$+\circlearrowleft \Sigma M_A = 0: \quad 5.96 \text{ kip}\cdot\text{ft} - P(2 \text{ ft}) - (1 \text{ kip})(2 \text{ ft}) - (0.720 \text{ kip})(4 \text{ ft}) = 0$$

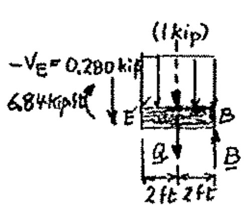
$$\mathbf{P} = 0.540 \text{ kip} \downarrow \blacktriangleleft$$

$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 1 \text{ kip} - 0.540 \text{ kip} - 0.720 \text{ kip} = 0$$

$$A_y = +2.26 \text{ kips} \qquad \mathbf{A} = 2.26 \text{ kips} \uparrow \triangleleft$$

Free body: Portion EB



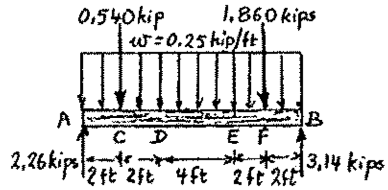
$$+\circlearrowleft \Sigma M_B = 0: \quad (0.280 \text{ kip})(4 \text{ ft}) + (1 \text{ kip})(2 \text{ ft}) + Q(2 \text{ ft}) - 6.84 \text{ kip}\cdot\text{ft} = 0$$

$$\mathbf{Q} = 1.860 \text{ kips} \downarrow \blacktriangleleft$$

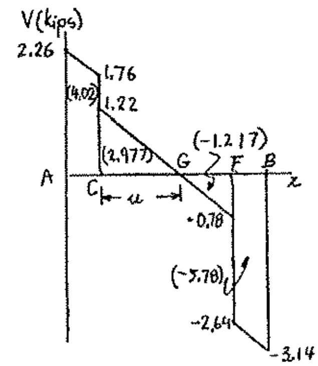
$$+\uparrow \Sigma F_y = 0: \quad B - 1.860 - 1 - 0.280 = 0$$

$$\mathbf{B} = 3.14 \text{ kips} \uparrow \blacktriangleleft$$

(*b*) Load diagram



Shear diagram



At A: $V_A = A = +2.26$ kips

To determine Point G where $V = 0$, we write

$$V_G - V_C = -w\mu$$

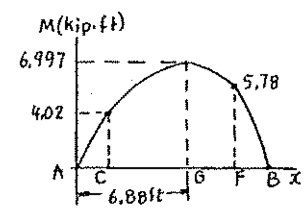
$$0 - (1.22 \text{ kips}) = -(0.25 \text{ kip/ft})\mu$$

$\mu = 4.88$ ft ◁

We next compute all areas

$|V|_{max} = 3.14$ kips at B ◀

Bending-moment diagram



At A: $M_A = 0$

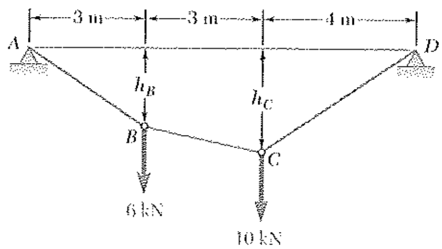
Largest value occurs at G with

$$AG = 2 + 4.88 = 6.88 \text{ ft}$$

$|M|_{max} = 6.997$ kip·ft ◀

6.88 ft from A ◀

Bending-moment diagram consists of 3 distinct arcs of parabolas.

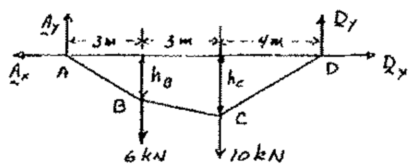


PROBLEM 7.93

Two loads are suspended as shown from the cable $ABCD$. Knowing that $h_B = 1.8$ m, determine (*a*) the distance h_C, (*b*) the components of the reaction at D, (*c*) the maximum tension in the cable.

SOLUTION

FBD Cable:



$$\rightarrow \Sigma F_x = 0: \quad -A_x + D_x = 0 \quad A_x = D_x$$

$$\circlearrowleft \Sigma M_A = 0: \quad (10 \text{ m})D_y - (6 \text{ m})(10 \text{ kN}) - (3 \text{ m})(6 \text{ kN}) = 0$$

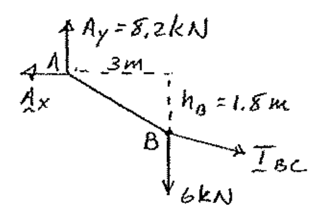
$$\mathbf{D}_y = 7.8 \text{ kN} \uparrow$$

$$\uparrow \Sigma F_y = 0: \quad A_y - 6 \text{ kN} - 10 \text{ kN} + 7.8 \text{ kN} = 0$$

$$\mathbf{A}_y = 8.2 \text{ kN} \uparrow$$

FDB AB:

$$\circlearrowleft \Sigma M_B = 0: \quad (1.8 \text{ m})A_x - (3 \text{ m})(8.2 \text{ kN}) = 0$$

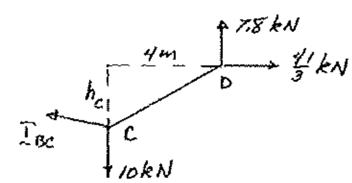


$$\mathbf{A}_x = \frac{41}{3} \text{ kN} \leftarrow$$

From above $\quad D_x = A_x = \dfrac{41}{3}$ kN

FBD CD:

$$\circlearrowleft \Sigma M_C = 0: \quad (4 \text{ m})(7.8 \text{ kN}) - h_C\left(\frac{41}{3} \text{ kN}\right) = 0$$



$$h_C = 2.283 \text{ m}$$

(*a*) $\quad h_C = 2.28$ m ◀

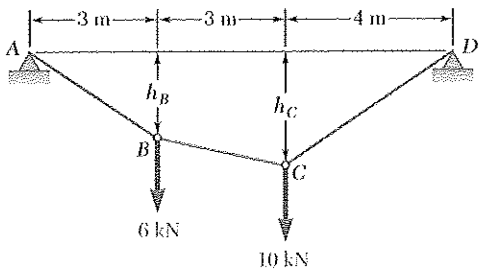
(*b*) $\quad \mathbf{D}_x = 13.67 \text{ kN} \rightarrow$ ◀

$\mathbf{D}_y = 7.80 \text{ kN} \uparrow$ ◀

Since $A_x = B_x$ and $A_y > B_y$, max T is T_{AB}

$$T_{AB} = \sqrt{A_x^2 + A_y^2} = \sqrt{\left(\frac{41}{3} \text{ kN}\right)^2 + (8.2 \text{ kN})^2}$$

(*c*) $\quad T_{max} = 15.94$ kN ◀

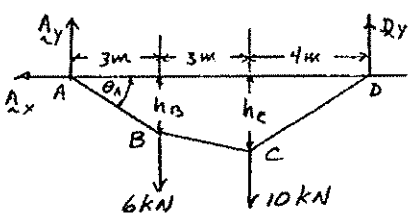


PROBLEM 7.94

Knowing that the maximum tension in cable *ABCD* is 15 kN, determine (*a*) the distance h_B, (*b*) the distance h_C.

SOLUTION

FBD Cable:



$$\rightarrow \Sigma F_x = 0: \quad -A_x + D_x = 0 \quad A_x = D_x$$

$$\circlearrowleft \Sigma M_A = 0: \quad (10 \text{ m}) D_y - (6 \text{ m})(10 \text{ kN}) - (3 \text{ m})(6 \text{ kN}) = 0$$

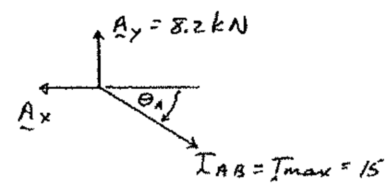
$$\mathbf{D}_y = 7.8 \text{ kN} \uparrow$$

$$\uparrow \Sigma F_y = 0: \quad A_y - 6 \text{ kN} - 10 \text{ kN} + 7.8 \text{ kN} = 0$$

$$\mathbf{A}_y = 8.2 \text{ kN} \uparrow$$

Since $\quad A_x = D_x \quad$ and $\quad A_y > D_y, \quad T_{max} = T_{AB}$

FBD Pt A:



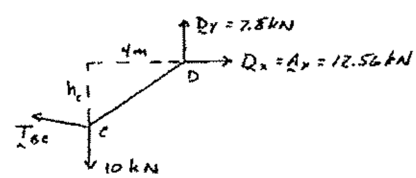
$$\uparrow \Sigma F_y = 0: \quad 8.2 \text{ kN} - (15 \text{ kN}) \sin\theta_A = 0$$

$$\theta_A = \sin^{-1} \frac{8.2 \text{ kN}}{15 \text{ kN}} = 33.139°$$

$$\rightarrow \Sigma F_x = 0: \quad -A_x + (15 \text{ kN}) \cos\theta_A = 0$$

$$A_x = (15 \text{ kN}) \cos(33.139°) = 12.56 \text{ kN}$$

FBD CD:



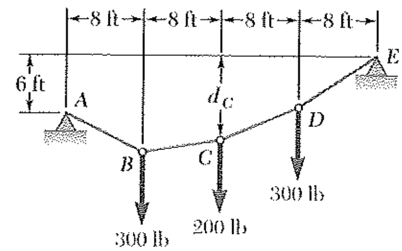
From *FBD* cable:

$$h_B = (3 \text{ m}) \tan\theta_A$$
$$= (3 \text{ m}) \tan(33.139°)$$

(*a*) $h_B = 1.959 \text{ m}$ ◀

$$\circlearrowleft \Sigma M_C = 0: \quad (4 \text{ m})(7.8 \text{ kN}) - h_C (12.56 \text{ kN}) = 0$$

(*b*) $h_C = 2.48 \text{ m}$ ◀

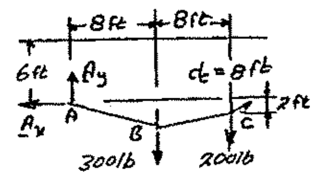


PROBLEM 7.95

If $d_C = 8$ ft, determine (*a*) the reaction at A, (*b*) the reaction at E.

SOLUTION

Free body: Portion ABC

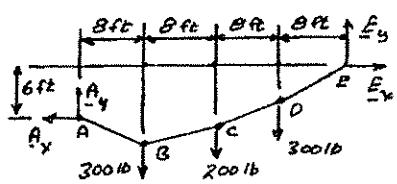


$$+\circlearrowleft \Sigma M_C = 0$$

$$2A_x - 16A_y + 300(8) = 0$$

$$A_x = 8A_y - 1200 \qquad (1)$$

Free body: Entire cable



$$+\circlearrowleft \Sigma M_E = 0: \quad +6A_x + 32A_y - (300 \text{ lb} + 200 \text{ lb} + 300 \text{ lb})16 \text{ ft} = 0$$

$$3A_x + 16A_y - 6400 = 0$$

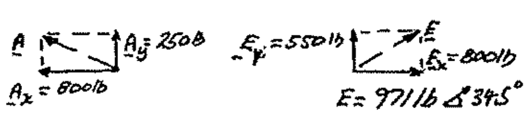
Substitute from Eq. (1):

$$3(8A_y - 1200) + 16A_y - 6400 = 0 \qquad \mathbf{A}_y = 250 \text{ lb}\uparrow$$

Eq. (1) $\quad A_x = 8(250) - 1200 \qquad \mathbf{A}_x = 800 \text{ lb} \leftarrow$

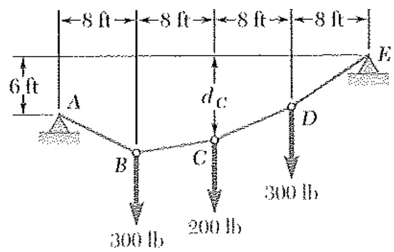
$$\xrightarrow{+} \Sigma F_x = 0: \quad -A_x + E_x = 0 \quad -800 \text{ lb} + E_x = 0 \qquad \mathbf{E}_x = 800 \text{ lb} \rightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad 250 + E_y - 300 - 200 - 300 = 0 \qquad \mathbf{E}_y = 550 \text{ lb}\uparrow$$



$\mathbf{A} = 838 \text{ lb} \searrow 17.4°$ ◀

$\mathbf{E} = 971 \text{ lb} \measuredangle 34.5°$ ◀

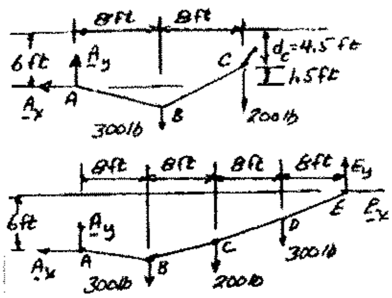


PROBLEM 7.96

If $d_C = 4.5$ ft, determine (*a*) the reaction at *A*, (*b*) the reaction at *E*.

SOLUTION

Free body: Portion *ABC*



$$+\circlearrowleft \Sigma M_C = 0: \quad -1.5A_x - 16A_y + 300 \times 8 = 0$$

$$A_x = \frac{(2400 - 16A_y)}{1.5} \qquad (1)$$

Free body: Entire cable

$$+\circlearrowleft \Sigma M_E = 0: \quad +6A_x + 32A_y - (300 \text{ lb} + 200 \text{ lb} + 300 \text{ lb})16 \text{ ft} = 0$$

$$3A_x + 16A_y - 6400 = 0$$

Substitute from Eq. (1):

$$\frac{3(2400 - 16A_y)}{1.5} + 16A_y - 6400 = 0$$

$$A_y = -100 \text{ lb}$$

Thus A_y acts downward $\qquad \mathbf{A}_y = 100 \text{ lb}\downarrow$

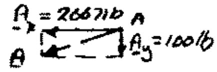
Eq. (1)

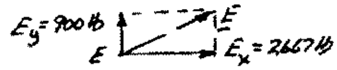
$$A_x = \frac{(2400 - 16(-100))}{1.5} = 2667 \text{ lb} \qquad \mathbf{A}_x = 2667 \text{ lb} \leftarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -A_x + E_x = 0 \quad -2667 + E_x = 0 \qquad \mathbf{E}_x = 2667 \text{ lb} \rightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + E_y - 300 - 200 - 300 = 0$$

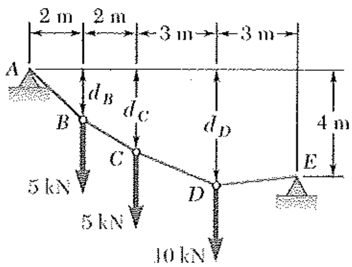
$$-100 \text{ lb} + E_y - 800 \text{ lb} = 0 \qquad \mathbf{E}_y = 900 \text{ lb}\uparrow$$





$\mathbf{A} = 2670 \text{ lb} \measuredangle 2.10°$ ◀

$\mathbf{E} = 2810 \text{ lb} \measuredangle 18.6°$ ◀

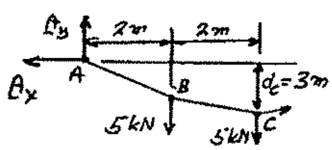


PROBLEM 7.97

Knowing that $d_C = 3$ m, determine (*a*) the distances d_B and d_D (*b*) the reaction at *E*.

SOLUTION

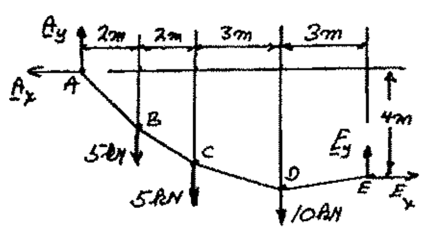
Free body: Portion *ABC*



$$+\circlearrowleft \Sigma M_C = 0: \quad 3A_x - 4A_y + (5\text{ kN})(2\text{ m}) = 0$$

$$A_x = \frac{4}{3}A_y - \frac{10}{3} \qquad (1)$$

Free body: Entire cable



$$+\circlearrowleft \Sigma M_E = 0: \quad 4A_x - 10A_y + (5\text{ kN})(8\text{ m}) + (5\text{ kN})(6\text{ m}) + (10\text{ kN})(3\text{ m}) = 0$$

$$4A_x - 10A_y + 100 = 0$$

Substitute from Eq. (1):

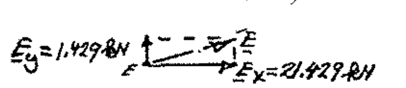
$$4\left(\frac{4}{3}A_y - \frac{10}{3}\right) - 10A_y + 100 = 0$$

$$A_y = +18.571\text{ kN} \qquad \mathbf{A}_y = 18.571\text{ kN}\uparrow$$

Eq. (1) $$A_x = \frac{4}{3}(18.511) - \frac{10}{3} = +21.429\text{ kN} \qquad \mathbf{A}_x = 21.429\text{ kN}\leftarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -A_x + E_x = 0 \quad -21.429 + E_x = 0 \qquad \mathbf{E}_x = 21.429\text{ kN}\rightarrow$$

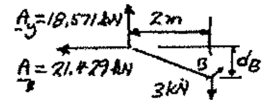
$$+\uparrow \Sigma F_y = 0: \quad 18.571\text{ kN} + E_y + 5\text{ kN} + 5\text{ kN} + 10\text{ kN} = 0 \qquad \mathbf{E}_y = 1.429\text{ kN}\uparrow$$



$$\mathbf{E} = 21.5\text{ kN} \measuredangle\ 3.81° \blacktriangleleft$$

PROBLEM 7.97 (Continued)

Portion AB

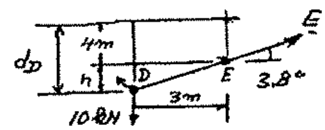


$$+\circlearrowleft \Sigma M_B = 0: \quad (18.571 \text{ kN})(2 \text{ m}) - (21.429 \text{ kN})d_B = 0$$

$d_B = 1.733 \text{ m}$ ◀

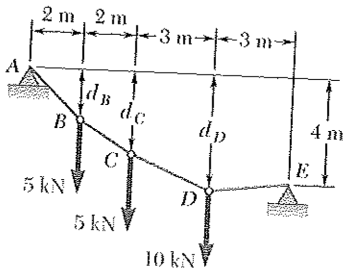
Portion DE

Geometry



$$h = (3 \text{ m})\tan 3.8°$$
$$= 0.199 \text{ m}$$
$$d_D = 4 \text{ m} + 0.199 \text{ m}$$

$d_D = 4.20 \text{ m}$ ◀

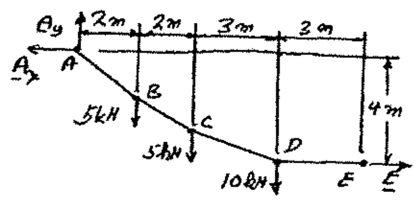


PROBLEM 7.98

Determine (*a*) distance d_C for which portion *DE* of the cable is horizontal, (*b*) the corresponding reactions at *A* and *E*.

SOLUTION

Free body: Entire cable



$+\uparrow \Sigma F_y = 0: \quad A_y - 5\text{ kN} - 5\text{ kN} - 10\text{ kN} = 0$

$\mathbf{A}_y = 20\text{ kN}\uparrow$

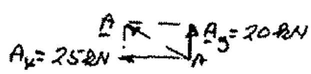
$+\circlearrowleft \Sigma M_A = 0: \quad E(4\text{ m}) - (5\text{ kN})(2\text{ m}) - (5\text{ kN})(4\text{ m}) - (10\text{ kN})(7\text{m}) = 0$

$E = +25\text{ kN}$

$\mathbf{E} = 25.0\text{ kN} \longrightarrow$ ◀

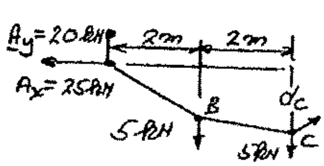
$\Sigma F_x = 0: \quad -A_x + 25\text{ kN} = 0$

$\mathbf{A}_x = 25\text{ kN} \longleftarrow$



$\mathbf{A} = 32.0\text{ kN} \searrow 38.7°$ ◀

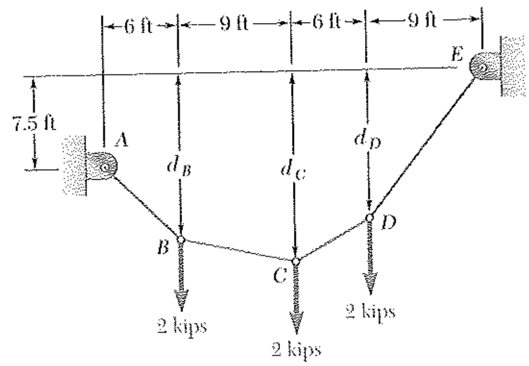
Free body: Portion *ABC*



$+\circlearrowleft \Sigma M_C = 0: \quad (25\text{ kN})d_C - (20\text{ kN})(4\text{ m}) + (5\text{ kN})(2\text{ m}) = 0$

$25d_C - 70 = 0$

$d_C = 2.80\text{ m}$ ◀

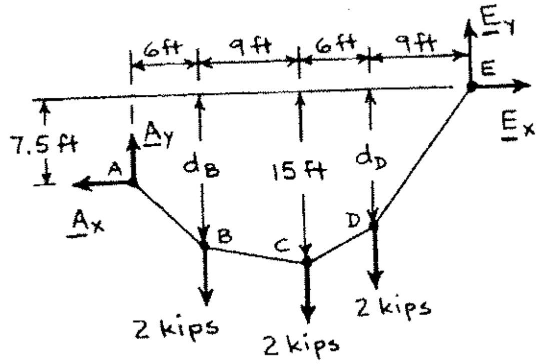


PROBLEM 7.99

If $d_C = 15$ ft, determine (*a*) the distances d_B and d_D, (*b*) the maximum tension in the cable.

SOLUTION

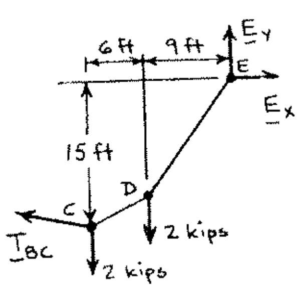
Free body: Entire cable



$$+\circlearrowleft \Sigma M_A = 0: \quad E_y(30\text{ ft}) - E_x(7.5\text{ ft}) - (2\text{ kips})(6\text{ ft}) - (2\text{ kips})(15\text{ ft}) - (2\text{ kips})(21\text{ ft}) = 0$$

$$7.5E_x - 30E_y + 84 = 0 \qquad (1)$$

Free body: Portion *CDE*



$$+\circlearrowleft \Sigma M_C = 0: \quad E_y(15\text{ ft}) - E_x(15\text{ ft}) - (2\text{ kips})(6\text{ ft}) = 0$$

$$15E_x - 15E_y + 12 = 0 \qquad (2)$$

$$\text{Eq. (1)} \times \frac{1}{2}: \quad 3.75E_x - 15E_y + 42 = 0 \qquad (3)$$

$$(2) - (3): \qquad 11.25E_x - 30 = 0$$

$$E_x = 2.6667\text{ kips}$$

Eq. (1):

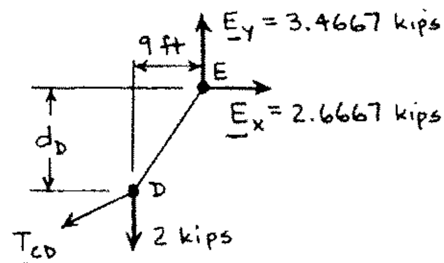
$$7.5(2.6667) - 30E_y + 84 = 0 \quad E_y = 3.4667\text{ kips}$$

$$T_m = \sqrt{E_x^2 + E_y^2} = \sqrt{(2.6667)^2 + (3.4667)^2}$$

$T_m = 4.37$ kips ◀

PROBLEM 7.99 (Continued)

Free body: Portion *DE*



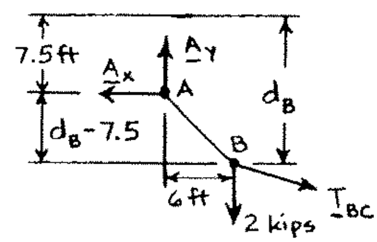
$+\circlearrowleft \Sigma M_D = 0$: $(3.4667 \text{ kips})(9 \text{ ft}) - (2.6667 \text{ kips})d_D = 0$ $\qquad d_D = 11.70 \text{ ft}$ ◀

Return to free body of entire cable (with $E_x = 2.6667$ kips, $E_y = 3.4667$ kips)

$+\uparrow \Sigma F_y = 0$: $A_y - 3(2 \text{ kips}) + 3.4667 \text{ kips} = 0$ $\qquad A_y = 2.5333$ kips

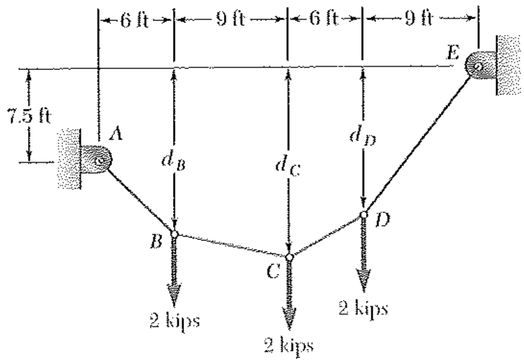
$\xrightarrow{+} \Sigma F_x = 0$: $2.6667 \text{ kips} - A_x = 0$ $\qquad A_x = 2.6667$ kips

Free body: Portion *AB*



$+\circlearrowleft \Sigma M_B = 0$: $A_x(d_B - 7.5) - A_y(6) = 0$

$(2.6667)(d_B - 7.5) - (2.5333)(6) = 0$ $\qquad d_B = 13.20 \text{ ft}$ ◀

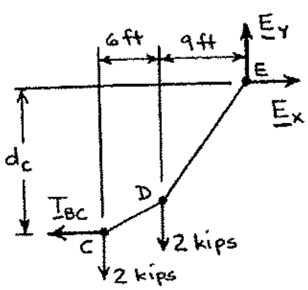


PROBLEM 7.100

Determine (*a*) the distance d_C for which portion *BC* of the cable is horizontal, (*b*) the corresponding components of the reaction at *E*.

SOLUTION

Free body: Portion *CDE*

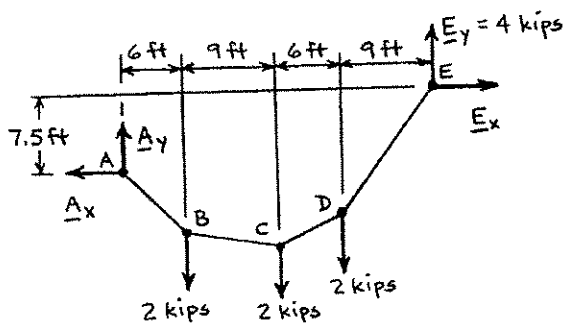


$$+\uparrow \Sigma F_y = 0: \quad E_y - 2(2\text{ kips}) = 0 \quad E_y = 4\text{ kips}$$

$$+\circlearrowleft \Sigma M_C = 0: \quad (4\text{ kips})(15\text{ ft}) - E_x d_C - (2\text{ kips})(6\text{ ft}) = 0$$

$$E_x d_C = 48\text{ kip}\cdot\text{ft} \qquad (1)$$

Free body: Entire cable

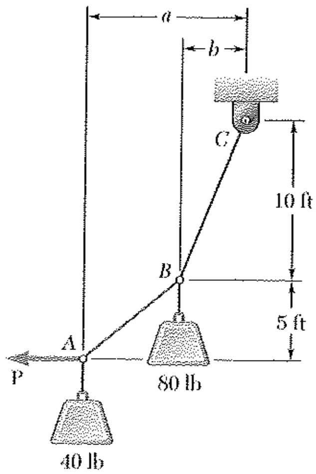


$$+\circlearrowleft \Sigma M_A = 0: \quad (4\text{ kips})(30\text{ ft}) - E_x(7.5\text{ ft}) - (2\text{ kips})(6\text{ ft}) - (2\text{ kips})(15\text{ ft}) - (2\text{ kips})(21\text{ ft}) = 0$$

$$E_x = 4.8\text{ kips}$$

From Eq. (1): $d_C = \dfrac{48}{E_x} = \dfrac{48}{4.8}$ $\qquad d_C = 10.00\text{ ft}$ ◀

Components of reaction at *E*: $\mathbf{E}_x = 4.80\text{ kips} \longrightarrow; \quad E_y = 4.00\text{ kips}\uparrow$ ◀

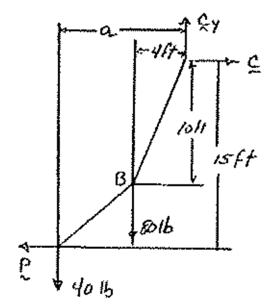


PROBLEM 7.101

Cable ABC supports two loads as shown. Knowing that $b = 4$ ft, determine (a) the required magnitude of the horizontal force **P**, (b) the corresponding distance a.

SOLUTION

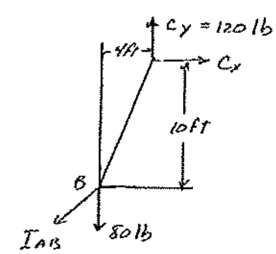
FBD ABC:



$$\uparrow \Sigma F_y = 0: \quad -40 \text{ lb} - 80 \text{ lb} + C_y = 0$$

$$\mathbf{C}_y = 120 \text{ lb} \uparrow$$

FBD BC:



$$\circlearrowleft \Sigma M_B = 0: \quad (4 \text{ ft})(120 \text{ lb}) - (10 \text{ ft})C_x = 0$$

$$\mathbf{C}_x = 48 \text{ lb} \longrightarrow$$

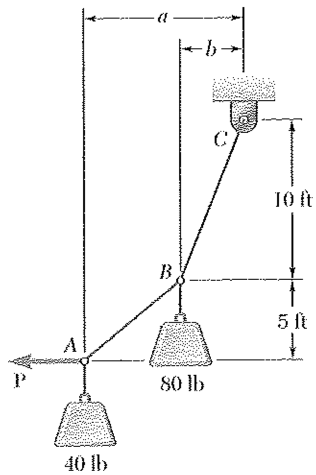
From ABC: $\quad \longrightarrow \Sigma F_x = 0: \quad -P + C_x = 0$

$$P = C_x = 48 \text{ lb}$$

(a) $\quad P = 48.0$ lb ◀

$$\circlearrowleft \Sigma M_C = 0: \quad (4 \text{ ft})(80 \text{ lb}) + a(40 \text{ lb}) - (15 \text{ ft})(48 \text{ lb}) = 0$$

(b) $\quad a = 10.00$ ft ◀

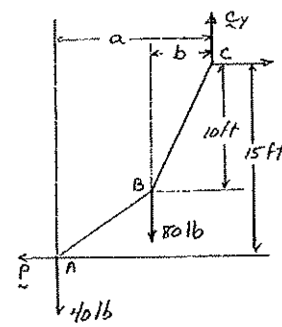


PROBLEM 7.102

Cable ABC supports two loads as shown. Determine the distances a and b when a horizontal force $\mathbf{P}$ of magnitude 60 lb is applied at A.

SOLUTION

FBD ABC:

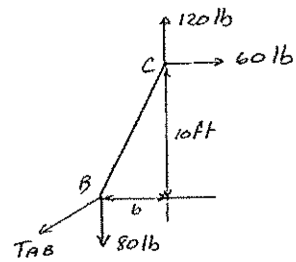


$$\xrightarrow{} \Sigma F_x = 0: \quad C_x - P = 0 \qquad C_x = 60 \text{ lb} \longrightarrow$$

$$\uparrow \Sigma F_y = 0: \quad C_y - 40 \text{ lb} - 80 \text{ lb} = 0$$

$$\mathbf{C}_y = 120 \text{ lb} \uparrow$$

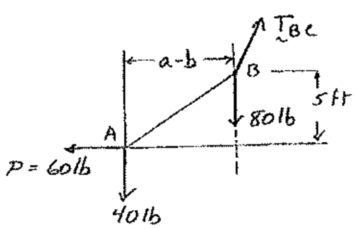
FBD BC:



$$\circlearrowleft \Sigma M_B = 0: \quad b(120 \text{ lb}) - (10 \text{ ft})(60 \text{ lb}) = 0$$

$b = 5.00$ ft ◀

FBD AB:



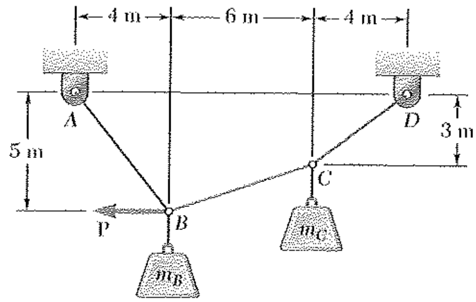
$$\Sigma M_B = 0: \quad (a - b)(40 \text{ lb}) - (5 \text{ ft})60 \text{ lb} = 0$$

$$a - b = 7.5 \text{ ft}$$

$$a = b + 7.5 \text{ ft}$$

$$= 5 \text{ ft} + 7.5 \text{ ft}$$

$a = 12.50$ ft ◀

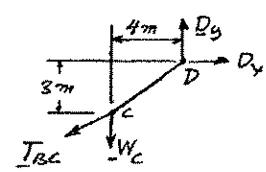


PROBLEM 7.103

Knowing that $m_B = 70$ kg and $m_C = 25$ kg, determine the magnitude of the force **P** required to maintain equilibrium.

SOLUTION

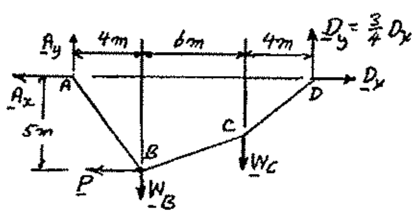
Free body: Portion CD



$$+\circlearrowleft \Sigma M_C = 0: \quad D_y(4 \text{ m}) - D_x(3 \text{ m}) = 0$$

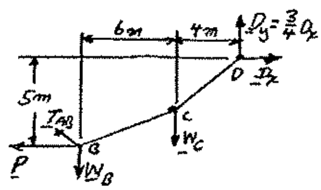
$$D_y = \frac{3}{4} D_x$$

Free body: Entire cable



$$+\circlearrowleft \Sigma M_A = 0: \quad \frac{3}{4} D_x(14 \text{ m}) - W_B(4 \text{ m}) - W_C(10 \text{ m}) - P(5 \text{ m}) = 0 \qquad (1)$$

Free body: Portion BCD



$$+\circlearrowleft \Sigma M_B = 0: \quad \frac{3}{4} D_x(10 \text{ m}) - D_x(5 \text{ m}) - W_C(6 \text{ m}) = 0$$

$$D_x = 2.4 W_C \qquad (2)$$

For $\quad m_B = 70 \text{ kg} \quad m_C = 25 \text{ kg}$

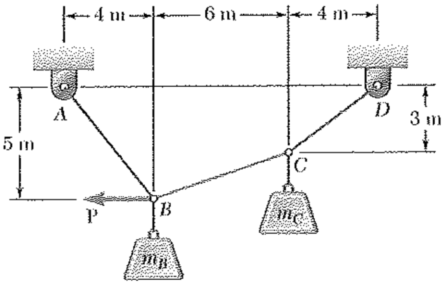
$g = 9.81 \text{ m/s}^2$: $\quad W_B = 70g \quad W_C = 25g$

Eq. (2): $\quad D_x = 2.4 W_C = 2.4(25g) = 60g$

Eq. (1): $\quad \frac{3}{4} 60g(14) - 70g(4) - 25g(10) - 5P = 0$

$$100g - 5P = 0: \quad P = 20g$$

$$P = 20(9.81) = 196.2 \text{ N} \qquad P = 196.2 \text{ N} \blacktriangleleft$$



PROBLEM 7.104

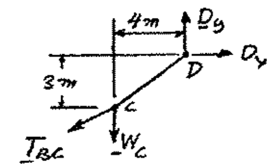
Knowing that $m_B = 18$ kg and $m_C = 10$ kg, determine the magnitude of the force **P** required to maintain equilibrium.

SOLUTION

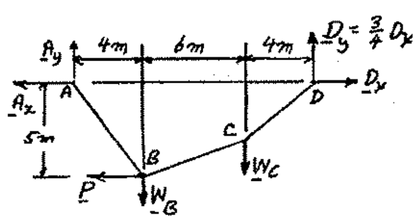
Free body: Portion CD

$$+\circlearrowleft \Sigma M_C = 0: \quad D_y(4\text{ m}) - D_x(3\text{ m}) = 0$$

$$D_y = \frac{3}{4} D_x$$

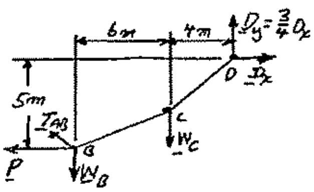


Free body: Entire cable



$$+\circlearrowleft \Sigma M_A = 0: \quad \frac{3}{4} D_x(14\text{ m}) - W_B(4\text{ m}) - W_C(10\text{ m}) - P(5\text{ m}) = 0 \qquad (1)$$

Free body: Portion BCD



$$+\circlearrowleft \Sigma M_B = 0: \quad \frac{3}{4} D_x(10\text{ m}) - D_x(5\text{ m}) - W_C(6\text{ m}) = 0$$

$$D_x = 2.4 W_C \qquad (2)$$

For $\quad m_B = 18\text{ kg} \quad m_C = 10\text{ kg}$

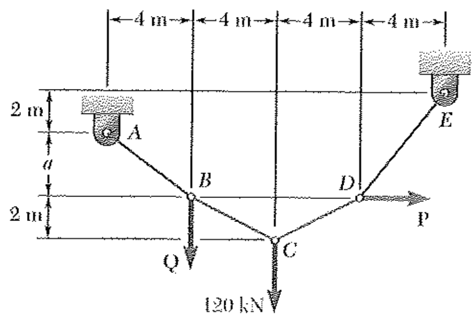
$g = 9.81\text{ m/s}^2$: $\quad W_B = 18g \quad W_C = 10g$

Eq. (2): $\quad D_x = 2.4W_C = 2.4(10g) = 24g$

Eq. (1): $\quad \frac{3}{4} 24g(14) - (18g)(4) - (10g)(10) - 5P = 0$

$80g - 5P$: $\quad P = 16g$

$P = 16(9.81) = 156.96$ N $\qquad P = 157.0$ N ◀

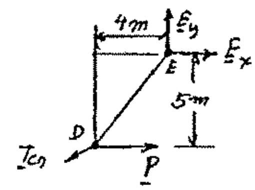


PROBLEM 7.105

If $a = 3$ m, determine the magnitudes of **P** and **Q** required to maintain the cable in the shape shown.

SOLUTION

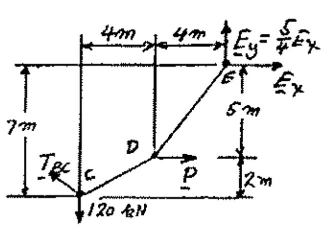
Free body: Portion DE



$$+\circlearrowleft \Sigma M_D = 0: \quad E_y(4\text{ m}) - E_x(5\text{ m}) = 0$$

$$E_y = \frac{5}{4}E_x$$

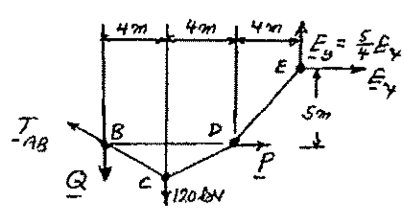
Free body: Portion CDE



$$+\circlearrowleft \Sigma M_C = 0: \quad \frac{5}{4}E_x(8\text{ m}) - E_x(7\text{ m}) - P(2\text{ m}) = 0$$

$$E_x = \frac{2}{3}P \qquad (1)$$

Free body: Portion $BCDE$

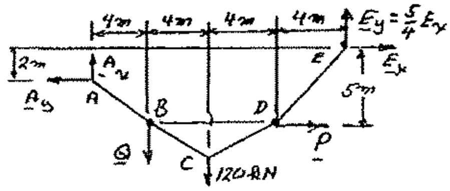


$$+\circlearrowleft \Sigma M_B = 0: \quad \frac{5}{4}E_x(12\text{ m}) - E_x(5\text{ m}) - (120\text{ kN})(4\text{ m}) = 0$$

$$10E_x - 480 = 0; \quad E_x = 48\text{ kN}$$

Eq. (1): $$48\text{ kN} = \frac{2}{3}P$$ $P = 72.0$ kN ◀

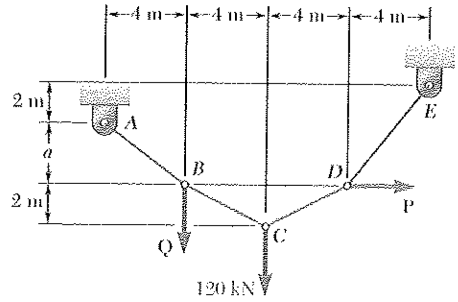
Free body: Entire cable



$$+\circlearrowleft \Sigma M_A = 0: \quad \frac{5}{4}E_x(16 \text{ m}) - E_x(2 \text{ m}) + P(3 \text{ m}) - Q(4 \text{ m}) - (120 \text{ kN})(8 \text{ m}) = 0$$

$$(48 \text{ kN})(20 \text{ m} - 2 \text{ m}) + (72 \text{ kN})(3 \text{ m}) - Q(4 \text{ m}) - 960 \text{ kN} \cdot \text{m} = 0$$

$$4Q = 120 \qquad Q = 30.0 \text{ kN} \blacktriangleleft$$

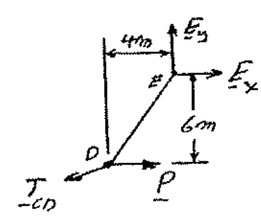


PROBLEM 7.106

If $a = 4$ m, determine the magnitudes of **P** and **Q** required to maintain the cable in the shape shown.

SOLUTION

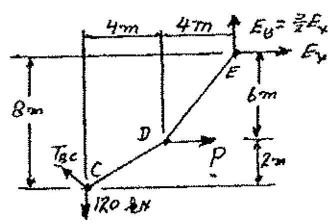
Free body: Portion *DE*



$$+\circlearrowleft \Sigma M_D = 0: \quad E_y(4 \text{ m}) - E_x(6 \text{ m}) = 0$$

$$E_y = \frac{3}{2}E_x$$

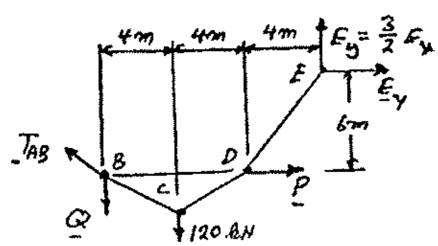
Free body: Portion *CDE*



$$+\circlearrowleft \Sigma M_C = 0: \quad \frac{3}{2}E_x(8 \text{ m}) - E_x(8 \text{ m}) - P(2 \text{ m}) = 0$$

$$E_x = \frac{1}{2}P \qquad (1)$$

Free body: Portion *BCDE*



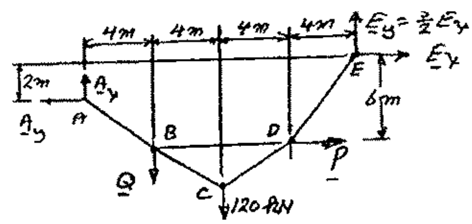
$$+\circlearrowleft \Sigma M_B = 0: \quad \frac{3}{2}E_x(12 \text{ m}) - E_x(6 \text{ m}) + (120 \text{ kN})(4 \text{ m}) = 0$$

$$12E_x = 480 \qquad E_x = 40 \text{ kN}$$

Eq (1): $\quad E_x = \frac{1}{2}P; \qquad 40 \text{ kN} = \frac{1}{2}P \qquad\qquad P = 80.0 \text{ kN}$ ◀

PROBLEM 7.106 (Continued)

Free body: Entire cable



$$+\circlearrowleft \Sigma M_A = 0:\ \ \frac{3}{2}E_x(16\text{ m}) - E_x(2\text{ m}) + P(4\text{ m}) - Q(4\text{ m}) - (120\text{ kN})(8\text{ m}) = 0$$

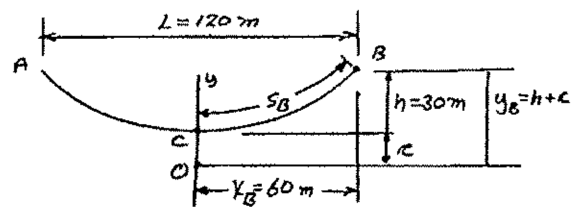
$$(40\text{ kN})(24\text{ m} - 2\text{ m}) + (80\text{ kN})(4\text{ m}) - Q(4\text{ m}) - 960\text{ kN}\cdot\text{m} = 0$$

$$4Q = 240 \qquad Q = 60.0\text{ kN} \blacktriangleleft$$

PROBLEM 7.107

A wire having a mass per unit length of 0.65 kg/m is suspended from two supports at the same elevation that are 120 m apart. If the sag is 30 m, determine (*a*) the total length of the wire, (*b*) the maximum tension in the wire.

SOLUTION



Eq. 7.16:

$$y_B = c\cosh\frac{x_B}{c}$$

$$30m + c = c\cosh\frac{60}{c}$$

Solve by trial and error: $c = 64.459 \text{ m}$

Eq. 7.15:

$$s_B = c\sin h\frac{x_B}{c}$$

$$s_B = (64.456 \text{ m})\sinh\frac{60 \text{ m}}{64.459 \text{ m}}$$

$$s_B = 69.0478 \text{ m}$$

Length $= 2s_B = 2(69.0478 \text{ m}) = 138.0956 \text{ m}$ $L = 138.1 \text{ m}$ ◀

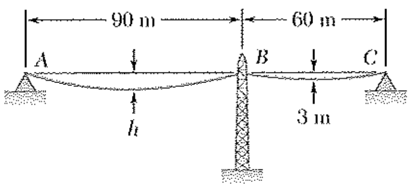
Eq. 7.18:

$$T_m = wy_B = w(h + c)$$

$$= (0.65 \text{ kg/m})(9.81 \text{ m/s}^2)(30 \text{ m} + 64.459 \text{ m})$$

$$T_m = 602.32 \text{ N}$$

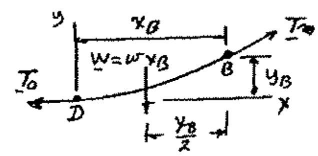
$T_m = 602 \text{ N}$ ◀



PROBLEM 7.108

Two cables of the same gauge are attached to a transmission tower at B. Since the tower is slender, the horizontal component of the resultant of the forces exerted by the cables at B is to be zero. Knowing that the mass per unit length of the cables is 0.4 kg/m, determine (*a*) the required sag h, (*b*) the maximum tension in each cable.

SOLUTION



$$W = wx_B$$

$$+\circlearrowleft \Sigma M_B = 0: \quad T_0 \; y_B - (wx_B)\frac{y_B}{2} = 0$$

(*a*) $\qquad$ Horiz. comp. $= T_0 = \dfrac{wx_B^2}{2y_B}$

<u>Cable *AB*</u> $\qquad x_B = 45 \text{ m}$

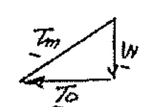
$$T_0 = \frac{w(45 \text{ m})^2}{2h}$$

<u>Cable *BC*</u> $\qquad x_B = 30 \text{ m}, \quad y_B = 3 \text{ m}$

$$T_0 = \frac{w(30 \text{ m})^2}{2(3 \text{ m})}$$

Equate $T_0 = T_0$ $\qquad \dfrac{w(45 \text{ m})^2}{2h} = \dfrac{w(30 \text{ m})^2}{2(3 \text{ m})}$ $\qquad$ $h = 6.75 \text{ m}$ ◀

(*b*) $\qquad T_m^2 = T_0^2 + W^2$



<u>Cable *AB*</u>: $\qquad w = (0.4 \text{ kg/m})(9.81 \text{ m/s}) = 3.924 \text{ N/m}$

$$x_B = 45 \text{ m}, \quad y_B = h = 6.75 \text{ m}$$

$$T_0 = \frac{wx_B^2}{2y_B} = \frac{(3.924 \text{ N/m})(45 \text{ m})^2}{2(6.75 \text{ m})} = 588.6 \text{ N}$$

$$W = wx_B = (3.924 \text{ N/m})(45 \text{ m}) = 176.58 \text{ N}$$

$$T_m^2 = (588.6 \text{ N})^2 + (176.58 \text{ N})^2$$

<u>For *AB*</u>: $\qquad$ $T_m = 615 \text{ N}$ ◀

Cable *BC* $\quad x_B = 30 \text{ m}, \quad y_B = 3 \text{ m}$

$$T_0 = \frac{wx_B^2}{2y_B} = \frac{(3.924 \text{ N/m})(30 \text{ m})^2}{2(3 \text{ m})} = 588.6 \text{ N} \quad \text{(Checks)}$$

$$W = wx_B = (3.924 \text{ N/m})(30 \text{ m}) = 117.72 \text{ N}$$

$$T_m^2 = (588.6 \text{ N})^2 + (117.72 \text{ N})^2$$

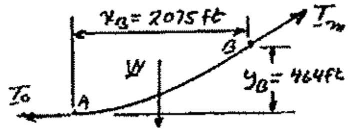
For *BC* $\quad T_m = 600 \text{ N}$ ◀

PROBLEM 7.109

Each cable of the Golden Gate Bridge supports a load $w = 11.1$ kips/ft along the horizontal. Knowing that the span L is 4150 ft and that the sag h is 464 ft, determine (*a*) the maximum tension in each cable, (*b*) the length of each cable.

SOLUTION

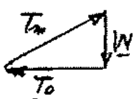
Eq. (7.8) Page 386:



At *B*: $$y_B = \frac{wx_B^2}{2T_0}$$

$$T_0 = \frac{wx_B^2}{2y_B} = \frac{(11.1\text{ kip/ft})(2075\text{ ft})^2}{2(464\text{ ft})}$$

(*a*)



$$T_0 = 51.500\text{ kips}$$

$$W = wx_B = (11.1\text{ kips/ft})(2075\text{ ft}) = 23.033\text{ kips}$$

$$T_m = \sqrt{T_0^2 + W^2} = \sqrt{(51.500\text{ kips})^2 + (23.033\text{ kips})^2}$$

$T_m = 56,400$ kips ◀

(*b*) $$s_B = x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2 - \frac{2}{5}\left(\frac{y_B}{y_B}\right)^4 + \cdots\right] \qquad \frac{y_B}{x_B} = \frac{464\text{ ft}}{2075\text{ ft}} = 0.22361$$

$$s_B = (2075\text{ ft})\left[1 + \frac{2}{3}(0.22361)^2 - \frac{2}{5}(0.22361)^4 + \cdots\right] = 2142.1\text{ ft}$$

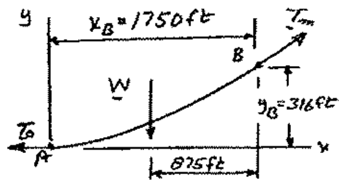
$\text{Length} = 2s_B = 2(2142.1\text{ ft})$

Length $= 4280$ ft ◀

PROBLEM 7.110

The center span of the George Washington Bridge, as originally constructed, consisted of a uniform roadway suspended from four cables. The uniform load supported by each cable was $w = 9.75$ kips/ft along the horizontal. Knowing that the span L is 3500 ft and that the sag h is 316 ft, determine for the original configuration (*a*) the maximum tension in each cable, (*b*) the length of each cable.

SOLUTION



$$W = wx_B = (9.75 \text{ kips/ft})(1750 \text{ ft})$$

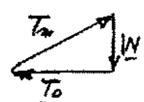
$$W = 17{,}063 \text{ kips}$$

$$+\circlearrowleft \Sigma M_B = 0: \quad T_0(316 \text{ ft}) - (17063 \text{ kips})(875 \text{ ft}) = c$$

$$T_0 = 47{,}247 \text{ kips}$$

(*a*)

$$T_m = \sqrt{T_0^2 + W^2}$$



$$= \sqrt{(47{,}247 \text{ kips})^2 + (17{,}063 \text{ kips})^2}$$

$$T_m = 50{,}200 \text{ kips} \blacktriangleleft$$

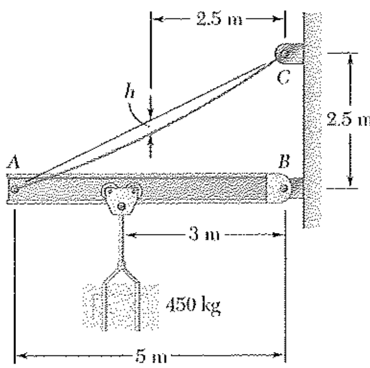
(*b*)

$$s_B = x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2 - \frac{2}{5}\left(\frac{y_B}{x_B}\right)^4 + \cdots\right]$$

$$\frac{y_B}{x_B} = \frac{316 \text{ ft}}{1750 \text{ ft}} = 0.18057$$

$$= (1750 \text{ ft})\left[1 + \frac{2}{3}(0.18057)^2 - \frac{2}{5}(0.18057)^4 + \cdots\right]$$

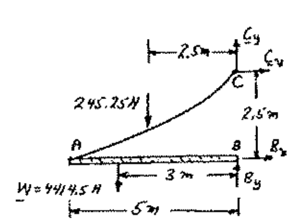
$$s_B = 1787.3 \text{ ft}; \text{ Length} = 2s_B = 3579.6 \text{ ft} \qquad \text{Length} = 3580 \text{ ft} \blacktriangleleft$$

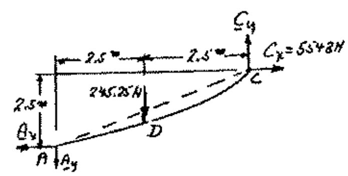


PROBLEM 7.111

The total mass of cable AC is 25 kg. Assuming that the mass of the cable is distributed uniformly along the horizontal, determine the sag h and the slope of the cable at A and C.

SOLUTION





Cable: $m = 25$ kg

$W = 25(9.81)$

$= 245.25$ N

Block: $m = 450$ kg

$W = 4414.5$ N

$+\circlearrowleft \Sigma M_B = 0: \quad (245.25)(2.5) + (4414.5)(3) - C_x(2.5) = 0$

$C_x = 5543$ N

$\Sigma F_x = 0: \quad A_x = C_x = 5543$ N

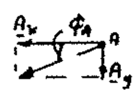
$+\circlearrowleft \Sigma M_A = 0: \quad C_y(5) - (5543)(2.5) - (245.25)(2.5) = 0$

$\mathbf{C}_y = 2894 \text{ N}\uparrow$

$+\uparrow \Sigma F_y = 0: \quad C_y - A_y - 245.25\text{N} = 0$

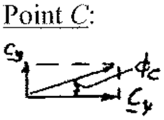
$2894\ N - A_y - 245.25 \text{ N} = 0 \qquad \mathbf{A}_y = 2649 \text{ N}\downarrow$

Point A:



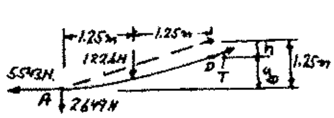
$\tan\phi_A = \dfrac{A_y}{A_x} = \dfrac{2649}{5543} = 0.4779; \qquad \phi_A = 25.5° ◀$

Point C:



$\tan\phi_C = \dfrac{C_y}{C_x} = \dfrac{2894}{5543} = 0.5221; \qquad \phi_C = 27.6° ◀$

Free body: Half cable $\qquad W = (12.5 \text{ kg})g = 122.6$



$+\circlearrowleft \Sigma M_0 = 0: \quad (122.6 \text{ N})(1.25 \text{ M}) + (2649 \text{ N})(2.5 \text{ m}) - (5543 \text{ N})y_d = 0$

$y_d = 1.2224$ m; sag $= h = 1.25 \text{ m} - 1.2224$ m

$h = 0.0276 \text{ m} = 27.6 \text{ mm}$ ◀

PROBLEM 7.112

A 50.5-m length of wire having a mass per unit length of 0.75 kg/m is used to span a horizontal distance of 50 m. Determine (*a*) the approximate sag of the wire, (*b*) the maximum tension in the wire. [*Hint:* Use only the first two terms of Eq. (7.10).]

SOLUTION

First two terms of Eq. 7.10

(*a*)

$$s_B = \frac{1}{2}(50.5\text{ m}) = 25.25\text{ m},$$

$$x_B = \frac{1}{2}(50\text{ m}) = 25\text{ m}$$

$$y_B = h$$

$$s_B = x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2\right]$$

$$25.25\text{ m} = 25\text{ m}\left[1 - \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2\right]$$

$$\left(\frac{y_B}{x_B}\right)^2 = 0.01\left(\frac{3}{2}\right)^2 = \sqrt{0.015}$$

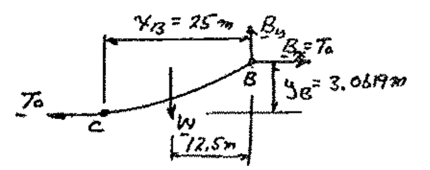
$$\frac{y_B}{x_B} = 0.12247$$

$$\frac{h}{25\text{m}} = 0.12247$$

$$h = 3.0619\text{ m}$$

$h = 3.06$ m ◁

(*b*) Free body: Portion *CB*



$$w = (0.75\text{ kg/m})(9.81\text{m}) = 7.3575\text{ N/m}$$

$$W = s_B\, w = (25.25\text{ m})(7.3575\text{ N/m})$$

$$W = 185.78\text{ N}$$

$$+\circlearrowleft \Sigma M_0 = 0: \quad T_0(3.0619\text{ m}) - (185.78\text{ N})(12.5\text{ m}) = 0$$

$$T_0 = 758.4\text{ N}$$

$$B_x = T_0 = 758.4\text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad B_y - 185.78\text{ N} = 0 \quad B_y = 185.78\text{ N}$$

$$T_m = \sqrt{B_x^2 + B_y^2} = \sqrt{(758.4\text{ N})^2 + (185.78\text{ N})^2}$$

$T_m = 781$ N ◁

PROBLEM 7.113

A cable of length $L + \Delta$ is suspended between two points that are at the same elevation and a distance L apart. (*a*) Assuming that Δ is small compared to L and that the cable is parabolic, determine the approximate sag in terms of L and Δ. (*b*) If $L = 100$ ft and $\Delta = 4$ ft, determine the approximate sag. [*Hint:* Use only the first two terms of Eq. (7.10).

SOLUTION

Eq. 7.10 (First two terms)

(*a*)

$$s_B = x_B\left[1+\frac{2}{3}\left(\frac{y_B}{x_B}\right)^2\right]$$

$$x_B = L/2$$

$$s_B = \frac{1}{2}(L+\Delta)$$

$$y_B = h$$

$$\frac{1}{2}(L+\Delta) = \frac{L}{2}\left[1+\frac{2}{3}\left(\frac{h}{\frac{L}{2}}\right)^2\right]$$

$$\frac{\Delta}{2} = \frac{4}{3}\frac{h^2}{L}; \quad h^2 = \frac{3}{8}L\Delta; \qquad h = \sqrt{\frac{3}{8}L\Delta} \blacktriangleleft$$

(*b*)

$$L = 100\text{ ft}, \quad h = 4\text{ ft}. \quad h = \sqrt{\frac{3}{8}(100)(4)}; \qquad h = 12.25\text{ ft} \blacktriangleleft$$

PROBLEM 7.114

The center span of the Verrazano-Narrows Bridge consists of two uniform roadways suspended from four cables. The design of the bridge allows for the effect of extreme temperature changes that cause the sag of the center span to vary from $h_w = 386$ ft in winter to $h_s = 394$ ft in summer. Knowing that the span is $L = 4260$ ft, determine the change in length of the cables due to extreme temperature changes.

SOLUTION

Eq. 7.10.

$$s_B = x_B\left[1+\frac{2}{3}\left(\frac{y_B}{x_B}\right)^2 - \frac{2}{5}\left(\frac{y_B}{x_B}\right)^4 + \cdots\right]$$

Winter:

$$y_B = h = 386 \text{ ft}, \quad x_B = \frac{1}{2}L = 2130 \text{ ft}$$

$$s_B = (2130)\left[1+\frac{2}{3}\left(\frac{386}{2130}\right)^2 - \frac{2}{5}\left(\frac{386}{2130}\right)^4 + \cdots\right] = 2175.715 \text{ ft}$$

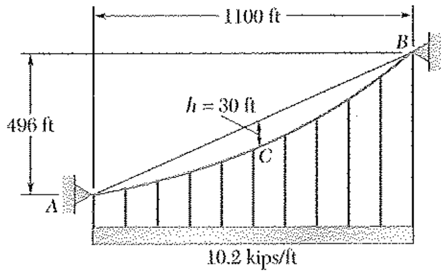
Summer:

$$y_B = h = 394 \text{ ft}, \quad x_B = \frac{1}{2}L = 2130 \text{ ft}$$

$$s_B = (2130)\left[1+\frac{2}{3}\left(\frac{394}{2130}\right)^2 - \frac{2}{5}\left(\frac{394}{2130}\right)^4 + \cdots\right] = 2177.59 \text{ ft}$$

$$\Delta = 2(\Delta s_B) = 2(2177.59 \text{ ft} - 2175.715 \text{ ft}) = 2(1.875 \text{ ft})$$

Change in length $= 3.75$ ft ◀

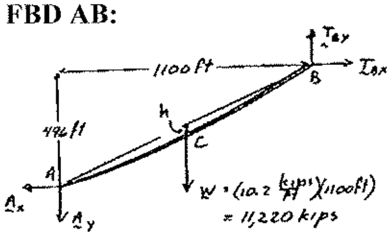


PROBLEM 7.115

Each cable of the side spans of the Golden Gate Bridge supports a load $w = 10.2$ kips/ft along the horizontal. Knowing that for the side spans the maximum vertical distance h from each cable to the chord AB is 30 ft and occurs at midspan, determine (*a*) the maximum tension in each cable, (*b*) the slope at B.

SOLUTION

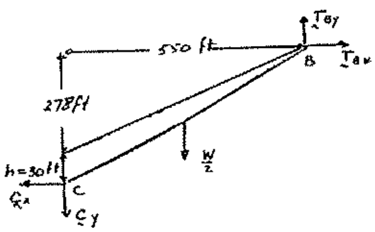
FBD AB:



$$\circlearrowleft \Sigma M_A = 0: \quad (1100\text{ ft})T_{By} - (496\text{ ft})T_{Bx} - (550\text{ ft})W = 0$$

$$11T_{By} - 4.96T_{Bx} = 5.5W \qquad (1)$$

FBD CB:



$$\circlearrowleft \Sigma M_C = 0: \quad (550\text{ ft})T_{By} - (278\text{ ft})T_{Bx} - (275\text{ ft})\frac{W}{2} = 0$$

$$11T_{By} - 5.56T_{Bx} = 2.75W \qquad (2)$$

Solving (1) and (2) $\quad T_{By} = 28,798$ kips

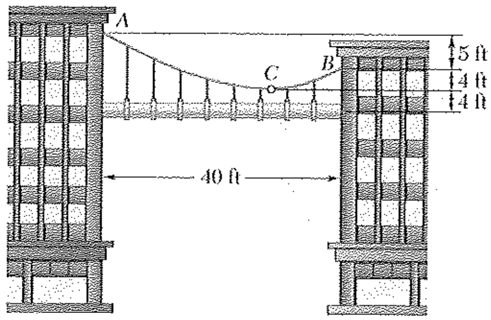
$$T_{Bx} = 51,425 \text{ kips}$$

$$T_{max} = T_B = \sqrt{T_{B_x}^2 + T_{B_y}^2} \qquad \tan\theta_B = \frac{T_{B_y}}{T_{B_x}}$$

So that

(*a*) $T_{max} = 58,940$ kips ◀

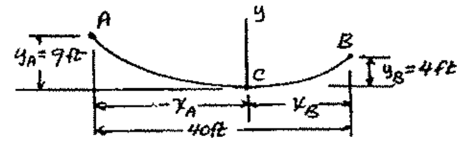
(*b*) $\theta_B = 29.2°$ ◀



PROBLEM 7.116

A steam pipe weighting 45 lb/ft that passes between two buildings 40 ft apart is supported by a system of cables as shown. Assuming that the weight of the cable system is equivalent to a uniformly distributed loading of 5 lb/ft, determine (*a*) the location of the lowest Point *C* of the cable, (*b*) the maximum tension in the cable.

SOLUTION



Note: $x_B - x_A = 40 \text{ ft}$

or $x_A = x_B - 40 \text{ ft}$

(*a*) Use Eq. 7.8

Point *A*: $y_A = \dfrac{wx_A^2}{2T_0}; \quad 9 = \dfrac{w(x_B - 40)^2}{2T_0}$ (1)

Point *B*: $y_B = \dfrac{wx_B^2}{2T_0}; \quad 4 = \dfrac{wx_B^2}{2T_0}$ (2)

Dividing (1) by (2): $\dfrac{9}{4} = \dfrac{(x_B - 40)^2}{x_B^2}; \quad x_B = 16 \text{ ft}$

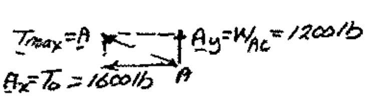
Point *C* is 16 ft to left of *B* ◀

(*b*) Maximum slope and thus $T_{\max}$ is at *A*

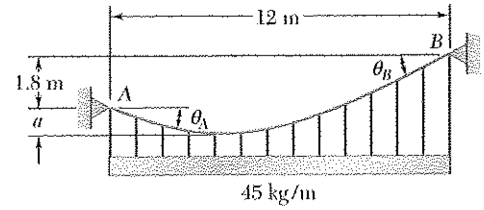
$$x_A = x_B - 40 = 16 - 40 = -24 \text{ ft}$$

$$y_A = \frac{wx_A^2}{2T_0}; \quad 9 \text{ ft} = \frac{(50 \text{ lb/ft})(-24 \text{ ft})^2}{2T_0}; \quad T_0 = 1600 \text{ lb}$$

$$W_{AC} = (50 \text{ lb/ft})(24 \text{ ft}) = 1200 \text{ lb}$$



$T_{\max} = 2000 \text{ lb}$ ◀



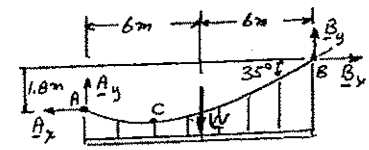
PROBLEM 7.117

Cable AB supports a load uniformly distributed along the horizontal as shown. Knowing that at B the cable forms an angle $\theta_B = 35°$ with the horizontal, determine (*a*) the maximum tension in the cable, (*b*) the vertical distance a from A to the lowest point of the cable.

SOLUTION

Free body: Entire cable

$$B_y = B_x \tan 35°$$



$$W = (45 \text{ kg/m})(12 \text{ m})(9.81 \text{ m/s}^2)$$

$$W = 5297.4 \text{ N}$$

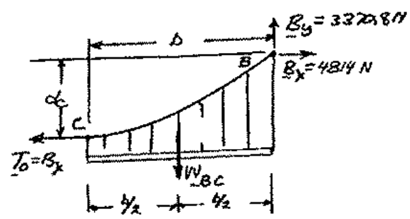
$$+\circlearrowleft \Sigma M_A = 0: \quad W(6 \text{ m}) + B_x(1.8 \text{ m}) - B_y(12 \text{ m}) = 0$$

$$(5297.4)(6) + 1.8B_x - B_x \tan 35°(12) = 0$$

$$B_x = 4814 \text{ N} \quad B_y = (4814 \text{ N})\tan 35° = 3370.8 \text{ N}$$

Free body: Portion CB

$$+\uparrow \Sigma F_y = 0: \quad B_y - W_{BC} = 0$$



$$W_{BC} = B_y = 3370.8 \text{ N}$$

$$W_{BC} = (45 \text{ kg/m})(9.81 \text{ m/s}^2)b$$

$$3370.8 \text{ N} = (441.45 \text{ N/m})b$$

$$b = 7.6357 \text{ m}$$

$$+\circlearrowleft \Sigma M_B = 0: \quad T_0 d_C - W_{BC}\left(\frac{1}{2}b\right) = 0$$

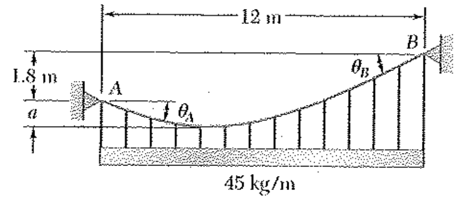
$$(4814 \text{ N})d_C - (3370.8 \text{ N})\frac{1}{2}(7.6357 \text{ m}) = 0$$

$$d_C = 2.6733 \text{ m}$$

(*a*) $d_C = 1.8 \text{ m} + a; \quad 2.6733 \text{ m} = 1.8 \text{ m} + a;$ $\qquad a = 0.873 \text{ m}$ ◀

(*b*) $T_m = B = \sqrt{B_x^2 + B_y^2} = \sqrt{(4814 \text{ N})^2 + (3370.8 \text{ N})}$

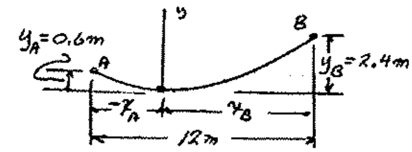
$T_m = 5877 \text{ N}$ $\qquad T_m = 5880 \text{ N}$ ◀



PROBLEM 7.118

Cable AB supports a load uniformly distributed along the horizontal as shown. Knowing that the lowest point of the cable is located at a distance $a = 0.6$ m below A, determine (*a*) the maximum tension in the cable, (*b*) the angle θ_B that the cable forms with the horizontal at B.

SOLUTION



Note: $x_B - x_A = 12 \text{ m}$

or $x_A = x_B - 12 \text{ m}$

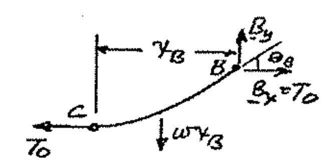
Point A: $y_A = \dfrac{wx_A^2}{2T_0}; \quad 0.6 = \dfrac{w(x_B - 12)^2}{2T_0}$ (1)

Point B: $y_B = \dfrac{wx_B^2}{2T_0}; \quad 2.4 = \dfrac{wx_B^2}{2T_0}$ (2)

Dividing (1) by (2): $\dfrac{0.6}{2.4} = \dfrac{(x_B - 12)^2}{x_B^2}; \quad x_B = 8 \text{ m}$

(*a*) Eq. (2): $2.4 = \dfrac{w(8)^2}{2T_0}; \quad T_0 = 13.333w$

Free body: Portion CB



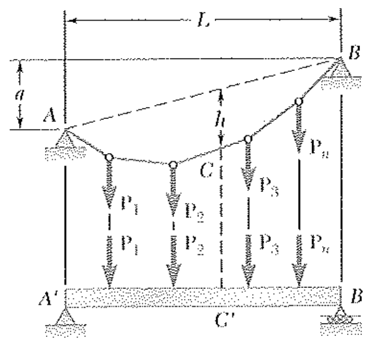
$\Sigma F_y = 0 \quad B_y = wx_B$

$B_y = 8w$

$T_m^2 = B_x^2 + B_y^2; \quad T_m^2 = (13.333w)^2 + (8w)^2$

$T_m = 15.549w = 15.549(45)(9.81)$ $\quad T_m = 6860 \text{ N}$ ◀

$\theta_B = \tan^{-1} B_y/B_x = \tan^{-1} 8w/13.333w$ $\quad \theta_B = 31.0°$ ◀



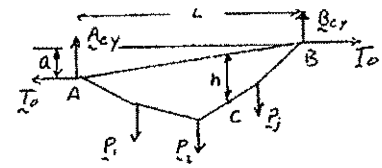
PROBLEM 7.119*

A cable AB of span L and a simple beam $A'B'$ of the same span are subjected to identical vertical loadings as shown. Show that the magnitude of the bending moment at a point C' in the beam is equal to the product T_0h, where T_0 is the magnitude of the horizontal component of the tension force in the cable and h is the vertical distance between Point C and the chord joining the points of support A and B.

SOLUTION

$$\circlearrowleft \Sigma M_B = 0: \quad LA_{Cy} + aT_0 - \Sigma M_{B\text{ loads}} = 0 \qquad (1)$$

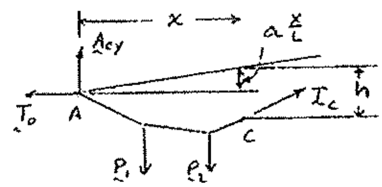
FBD Cable:



(Where $\Sigma M_{B\text{ loads}}$ includes all applied loads)

$$\circlearrowleft \Sigma M_C = 0: \quad xA_{Cy} - \left(h - a\frac{x}{L}\right)T_0 - \Sigma M_{C\text{ left}} = 0 \qquad (2)$$

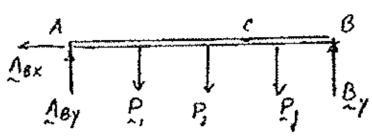
FBD AC:



(Where $\Sigma M_{C\text{ left}}$ includes all loads left of C)

$$\frac{x}{L}(1) - (2): \quad hT_0 - \frac{x}{L}\Sigma M_{B\text{ loads}} + \Sigma M_{C\text{ left}} = 0 \qquad (3)$$

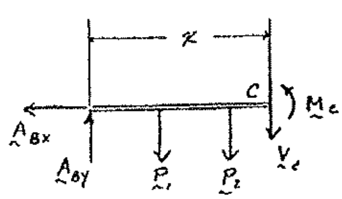
FBD Beam:



$$\circlearrowleft \Sigma M_B = 0: \quad LA_{By} - \Sigma M_{B\text{ loads}} = 0 \qquad (4)$$

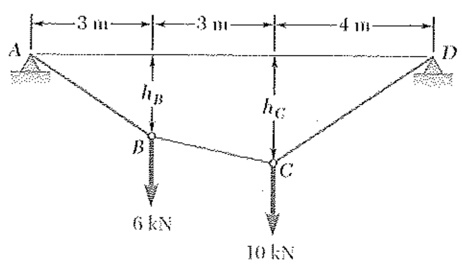
$$\circlearrowleft \Sigma M_C = 0: \quad xA_{By} - \Sigma M_{C\text{ left}} - M_C = 0 \qquad (5)$$

FBD AC:



$$\frac{x}{L}(4) - (5): \quad -\frac{x}{L}\Sigma M_{B\text{ loads}} + \Sigma M_{C\text{ left}} + M_C = 0 \qquad (6)$$

Comparing (3) and (6) $\qquad M_C = hT_0 \qquad$ Q.E.D.

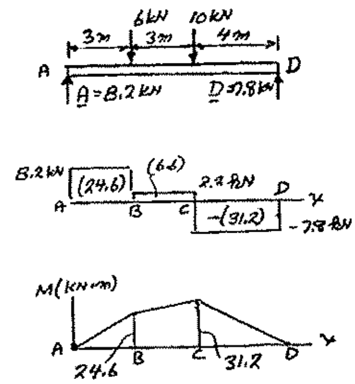


PROBLEM 7.120

Making use of the property established in Problem 7.119, solve the problem indicated by first solving the corresponding beam problem.

PROBLEM 7.94 (*a*) Knowing that the maximum tension in cable *ABCD* is 15 kN, determine the distance h_B.

SOLUTION



$$+\circlearrowleft \Sigma M_B = 0: \quad A(10\text{ m}) - (6\text{ kN})(7\text{ m}) - (10\text{ kN})(4\text{ m}) = 0$$

$$A = 8.2\text{ kN}$$

$$+\!\uparrow \Sigma F_y = 0: \quad 8.2\text{ kN} - 6\text{ kN} - 10\text{ kN} + B = 0$$

$$B = 7.8\text{ kN}$$

At A:

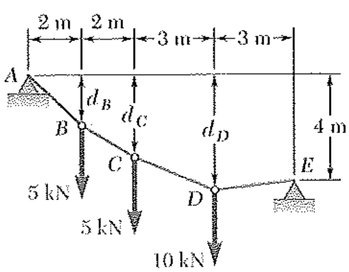
$$T_m^2 = T_0^2 + A^2$$

$$15^2 = T_0^2 + 8.2$$

$$T_0 = 12.56\text{ kN}$$

At B: $\quad M_B = T_0 h_B; \quad 24.6\text{ kN}\cdot\text{m} = (12.56\text{ kN})h_B; \qquad h_B = 1.959\text{ m}$ ◀

At C: $\quad M_C = T_0 h_C; \quad 31.2\text{ kN}\cdot\text{m} = (12.56\text{ kN})h_C; \qquad h_C = 2.48\text{ m}$ ◀

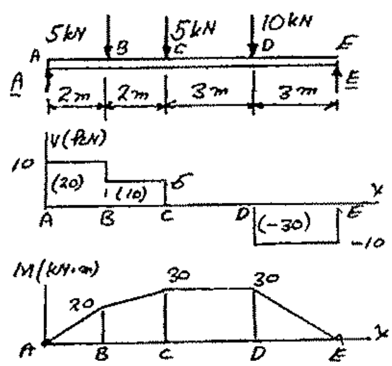


PROBLEM 7.121

Making use of the property established in Problem 7.119, solve the problem indicated by first solving the corresponding beam problem.

PROBLEM 7.97 (*a*) Knowing that $d_C = 3$ m, determine the distances d_B and d_D.

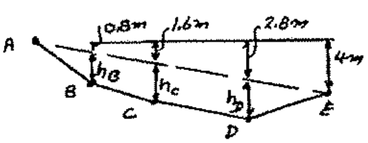
SOLUTION



$$+\circlearrowleft \Sigma M_E = 0: \quad A(10 \text{ m}) - (5 \text{ kN})(8 \text{ m}) - (5 \text{ kN})(6 \text{ m}) - (10 \text{ kN})(3 \text{ m}) = 0$$

$$A = 10 \text{ kN}$$

Geometry:



$$d_C = 1.6 \text{ m} + h_C$$
$$3 \text{ m} = 1.6 \text{ m} + h_C$$
$$h_C = 1.4 \text{ m}$$

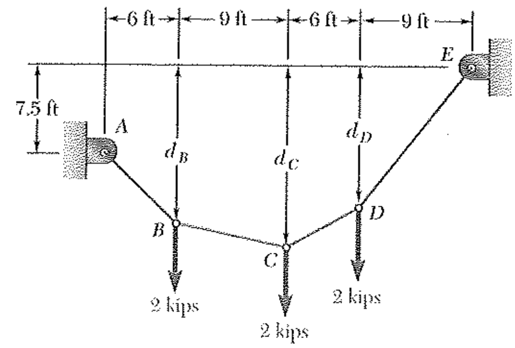
Since $M = T_0 h$, h is proportional to M, thus

$$\frac{h_B}{M_B} = \frac{h_C}{M_C} = \frac{h_D}{M_D}; \quad \frac{h_B}{20 \text{ kN}\cdot\text{m}} = \frac{1.4 \text{ m}}{30 \text{ kN}\cdot\text{m}} = \frac{h_D}{30 \text{ kN}\cdot\text{m}}$$

$$h_B = 1.4\left(\frac{20}{30}\right) = 0.9333 \text{ m} \qquad \| \qquad h_D = 1.4\left(\frac{30}{30}\right) = 1.4 \text{ m}$$

$$d_B = 0.8 \text{ m} + 0.9333 \text{ m} \qquad \| \qquad d_D = 2.8 \text{ m} + 1.4 \text{ m}$$

$d_B = 1.733$ m ◀ $\qquad$ $d_D = 4.20$ m ◀



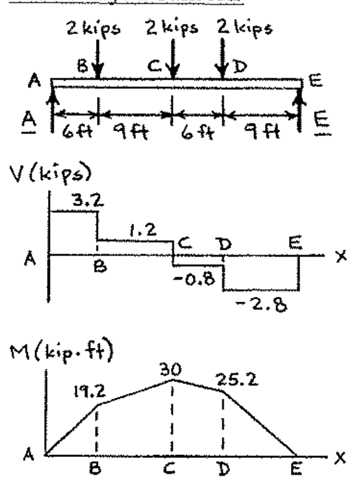
PROBLEM 7.122

Making use of the property established in Problem 7.119, solve the problem indicated by first solving the corresponding beam problem.

PROBLEM 7.99 (*a*) If $d_C = 15$ ft, determine the distances d_B and d_D.

SOLUTION

Free body: Beam AE



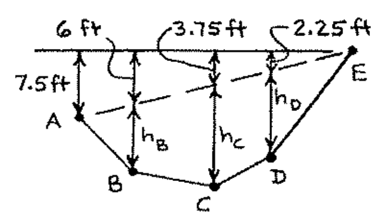
$$+\circlearrowleft \Sigma M_E = 0: \quad -A(30) + 2(24) + 2(15) + 2(9) = 0$$

$$\mathbf{A} = 3.2 \text{ kips} \uparrow$$

$$+\Sigma F_y = 0: \quad 3.2 - 3(2) + B = 0$$

$$\mathbf{B} = 2.8 \text{ kips} \uparrow$$

Geometry:



Given: $d_C = 15$ ft

Then, $h_C = d_C - 3.75 \text{ ft} = 11.25$ ft

Since $M = T_0 h$, h is proportional to M. Thus,

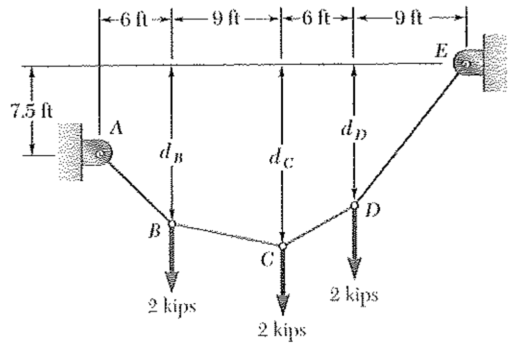
$$\frac{h_B}{M_B} = \frac{h_C}{M_C} = \frac{h_D}{M_D}$$

$$\text{or,} \quad \frac{h_B}{19.2} = \frac{11.25 \text{ ft}}{30} = \frac{h_D}{25.2}$$

$$\text{or,} \quad h_B = 7.2 \text{ ft} \quad h_D = 9.45 \text{ ft}$$

Then, $d_B = 6 + h_B = 6 + 7.2$ $\qquad d_B = 13.20$ ft ◀

$d_D = 2.25 + h_D = 2.25 + 9.45$ $\qquad d_D = 11.70$ ft ◀



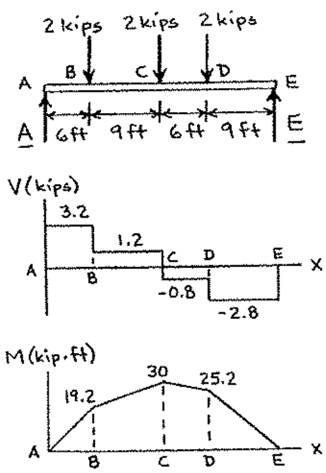
PROBLEM 7.123

Making use of the property established in Problem 7.119, solve the problem indicated by first solving the corresponding beam problem.

PROBLEM 7.100 (*a*) Determine the distance d_C for which portion BC of the cable is horizontal.

SOLUTION

Free body: Beam AE



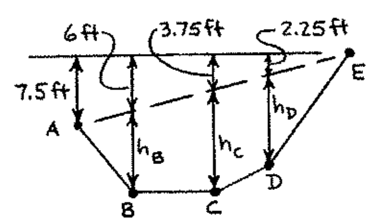
$$+\circlearrowleft \Sigma M_E = 0: \quad -A(30) + 2(24) + 2(15) + 2(9) = 0$$

$$\mathbf{A} = 3.2 \text{ kips} \uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad 3.2 - 3(2) + B = 0$$

$$\mathbf{B} = 2.8 \text{ kips} \uparrow$$

Geometry:



Given: $d_C = d_B$

Then, $3.75 + h_C = 6 + h_B$

$$h_C = 2.25 + h_B \qquad (1)$$

Since $M = T_0 h$, h is proportional to M. Thus,

$$\frac{h_B}{M_B} = \frac{h_C}{M_C} \quad \text{or,} \quad \frac{h_B}{19.2} = \frac{h_C}{30}$$

$$h_B = 0.64 h_C \qquad (2)$$

Substituting (2) into (1):

$$h_C = 2.25 + 0.64 h_C \quad h_C = 6.25 \text{ ft}$$

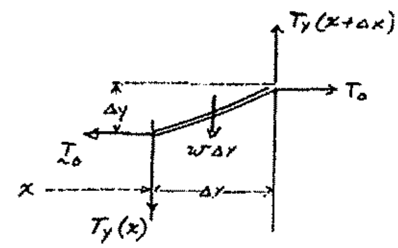
Then, $d_C = 3.75 + h_C = 3.75 + 6.25$ $\qquad d_C = 10.00 \text{ ft}$ ◄

PROBLEM 7.124*

Show that the curve assumed by a cable that carries a distributed load $w(x)$ is defined by the differential equation $d^2y/dx^2 = w(x)/T_0$, where T_0 is the tension at the lowest point.

SOLUTION

FBD Elemental segment:



$$\uparrow \Sigma F_y = 0: \quad T_y(x+\Delta x) - T_y(x) - w(x)\Delta x = 0$$

So
$$\frac{T_y(x+\Delta x)}{T_0} - \frac{T_y(x)}{T_0} = \frac{w(x)}{T_0}\Delta x$$

But
$$\frac{T_y}{T_0} = \frac{dy}{dx}$$

So
$$\frac{\left.\frac{dy}{dx}\right|_{x+\Delta x} - \left.\frac{dy}{dx}\right|_{x}}{\Delta x} = \frac{w(x)}{T_0}$$

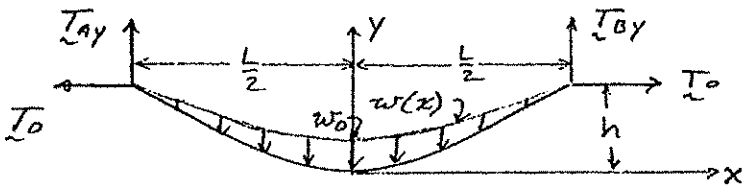
In $\lim\limits_{\Delta x \to 0}$:
$$\frac{d^2y}{dx^2} = \frac{w(x)}{T_0} \qquad \text{Q.E.D.}$$

PROBLEM 7.125*

Using the property indicated in Problem 7.124, determine the curve assumed by a cable of span L and sag h carrying a distributed load $w = w_0 \cos(\pi x/L)$, where x is measured from mid-span. Also determine the maximum and minimum values of the tension in the cable.

PROBLEM 7.124 Show that the curve assumed by a cable that carries a distributed load $w(x)$ is defined by the differential equation $d^2y/dx^2 = w(x)/T_0$, where T_0 is the tension at the lowest point.

SOLUTION



$$w(x) = w_0 \cos\frac{\pi x}{L}$$

From Problem 7.124

$$\frac{d^2y}{dx^2} = \frac{w(x)}{T_0} = \frac{w_0}{T_0}\cos\frac{\pi x}{L}$$

So

$$\frac{dy}{dx} = \frac{W_0 L}{T_0 \pi}\sin\frac{\pi x}{L} \quad \left(\text{using } \left.\frac{dy}{dx}\right|_0 = 0\right)$$

$$y = \frac{w_0 L^2}{T_0 \pi^2}\left(1 - \cos\frac{\pi x}{L}\right) \quad [\text{using } y(0) = 0] \blacktriangleleft$$

But

$$y\left(\frac{L}{2}\right) = h = \frac{w_0 L^2}{T_0 \pi^2}\left(1 - \cos\frac{\pi}{2}\right) \quad \text{so} \quad T_0 = \frac{w_0 L^2}{\pi^2 h}$$

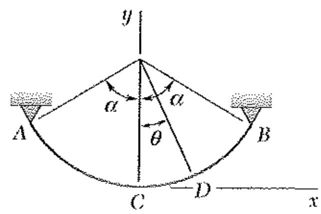
And

$$T_0 = T_{\min} \quad \text{so} \qquad T_{\min} = \frac{w_0 L^2}{\pi^2 h} \blacktriangleleft$$

$$T_{\max} = T_A = T_B: \quad \frac{T_{By}}{T_0} = \left.\frac{dy}{dx}\right|_{x=L/2} = \frac{w_0 L}{T_0 \pi}$$

$$T_{By} = \frac{w_0 L}{\pi} \qquad T_B = \sqrt{T_{By}^2 + T_0^2} = \frac{w_0 L}{\pi}\sqrt{1 + \left(\frac{L}{\pi h}\right)^2} \blacktriangleleft$$

PROBLEM 7.126

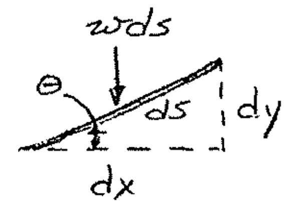


If the weight per unit length of the cable AB is $w_0/\cos^2\theta$, prove that the curve formed by the cable is a circular arc. (*Hint:* Use the property indicated in Problem 7.124.)

PROBLEM 7.124 Show that the curve assumed by a cable that carries a distributed load $w(x)$ is defined by the differential equation $d^2y/dx^2 = w(x)/T_0$, where T_0 is the tension at the lowest point.

SOLUTION

Elemental Segment:



Load on segment* $\qquad w(x)dx = \dfrac{w_0}{\cos^2\theta}ds$

But $\qquad dx = \cos\theta ds, \quad \text{so} \quad w(x) = \dfrac{w_0}{\cos^3\theta}$

From Problem 7.119 $\qquad \dfrac{d^2y}{dx^2} = \dfrac{w(x)}{T_0} = \dfrac{w_0}{T_0\cos^3\theta}$

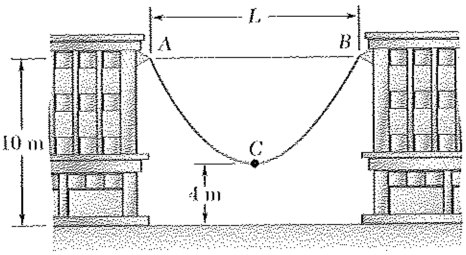
In general $\qquad \dfrac{d^2y}{dx^2} = \dfrac{d}{dx}\left(\dfrac{dy}{dx}\right) = \dfrac{d}{dx}(\tan\theta) = \sec^2\theta\dfrac{d\theta}{dx}$

So $\qquad \dfrac{d\theta}{dx} = \dfrac{w_0}{T_0\cos^3\theta\sec^2\theta} = \dfrac{w_0}{T_0\cos\theta}$

or $\qquad \dfrac{T_0}{w_0}\cos\theta d\theta = dx = rd\theta\cos\theta$

Giving $r = \dfrac{T_0}{w_0} =$ constant. So curve is circular arc $\qquad$ Q.E.D.

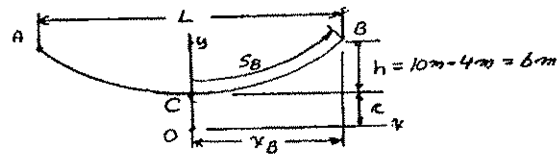
*For large sag, it is not appropriate to approximate ds by dx.



PROBLEM 7.127

A 30-m cable is strung as shown between two buildings. The maximum tension is found to be 500 N, and the lowest point of the cable is observed to be 4 m above the ground. Determine (*a*) the horizontal distance between the buildings, (*b*) the total mass of the cable.

SOLUTION



$$s_B = 15 \text{ m}$$
$$T_m = 500 \text{ N}$$

Eq. 7.17: $y_B^2 - s_B^2 = c^2; \quad (6+c)^2 - 15^2 = c^2$

$$36 + 12c + c^2 - 225 = c^2$$
$$12c = 189 \quad c = 15.75 \text{ m}$$

Eq. 7.15: $s_B = c \sinh \frac{x_B}{c}; \quad 15 = (15.75)\sinh \frac{x_B}{c}$

$$\sinh \frac{x_B}{c} = 0.95238 \quad \frac{x_B}{c} = 0.8473$$

(*a*) $x_B = 0.8473(15.75) = 13.345 \text{ m}; \quad L = 2x_B$ $\qquad L = 26.7 \text{ m}$ ◀

(*b*) Eq. 7.18: $T_m = wy_B; \quad 500 \text{ N} = w(6 + 15.75)$

$$w = 22.99 \text{ N/m}$$

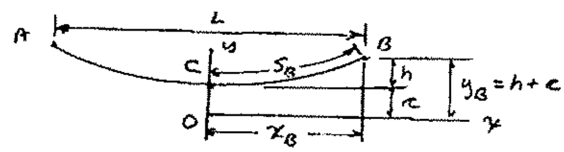
$$W = 2s_B w = (30 \text{ m})(22.99 \text{ N/m})$$
$$= 689.7 \text{ N}$$
$$m = \frac{W}{g} = \frac{689.7 \text{ N}}{9.81 \text{ m/s}^2}$$

Total mass = 70.3 kg ◀

PROBLEM 7.128

A 200-ft steel surveying tape weighs 4 lb. If the tape is stretched between two points at the same elevation and pulled until the tension at each end is 16 lb, determine the horizontal distance between the ends of the tape. Neglect the elongation of the tape due to the tension.

SOLUTION



$$s_B = 100 \text{ ft}$$

$$w = \left(\frac{4 \text{ lb}}{200 \text{ ft}}\right) = 0.02 \text{ lb/ft} \quad T_m = 16 \text{ m}$$

Eq. 7.18: $T_m = wy_B;\quad 16 \text{ lb} = (0.02 \text{ lb/ft})y_B;\quad y_B = 800 \text{ ft}$

Eq. 7.17: $y_B^2 - s_B^2 = c^2;\quad (800)^2 - (100)^2 = c^2;\quad c = 793.73 \text{ ft}$

Eq. 7.15: $s_B = c \sinh\frac{x_B}{c};\quad 100 = 793.73 \sinh\frac{x_B}{c}$

$$\frac{x_B}{c} = 0.12566;\quad x_B = 99.737 \text{ ft}$$

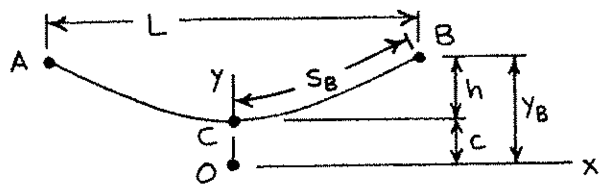
$$L = 2x_B = 2(99.737 \text{ ft})$$

$L = 199.5$ ft ◀

PROBLEM 7.129

A 200-m-long aerial tramway cable having a mass per unit length of 3.5 kg/m is suspended between two points at the same elevation. Knowing that the sag is 50 m, find (*a*) the horizontal distance between the supports, (*b*) the maximum tension in the cable.

SOLUTION



Given:

$$\text{Length} = 200 \text{ m}$$
$$\text{Unit mass} = 3.5 \text{ kg/m}$$
$$h = 50 \text{ m}$$

Then,

$$w = (3.5 \text{ kg/m})(9.81 \text{ m/s}^2) = 34.335 \text{ N/m}$$
$$s_B = 100 \text{ m}$$
$$y_B = h + c = 50 \text{ m} + c$$
$$y_B^2 - s_B^2 = c^2; \quad (50 + c)^2 - (100)^2 = c^2$$
$$50^2 + 100c + c^2 - 100^2 = c^2$$
$$c = 75 \text{ m}$$

$$s_B = c \sinh \frac{x_B}{c}; \quad 100 = 75 \sinh \frac{x_B}{75}$$

$$x_B = 82.396 \text{ m}$$

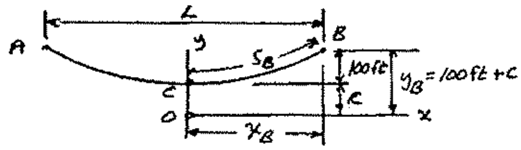
$$\text{span} = L = 2x_B = 2(82.396 \text{ m}) \qquad L = 164.8 \text{ m} \blacktriangleleft$$

$$T_m = wy_B = (34.335 \text{ N/m})(50 \text{ m} + 75 \text{ m}) \qquad T_m = 4290 \text{ N} \blacktriangleleft$$

PROBLEM 7.130

An electric transmission cable of length 400 ft weighing 2.5 lb/ft is suspended between two points at the same elevation. Knowing that the sag is 100 ft, determine the horizontal distance between the supports and the maximum tension.

SOLUTION



$$s_B = 200 \text{ ft}$$

Eq. 7.17:

$$y_B^2 - s_B^2 = c^2; \quad (100 + c)^2 - 200^2 = c^2$$

$$10000 + 200c + c^2 - 40000 = c^2; \quad c = 150 \text{ ft}$$

Eq. 7.15:

$$s_B = c \sinh \frac{x_B}{c}; \quad 200 = 150 \sinh \frac{x_B}{c}$$

$$\sinh \frac{x_B}{c} = \frac{4}{3}; \quad \frac{x_B}{c} = 1.0986$$

$$x_B = (150)(1.0986) = 164.79 \text{ ft}$$

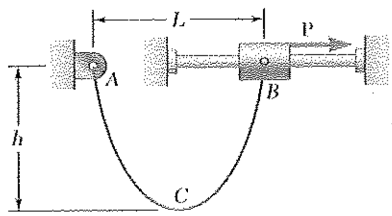
$$L = 2x_B = 2(164.79 \text{ ft}) = 329.58 \text{ ft}$$

$L = 330$ ft ◀

Eq. 7.18:

$$T_m = wy_B = (2.5 \text{ lb/ft})(100 \text{ ft} + 150 \text{ ft})$$

$T_m = 625$ lb ◀

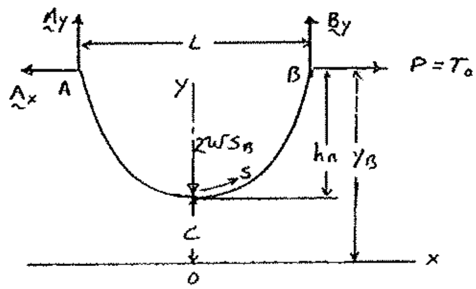


PROBLEM 7.131

A 20-m length of wire having a mass per unit length of 0.2 kg/m is attached to a fixed support at A and to a collar at B. Neglecting the effect of friction, determine (a) the force $\mathbf{P}$ for which $h = 8$ m, (b) the corresponding span L.

SOLUTION

FBD Cable:



$$s_T = 20 \text{ m} \quad \left(\text{so } s_B = \frac{20 \text{ m}}{2} = 10 \text{ m}\right)$$

$$w = (0.2 \text{ kg/m})(9.81 \text{ m/s}^2)$$
$$= 1.96200 \text{ N/m}$$

$$h_B = 8 \text{ m}$$

$$y_B^2 = (c + h_B)^2 = c^2 + s_B^2$$

So
$$c = \frac{s_B^2 - h_B^2}{2h_B}$$

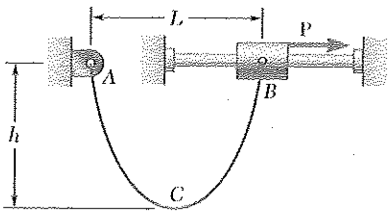
$$c = \frac{(10 \text{ m})^2 - (8 \text{ m})^2}{2(8 \text{ m})}$$
$$= 2.250 \text{ m}$$

Now
$$s_B = c\sinh\frac{x_B}{c} \rightarrow x_B = c\sinh^{-1}\frac{s_B}{c}$$
$$= (2.250 \text{ m})\sinh^{-1}\left(\frac{10 \text{ m}}{2.250 \text{ m}}\right)$$

$$x_B = 4.9438 \text{ m}$$

$$P = T_0 = wc = (1.96200 \text{ N/m})(2.250 \text{ m}) \qquad (a) \qquad \mathbf{P} = 4.41 \text{ N} \longrightarrow \blacktriangleleft$$

$$L = 2x_B = 2(4.9438 \text{ m}) \qquad (b) \qquad L = 9.89 \text{ m} \blacktriangleleft$$

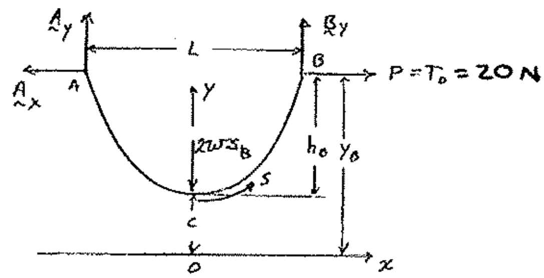


PROBLEM 7.132

A 20-m length of wire having a mass per unit length of 0.2 kg/m is attached to a fixed support at A and to a collar at B. Knowing that the magnitude of the horizontal force applied to the collar is $P = 20$ N, determine (a) the sag h, (b) the span L.

SOLUTION

FBD Cable:



$$s_T = 20 \text{ m}, \quad w = (0.2 \text{ kg/m})(9.81 \text{ m/s}^2) = 1.96200 \text{ N/m}$$

$$P = T_0 = wc \quad c = \frac{P}{w}$$

$$c = \frac{20 \text{ N}}{1.9620 \text{ N/m}} = 10.1937 \text{ m}$$

$$y_B^2 = (h_B + c)^2 = c^2 + s_B^2$$

$$h^2 + 2ch - s_B^2 = 0 \quad s_B = \frac{20 \text{ m}}{2} = 10 \text{ m}$$

$$h^2 + 2(10.1937 \text{ m})h - 100 \text{ m}^2 = 0$$

$$h = 4.0861 \text{ m}$$

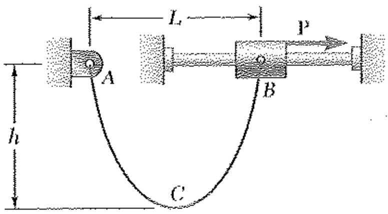
(a) $h = 4.09$ m ◀

$$s_B = c\sinh\frac{x_A}{c} \rightarrow x_B = c\sinh^{-1}\frac{s_B}{c} = (10.1937 \text{ m})\sinh^{-1}\left(\frac{10 \text{ m}}{10.1937 \text{ m}}\right)$$

$$= 8.8468 \text{ m}$$

$$L = 2x_B = 2(8.8468 \text{ m})$$

(b) $L = 17.69$ m ◀

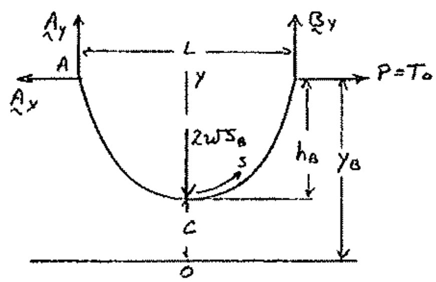


PROBLEM 7.133

A 20-m length of wire having a mass per unit length of 0.2 kg/m is attached to a fixed support at A and to a collar at B. Neglecting the effect of friction, determine (*a*) the sag h for which $L = 15$ m, (*b*) the corresponding force **P**.

SOLUTION

FBD Cable:



$$s_T = 20 \text{ m} \rightarrow s_B = \frac{20 \text{ m}}{2} = 10 \text{ m}$$

$$w = (0.2 \text{ kg/m})(9.81 \text{ m/s}^2) = 1.96200 \text{ N/m}$$

$$L = 15 \text{ m}$$

$$s_B = c \sinh\frac{x_B}{c} = c \sinh\frac{\frac{L}{2}}{c}$$

$$10 \text{ m} = c \sinh\frac{7.5 \text{ m}}{c}$$

Solving numerically: $c = 5.5504$ m

$$y_B = c\cosh\left(\frac{x_B}{c}\right) = (5.5504)\cosh\left(\frac{7.5}{5.5504}\right)$$

$$y_B = 11.4371 \text{ m}$$

$$h_B = y_B - c = 11.4371 \text{ m} - 5.5504 \text{ m}$$

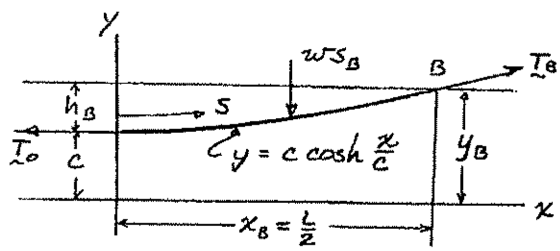
(*a*) $h_B = 5.89$ m ◀

$P = wc = (1.96200 \text{ N/m})(5.5504 \text{ m})$ (*b*) $\mathbf{P} = 10.89$ N ⟶ ◀

PROBLEM 7.134

Determine the sag of a 30-ft chain that is attached to two points at the same elevation that are 20 ft apart.

SOLUTION



$$s_B = \frac{30 \text{ ft}}{2} = 15 \text{ ft} \quad L = 20 \text{ ft}$$

$$x_B = \frac{L}{2} = 10 \text{ ft}$$

$$s_B = c \sinh \frac{x_B}{c}$$

$$15 \text{ ft} = c \sinh \frac{10 \text{ ft}}{c}$$

Solving numerically: $c = 6.1647 \text{ ft}$

$$y_B = c \cosh \frac{x_B}{c}$$

$$= (6.1647 \text{ ft}) \cosh \frac{10 \text{ ft}}{6.1647 \text{ ft}}$$

$$= 16.2174 \text{ ft}$$

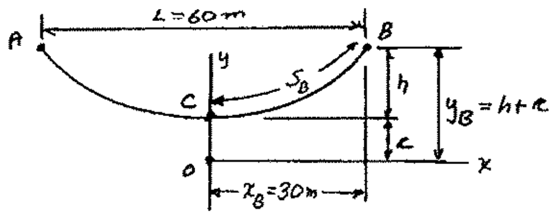
$$h_B = y_B - c = 16.2174 \text{ ft} - 6.1647 \text{ ft}$$

$h_B = 10.05 \text{ ft}$ ◀

PROBLEM 7.135

A 90-m wire is suspended between two points at the same elevation that are 60 m apart. Knowing that the maximum tension is 300 N, determine (*a*) the sag of the wire, (*b*) the total mass of the wire.

SOLUTION



$$s_B = 45 \text{ m}$$

Eq. 7.17:

$$s_B = c\sinh\frac{x_B}{c}$$

$$45 = c\sinh\frac{30}{c}; \quad c = 18.494 \text{ m}$$

Eq. 7.16:

$$y_B = c\cosh\frac{x_B}{c}$$

$$y_B = (18.494)\cosh\frac{30}{18.494}$$

$$y_B = 48.652 \text{ m}$$

$$y_B = h + c$$

$$48.652 = h + 18.494$$

$$h = 30.158 \text{ m}$$

$h = 30.2$ m ◀

Eq. 7.18:

$$T_m = wy_B$$

$$300 \text{ N} = w(48.652 \text{ m})$$

$$w = 6.166 \text{ N/m}$$

Total weight of cable

$$W = w(\text{Length})$$

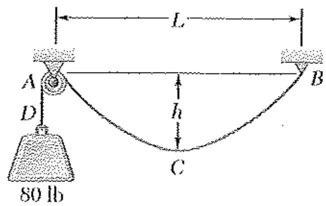
$$= (6.166 \text{ N/m})(90 \text{ m})$$

$$= 554.96 \text{ N}$$

Total mass of cable

$$m = \frac{W}{g} = \frac{554.96 \text{ N}}{9.81 \text{ m/s}} = 56.57 \text{ kg}$$

$m = 56.6$ kg ◀



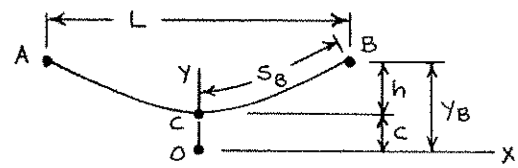
PROBLEM 7.136

A counterweight D is attached to a cable that passes over a small pulley at A and is attached to a support at B. Knowing that $L = 45$ ft and $h = 15$ ft, determine (a) the length of the cable from A to B, (b) the weight per unit length of the cable. Neglect the weight of the cable from A to D.

SOLUTION

Given:

$$L = 45 \text{ ft}$$
$$h = 15 \text{ ft}$$
$$T_A = 80 \text{ lb}$$
$$x_B = 22.5 \text{ ft}$$



By symmetry:

$$T_B = T_A = T_m = 80 \text{ lb}$$

We have

$$y_B = c\cosh\frac{x_B}{c} = c\cosh\frac{22.5}{c}$$

and

$$y_B = h + c = 15 + c$$

Then

$$c\cosh\frac{22.5}{c} = 15 + c$$

or

$$\cosh\frac{22.5}{c} = \frac{15}{c} + 1$$

Solve by trial for c:

$$c = 18.9525 \text{ ft}$$

(a)

$$s_B = c\sinh\frac{x_B}{c}$$
$$= (18.9525 \text{ ft})\sinh\frac{22.5}{18.9525}$$
$$= 28.170 \text{ ft}$$

Length $= 2s_B = 2(28.170 \text{ ft}) = 56.3$ ft ◀

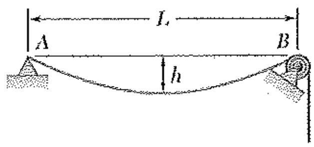
(b)

$$T_m = wy_B = w(h + c)$$

$$80 \text{ lb} = w(15 \text{ ft} + 18.9525 \text{ ft})$$

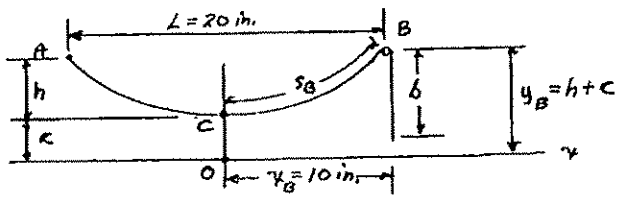
$w = 2.36$ lb/ft ◀

PROBLEM 7.137



A uniform cord 50 in. long passes over a pulley at B and is attached to a pin support at A. Knowing that $L = 20$ in. and neglecting the effect of friction, determine the smaller of the two values of h for which the cord is in equilibrium.

SOLUTION



Length of overhang: $b = 50\text{ in.} - 2s_B$

Weight of overhang equals max. tension

$$T_m = T_B = wb = w(50\text{ in.} - 2s_B)$$

Eq. 7.15: $s_B = c\sinh\dfrac{x_B}{c}$

Eq. 7.16: $y_B = c\cosh\dfrac{x_B}{c}$

Eq. 7.18: $T_m = wy_B$

$$w(50\text{ in.} - 2s_B) = wy_B$$

$$w\left(50\text{ in.} - 2c\sinh\frac{x_B}{c}\right) = wc\cosh\frac{x_B}{c}$$

$$x_B = 10:\quad 50 - 2c\sinh\frac{10}{c} = c\cosh\frac{10}{c}$$

Solve by trial and error: $c = 5.549$ in. and $c = 27.742$ in.

For $c = 5.549$ in.

$$y_B = (5.549\text{ in.})\cosh\frac{10\text{ in.}}{5.549\text{ in.}} = 17.277\text{ in.}$$

$$y_B = h + c;\quad 17.277\text{ in.} = h + 5.549\text{ in.}$$

$$h = 11.728\text{ in.}$$

$h = 11.73$ in. ◀

For $c = 27.742$ in.

$$y_B = (27.742\text{ in.})\cosh\frac{10\text{ in.}}{27.742\text{ in.}} = 29.564\text{ in.}$$

$$y_B = h + c;\quad 29.564\text{ in.} = h + 27.742\text{ in.}$$

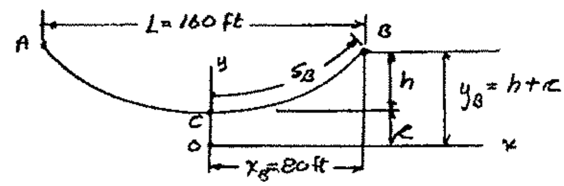
$$h = 1.8219\text{ in.}$$

$h = 1.822$ in. ◀

PROBLEM 7.138

A cable weighing 2 lb/ft is suspended between two points at the same elevation that are 160 ft apart. Determine the smallest allowable sag of the cable if the maximum tension is not to exceed 400 lb.

SOLUTION



Eq. 7.18: $T_m = wy_B$; 400 lb = (2 lb/ft)y_B; $y_B = 200$ ft

Eq. 7.16:

$$y_B = c\cosh\frac{x_B}{c}$$

$$200\text{ ft} = c\cosh\frac{80\text{ ft}}{c}$$

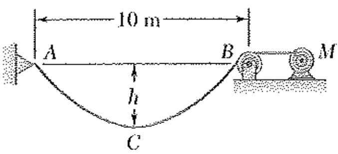
Solve for c: $c = 182.148$ ft and $c = 31.592$ ft

$$y_B = h + c; \quad h = y_B - c$$

For $c = 182.148$ ft; $h = 200 - 182.147 = 17.852$ ft ◁

For $c = 31.592$ ft; $h = 200 - 31.592 = 168.408$ ft ◁

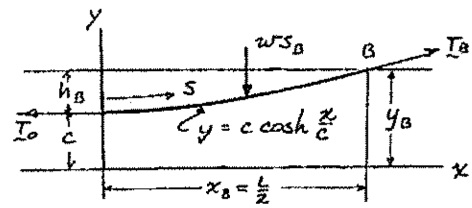
For $T_m \leq 400$ lb: smallest $h = 17.85$ ft ◀



PROBLEM 7.139

A motor M is used to slowly reel in the cable shown. Knowing that the mass per unit length of the cable is 0.4 kg/m, determine the maximum tension in the cable when $h = 5$ m.

SOLUTION



$$w = 0.4 \text{ kg/m} \quad L = 10 \text{ m} \quad h_B = 5 \text{ m}$$

$$y_B = c \cosh \frac{x_B}{c}$$

$$h_B + c = c \cosh \frac{L}{2c}$$

$$5 \text{ m} = c\left(\cosh \frac{5 \text{ m}}{c} - 1\right)$$

Solving numerically:

$$c = 3.0938 \text{ m}$$

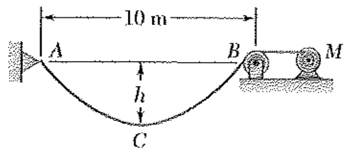
$$y_B = h_B + c = 5 \text{ m} + 3.0938 \text{ m}$$

$$= 8.0938 \text{ m}$$

$$T_{\max} = T_B = wy_B$$

$$= (0.4 \text{ kg/m})(9.81 \text{ m/s}^2)(8.0938 \text{ m})$$

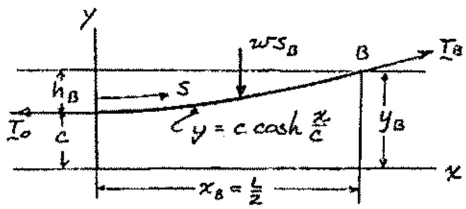
$T_{\max} = 31.8$ N ◀



PROBLEM 7.140

A motor M is used to slowly reel in the cable shown. Knowing that the mass per unit length of the cable is 0.4 kg/m, determine the maximum tension in the cable when $h = 3$ m.

SOLUTION



$$w = 0.4 \text{ kg/m}, \quad L = 10 \text{ m}, \quad h_B = 3 \text{ m}$$

$$y_B = h_B + c = c\cosh\frac{x_B}{c} = c\cosh\frac{L}{2c}$$

$$3 \text{ m} = c\left(c\cosh\frac{5 \text{ m}}{c} - 1\right)$$

Solving numerically:

$$c = 4.5945 \text{ m}$$

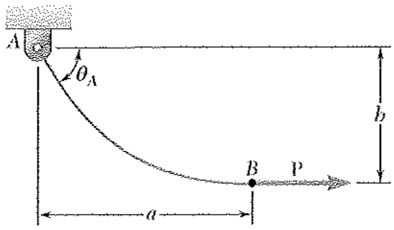
$$y_B = h_B + c = 3 \text{ m} + 4.5945 \text{ m}$$

$$= 7.5945 \text{ m}$$

$$T_{\max} = T_B = wy_B$$

$$= (0.4 \text{ kg/m})(9.81 \text{ m/s}^2)(7.5945 \text{ m})$$

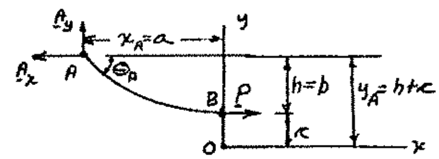
$T_{\max} = 29.8$ N ◀



PROBLEM 7.141

A uniform cable weighing 3 lb/ft is held in the position shown by a horizontal force **P** applied at *B*. Knowing that $P = 180$ lb and $\theta_A = 60°$, determine (*a*) the location of Point *B*, (*b*) the length of the cable.

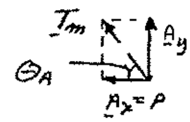
SOLUTION



Eq. 7.18: $$T_0 = P = cw$$

$$c = \frac{P}{w} = \frac{180 \text{ lb}}{3 \text{ lb/ft}} \qquad c = 60 \text{ ft}$$

At *A*: $$T_m = \frac{P}{\cos 60°}$$



$$= \frac{cw}{0.5} = 2cw$$

(*a*) Eq. 7.18: $$T_m = w(h+c)$$

$$2cw = w(h+c)$$

$$2c = h + c \quad h = b = c$$

$b = 60.0$ ft ◀

Eq. 7.16: $$y_A = c\cosh\frac{x_A}{c}$$

$$h + c = c\cosh\frac{x_A}{c}$$

$$(60 \text{ ft} + 60 \text{ ft}) = (60 \text{ ft}) \quad \cosh\frac{x_A}{60}$$

$$\cosh\frac{x_A}{60 \text{ m}} = 2 \quad \frac{x_A}{60 \text{ m}} = 1.3170$$

$$x_A = 79.02 \text{ ft}$$

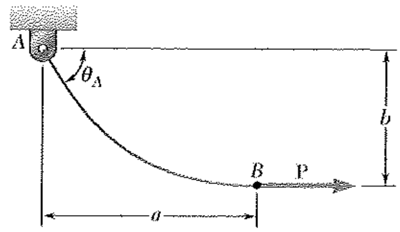
$a = 79.0$ ft ◀

(*b*) Eq. 7.15: $$s_A = c\sinh\frac{x_B}{c} = (60 \text{ ft})\sinh\frac{79.02 \text{ ft}}{60 \text{ ft}}$$

$$s_A = 103.92 \text{ ft}$$

$$\text{length} = s_A$$

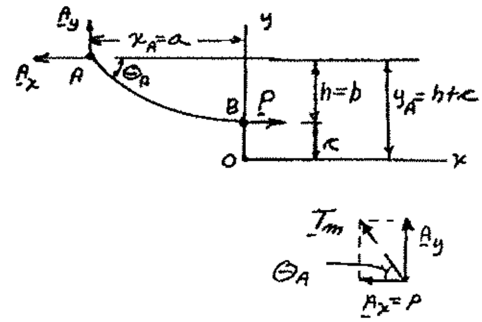
$s_A = 103.9$ ft ◀



PROBLEM 7.142

A uniform cable weighing 3 lb/ft is held in the position shown by a horizontal force **P** applied at B. Knowing that $P = 150$ lb and $\theta_A = 60°$, determine (a) the location of Point B, (b) the length of the cable.

SOLUTION



Eq. 7.18:
$$T_0 = P = cw$$
$$c = \frac{P}{w} = \frac{150 \text{ lb}}{3 \text{ lb/ft}} = 50 \text{ ft}$$

At A:
$$T_m = \frac{P}{\cos 60°} = \frac{cw}{0.5} = 2\,cw$$

(a) Eq. 7.18:
$$T_m = w(h + c)$$
$$2cw = w(h + c)$$
$$2c = h + c \quad h = c = b$$

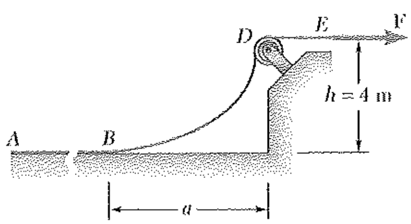
$b = 50.0$ ft ◀

Eq. 7.16:
$$y_A = c\cosh\frac{x_A}{c}$$
$$h + c = c\cosh\frac{x_A}{c}$$
$$(50 \text{ ft} + 50 \text{ ft}) = (50 \text{ ft})\cosh\frac{x_A}{c}$$
$$\cosh\frac{x_A}{c} = 2 \quad \frac{x_A}{c} = 1.3170$$
$$x_A = 1.3170(50 \text{ ft}) = 65.85 \text{ ft}$$

$a = 65.8$ ft ◀

(b) Eq. 7.15:
$$s_A = c\sinh\frac{x_A}{c} = (50 \text{ ft})\sinh\frac{65.85 \text{ ft}}{50 \text{ ft}}$$
$$s_A = 86.6 \text{ ft}$$

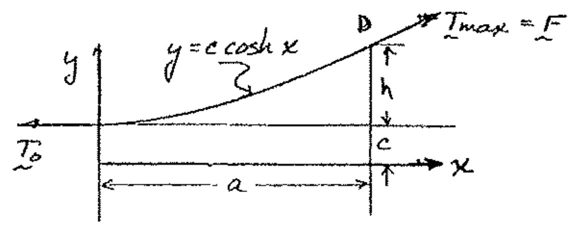
$\text{length} = s_A = 86.6$ ft ◀



PROBLEM 7.143

To the left of Point B the long cable $ABDE$ rests on the rough horizontal surface shown. Knowing that the mass per unit length of the cable is 2 kg/m, determine the force **F** when $a = 3.6$ m.

SOLUTION



$$x_D = a = 3.6 \text{ m} \quad h = 4 \text{ m}$$

$$y_D = c\cosh\frac{x_D}{c}$$

$$h + c = c\cosh\frac{a}{c}$$

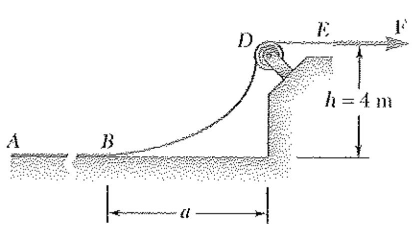
$$4 \text{ m} = c\left(\cosh\frac{3.6 \text{ m}}{c} - 1\right)$$

Solving numerically $\quad c = 2.0712$ m

Then $\quad y_B = h + c = 4 \text{ m} + 2.0712 \text{ m} = 6.0712 \text{ m}$

$$F = T_{\max} = wy_B = (2 \text{ kg/m})(9.81 \text{ m/s}^2)(6.0712 \text{ m})$$

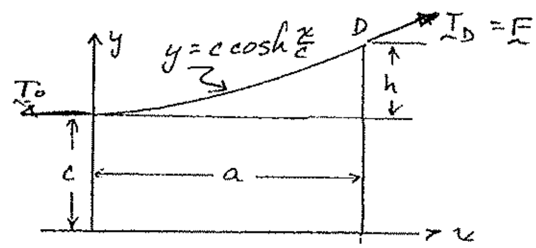
$\mathbf{F} = 119.1 \text{ N} \longrightarrow$ ◀



PROBLEM 7.144

To the left of Point B the long cable $ABDE$ rests on the rough horizontal surface shown. Knowing that the mass per unit length of the cable is 2 kg/m, determine the force **F** when $a = 6$ m.

SOLUTION



$$x_D = a = 6 \text{ m} \quad h = 4 \text{ m}$$

$$y_D = c\cosh\frac{x_D}{c}$$

$$h + c = c\cosh\frac{a}{c}$$

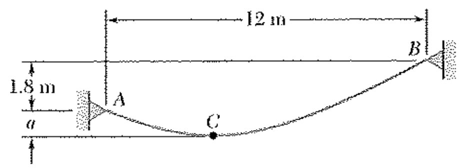
$$4 \text{ m} = c\left(\cosh\frac{6 \text{ m}}{c} - 1\right)$$

Solving numerically $\quad c = 5.054$ m

$$y_B = h + c = 4 \text{ m} + 5.054 \text{ m} = 9.054 \text{ m}$$

$$F = T_D = wy_D = (2 \text{ kg/m})(9.81 \text{ m/s}^2)(9.054 \text{ m})$$

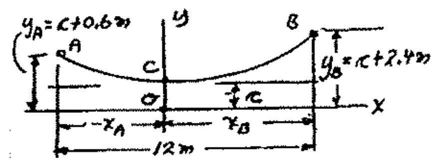
$\mathbf{F} = 177.6 \text{ N} \longrightarrow$ ◀



PROBLEM 7.145

The cable ACB has a mass per unit length of 0.45 kg/m. Knowing that the lowest point of the cable is located at a distance $a = 0.6$ m below the support A, determine (a) the location of the lowest Point C, (b) the maximum tension in the cable.

SOLUTION



Note: $$x_B - x_A = 12 \text{ m}$$

or, $$-x_A = 12 \text{ m} - x_B$$

Point A: $$y_A = c\cosh\frac{-x_A}{c}; \quad c + 0.6 = c\cosh\frac{12 - x_B}{c} \qquad (1)$$

Point B: $$y_B = c\cosh\frac{x_B}{c}; \quad c + 2.4 = c\cosh\frac{x_B}{c} \qquad (2)$$

From (1): $$\frac{12}{c} - \frac{x_B}{c} = \cosh^{-1}\left(\frac{c+0.6}{c}\right) \qquad (3)$$

From (2): $$\frac{x_B}{c} = \cosh^{-1}\left(\frac{c+2.4}{c}\right) \qquad (4)$$

Add (3) + (4): $$\frac{12}{c} = \cosh^{-1}\left(\frac{c+0.6}{c}\right) + \cosh^{-1}\left(\frac{c+2.4}{c}\right)$$

Solve by trial and error: $$c = 13.6214 \text{ m}$$

Eq. (2) $$13.6214 + 2.4 = 13.6214\cosh\frac{x_B}{c}$$

$$\cosh\frac{x_B}{c} = 1.1762; \quad \frac{x_B}{c} = 0.58523$$

$$x_B = 0.58523(13.6214 \text{ m}) = 7.9717 \text{ m}$$

Point C is 7.97 m to left of B ◀

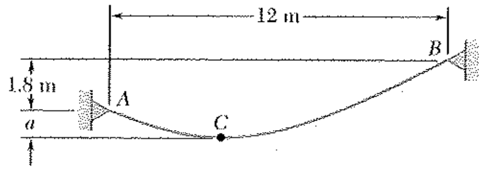
$$y_B = c + 2.4 = 13.6214 + 2.4 = 16.0214 \text{ m}$$

Eq. 7.18: $$T_m = wy_B = (0.45 \text{ kg/m})(9.81 \text{ m/s}^2)(16.0214 \text{ m})$$

$$T_m = 70.726 \text{ N}$$

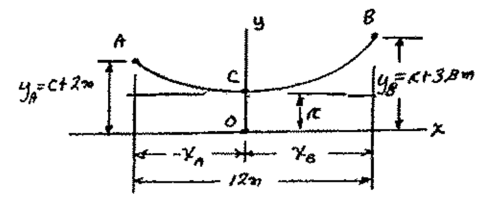
$T_m = 70.7$ N ◀

PROBLEM 7.146



The cable *ACB* has a mass per unit length of 0.45 kg/m. Knowing that the lowest point of the cable is located at a distance $a = 2$ m below the support *A*, determine (*a*) the location of the lowest Point *C*, (*b*) the maximum tension in the cable.

SOLUTION



Note:

$$x_B - x_A = 12 \text{ m}$$

or

$$-x_A = 12 \text{ m} - x_B$$

Point *A*: $$y_A = c\cosh\frac{-x_A}{c}; \quad c + 2 = c\cosh\frac{12 - x_B}{c} \tag{1}$$

Point *B*: $$y_B = c\cosh\frac{x_B}{c}; \quad c + 3.8 = c\cosh\frac{x_B}{c} \tag{2}$$

From (1): $$\frac{12}{c} - \frac{x_B}{c} = \cosh^{-1}\left(\frac{c+2}{c}\right) \tag{3}$$

From (2): $$\frac{x_B}{c} = \cosh^{-1}\left(\frac{c+3.8}{c}\right) \tag{4}$$

Add (3) + (4): $$\frac{12}{c} = \cosh^{-1}\left(\frac{c+2}{c}\right) + \cosh^{-1}\left(\frac{c+3.8}{c}\right)$$

Solve by trial and error: $c = 6.8154$ m

Eq. (2): $$6.8154 \text{ m} + 3.8 \text{ m} = (6.8154 \text{ m})\cosh\frac{x_B}{c}$$

$$\cosh\frac{x_B}{c} = 1.5576 \quad \frac{x_B}{c} = 1.0122$$

$$x_B = 1.0122(6.8154 \text{ m}) = 6.899 \text{ m}$$

Point *C* is 6.90 m to left of *B* ◀

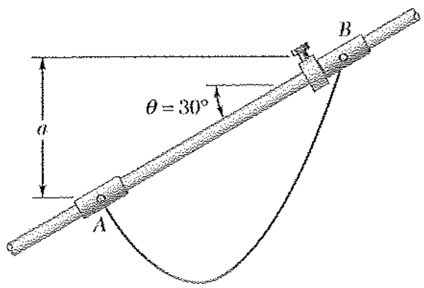
$$y_B = c + 3.8 = 6.8154 + 3.8 = 10.6154 \text{ m}$$

Eq. (7.18): $$T_m = wy_B = (0.45 \text{ kg/m})(9.81 \text{ m/s}^2)(10.6154 \text{ m})$$

$$T_m = 46.86 \text{ N}$$

$T_m = 46.9$ N ◀

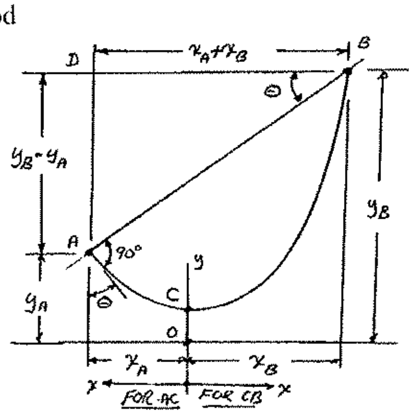
PROBLEM 7.147*



The 10-ft cable AB is attached to two collars as shown. The collar at A can slide freely along the rod; a stop attached to the rod prevents the collar at B from moving on the rod. Neglecting the effect of friction and the weight of the collars, determine the distance a.

SOLUTION

Collar at A: Since $\mu = 0$, cable $\perp$ rod



Point A:

$$y = c\cosh\frac{x}{c}; \quad \frac{dy}{dx} = \sinh\frac{x}{c}$$

$$\tan\theta = \left|\frac{dy}{dx}\right|_A = \sinh\frac{x_A}{c}$$

$$\frac{x_A}{c} = \sinh(\tan(90° - \theta))$$

$$x_A = c\sinh(\tan(90° - \theta)) \qquad (1)$$

Length of cable = 10 ft

$$10 \text{ ft} = AC + CB$$

$$10 = c\sinh\frac{x_A}{c} + c\sinh\frac{x_B}{c}$$

$$\sinh\frac{x_B}{c} = \frac{10}{c} - \sinh\frac{x_A}{c}$$

$$x_B = c\sinh^{-1}\left[\frac{10}{c} - \sinh\frac{x_A}{c}\right] \qquad (2)$$

$$y_A = c\cosh\frac{x_A}{c} \quad y_B = c\cosh\frac{x_B}{c} \qquad (3)$$

In ΔABD:

$$\tan\theta = \frac{y_B - y_A}{x_B + x_A} \qquad (4)$$

PROBLEM 7.147* (Continued)

Method of solution:

For given value of θ, choose trial value of c and calculate:

From Eq. (1): x_A

Using value of x_A and c, calculate:

From Eq. (2): x_B

From Eq. (3): y_A and y_B

Substitute values obtained for x_A, x_B, y_A, y_B into Eq. (4) and calculate θ

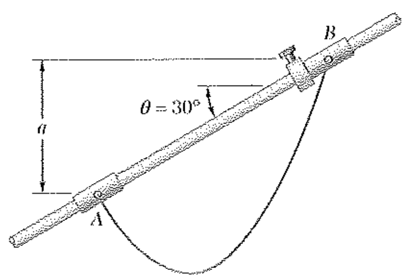
Choose new trial value of θ and repeat above procedure until calculated value of θ is equal to given value of θ.

For $\theta = 30°$

Result of trial and error procedure:

$$c = 1.803 \text{ ft}$$
$$x_A = 2.3745 \text{ ft}$$
$$x_B = 3.6937 \text{ ft}$$
$$y_A = 3.606 \text{ ft}$$
$$y_B = 7.109 \text{ ft}$$
$$\begin{aligned} a &= y_B - y_A \\ &= 7.109 \text{ ft} - 3.606 \text{ ft} \\ &= 3.503 \text{ ft} \end{aligned}$$

$a = 3.50$ ft ◀



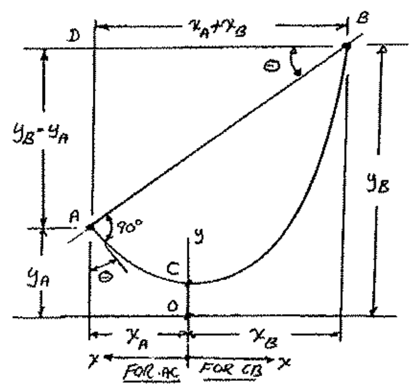
PROBLEM 7.148*

Solve Problem 7.147 assuming that the angle θ formed by the rod and the horizontal is 45°.

PROBLEM 7.147 The 10-ft cable AB is attached to two collars as shown. The collar at A can slide freely along the rod; a stop attached to the rod prevents the collar at B from moving on the rod. Neglecting the effect of friction and the weight of the collars, determine the distance a.

SOLUTION

Collar at A: Since $\mu = 0$, cable $\perp$ rod



Point A:

$$y = c\cosh\frac{x}{c};\quad \frac{dy}{dx} = \sinh\frac{x}{c}$$

$$\tan\theta = \left|\frac{dy}{dx}\right|_A = \sinh\frac{x_A}{c}$$

$$\frac{x_A}{c} = \sinh(\tan(90° - \theta))$$

$$x_A = c\sinh(\tan(90° - \theta)) \tag{1}$$

Length of cable = 10 ft

$$10\text{ ft} = AC + CB$$

$$10 = c\sinh\frac{x_A}{c} + c\sinh\frac{x_B}{c}$$

$$\sinh\frac{x_B}{c} = \frac{10}{c} - \sinh\frac{x_A}{c}$$

$$x_B = c\sinh^{-1}\left[\frac{10}{c} - \sinh\frac{x_A}{c}\right] \tag{2}$$

$$y_A = c\cosh\frac{x_A}{c}\quad y_B = c\cosh\frac{x_B}{c} \tag{3}$$

PROBLEM 7.148* (Continued)

<u>In Δ *ABD*</u>: $$\tan\theta = \frac{y_B - y_A}{x_B + x_A} \qquad (4)$$

<u>Method of solution</u>:

For given value of θ, choose trial value of c and calculate:

From Eq. (1): x_A

Using value of x_A and c, calculate:

From Eq. (2): x_B

From Eq. (3): y_A and y_B

Substitute values obtained for x_A, x_B, y_A, y_B into Eq. (4) and calculate θ

Choose new trial value of θ and repeat above procedure until calculated value of θ is equal to given value of θ.

For $\theta = 45°$

Result of trial and error procedure:

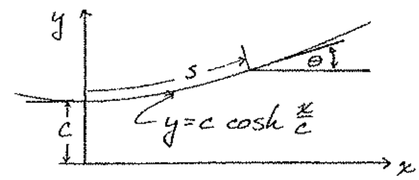
$$c = 1.8652 \text{ ft}$$
$$x_A = 1.644 \text{ ft}$$
$$x_B = 4.064 \text{ ft}$$
$$y_A = 2.638 \text{ ft}$$
$$y_B = 8.346 \text{ ft}$$
$$a = y_B - y_A$$
$$= 8.346 \text{ ft} - 2.638 \text{ ft}$$
$$= 5.708 \text{ ft}$$

$a = 5.71$ ft ◀

PROBLEM 7.149

Denoting by θ the angle formed by a uniform cable and the horizontal, show that at any point (*a*) $s = c \tan \theta$, (*b*) $y = c \sec \theta$.

SOLUTION



(*a*) $$\tan\theta = \frac{dy}{dx} = \sinh\frac{x}{c}$$

$$s = c\sinh\frac{x}{c} = c\tan\theta \qquad \text{Q.E.D.}$$

(*b*) Also $$y^2 = s^2 + c^2 (\cosh^2 x = \sinh^2 x + 1)$$

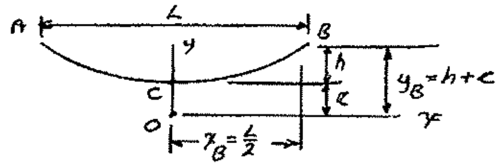
so $$y^2 = c^2(\tan^2\theta + 1)c^2\sec^2\theta$$

and $$y = c\sec\theta \qquad \text{Q.E.D.}$$

PROBLEM 7.150*

(a) Determine the maximum allowable horizontal span for a uniform cable of weight per unit length w if the tension in the cable is not to exceed a given value T_m. (b) Using the result of part a, determine the maximum span of a steel wire for which $w = 0.25$ lb/ft and $T_m = 8000$ lb.

SOLUTION



(a)

$$T_m = wy_B$$

$$= wc\cosh\frac{x_B}{c}$$

$$= wx_B\left(\frac{1}{\frac{x_B}{c}}\right)\cosh\frac{x_B}{c}$$

We shall find ratio $\left(\frac{x_B}{c}\right)$ for when T_m is minimum

$$\frac{dT_m}{d\left(\frac{x_B}{c}\right)} = wx_B\left[\frac{1}{\frac{x_B}{c}}\sinh\frac{x_B}{c} - \left(\frac{1}{\frac{x_B}{c}}\right)^2\cosh\frac{x_B}{c}\right] = 0$$

$$\frac{\sinh\frac{x_B}{c}}{\cosh\frac{x_B}{c}} = \frac{1}{\frac{x_B}{c}}$$

$$\tanh\frac{x_B}{c} = \frac{c}{x_B}$$

Solve by trial and error for: $\frac{x_B}{c} = 1.200$ (1)

$s_B = c\sinh\frac{x_B}{c} = c\sinh(1.200)$: $\frac{s_B}{c} = 1.509$

Eq. 7.17: $y_B^2 - s_B^2 = c^2$

$$y_B^2 = c^2\left[1 + \left(\frac{s_B}{c}\right)^2\right] = c^2(1 + 1.509^2)$$

$$y_B = 1.810c$$

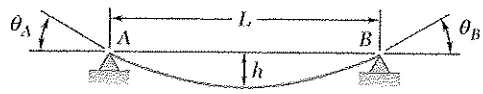
Eq. 7.18:
$$T_m = wy_B$$
$$= 1.810\,wc$$
$$c = \frac{T_m}{1.810\,w}$$

Eq. (1):
$$x_B = 1.200c = 1.200\frac{T_m}{1.810w} = 0.6630\frac{T_m}{w}$$

Span:
$$L = 2x_B = 2(0.6630)\frac{T_m}{w}$$
$$L = 1.326\frac{T_m}{w} \blacktriangleleft$$

(*b*) For $w = 0.25\text{ lb/ft}$ and $T_m = 8000\text{ lb}$

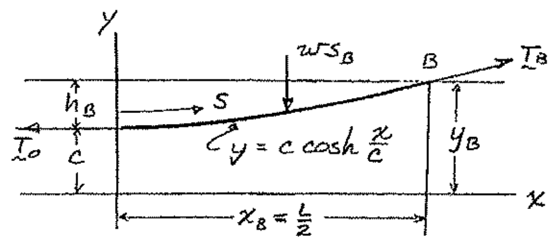
$$L = 1.326\frac{8000\text{ lb}}{0.25\text{ lb/ft}}$$
$$= 42{,}432\text{ ft}$$
$$L = 8.04\ \text{miles} \blacktriangleleft$$



PROBLEM 7.151*

A cable has a mass per unit length of 3 kg/m and is supported as shown. Knowing that the span L is 6 m, determine the *two* values of the sag h for which the maximum tension is 350 N.

SOLUTION



$$y_{max} = c\cosh\frac{L}{2c} = h + c$$

$$w = (3\text{ kg/m})(9.81\text{ m/s}^2) = 29.43\text{ N/m}$$

$$T_{max} = wy_{max}$$

$$y_{max} = \frac{T_{max}}{w}$$

$$y_{max} = \frac{350\text{ N}}{29.43\text{ N/m}} = 11.893\text{ m}$$

$$c\cosh\frac{3\text{ m}}{c} = 11.893\text{ m}$$

Solving numerically

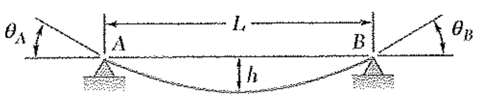
$$c_1 = 0.9241\text{ m}$$

$$c_2 = 11.499\text{ m}$$

$$h = y_{max} - c$$

$$h_1 = 11.893\text{ m} - 0.9241\text{ m}$$ $h_1 = 10.97\text{ m}$ ◀

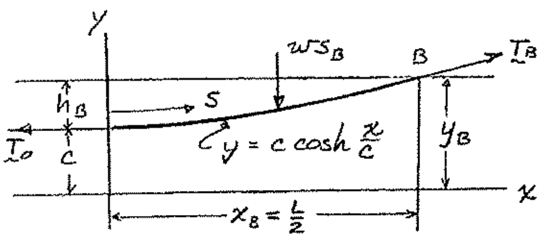
$$h_2 = 11.893\text{ m} - 11.499\text{ m}$$ $h_2 = 0.394\text{ m}$ ◀



PROBLEM 7.152*

Determine the sag-to-span ratio for which the maximum tension in the cable is equal to the total weight of the entire cable AB.

SOLUTION



$$T_{\max} = wy_B = 2ws_B$$

$$y_B = 2s_B$$

$$c\cosh\frac{L}{2c} = 2c\sinh\frac{L}{2c}$$

$$\tanh\frac{L}{2c} = \frac{1}{2}$$

$$\frac{L}{2c} = \tanh^{-1}\frac{1}{2} = 0.549306$$

$$\frac{h_B}{c} = \frac{y_B - c}{c} = \cosh\frac{L}{2c} - 1 = 0.154701$$

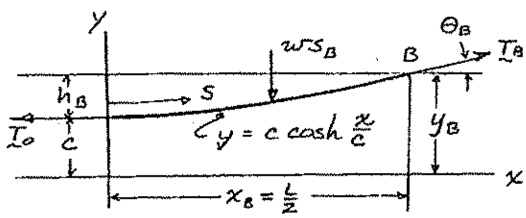
$$\frac{h_B}{L} = \frac{\frac{h_B}{c}}{2\left(\frac{L}{2c}\right)} = \frac{0.5(0.154701)}{0.549306} = 0.14081$$

$$\frac{h_B}{L} = 0.1408 \blacktriangleleft$$

PROBLEM 7.153*

A cable of weight w per unit length is suspended between two points at the same elevation that are a distance L apart. Determine (*a*) the sag-to-span ratio for which the maximum tension is as small as possible, (*b*) the corresponding values of θ_B and T_m.

SOLUTION



(*a*)

$$T_{\max} = wy_B = wc\cosh\frac{L}{2c}$$

$$\frac{dT_{\max}}{dc} = w\left(\cosh\frac{L}{2c} - \frac{L}{2c}\sinh\frac{L}{2c}\right)$$

For $\min T_{\max}$, $\dfrac{dT_{\max}}{dc} = 0$

$$\tanh\frac{L}{2c} = \frac{2c}{L} \rightarrow \frac{L}{2c} = 1.1997$$

$$\frac{y_B}{c} = \cosh\frac{L}{2c} = 1.8102$$

$$\frac{h}{c} = \frac{y_B}{c} - 1 = 0.8102$$

$$\frac{h}{L} = \left[\frac{1}{2}\frac{h}{c}\left(\frac{2c}{L}\right)\right] = \frac{0.8102}{2(1.1997)} = 0.3375$$

$\dfrac{h}{L} = 0.338$ ◀

(*b*)

$$T_0 = wc \quad T_{\max} = wc\cosh\frac{L}{2c} \quad \frac{T_{\max}}{T_0} = \cosh\frac{L}{2c} = \frac{y_B}{c}$$

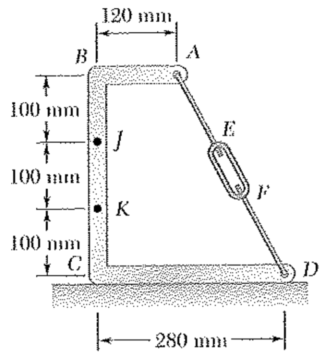
But $T_0 = T_{\max}\cos\theta_B \quad \dfrac{T_{\max}}{T_0} = \sec\theta_B$

So $\theta_B = \sec^{-1}\left(\dfrac{y_B}{c}\right) = \sec^{-1}(1.8102) = 56.46°$

$\theta_B = 56.5°$ ◀

$$T_{\max} = wy_B = w\frac{y_B}{c}\left(\frac{2c}{L}\right)\left(\frac{L}{2}\right) = w(1.8102)\frac{L}{2(1.1997)}$$

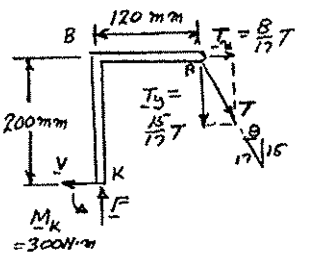
$T_{\max} = 0.755wL$ ◀



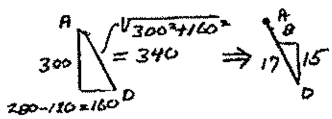
PROBLEM 7.154

It has been experimentally determined that the bending moment at Point K of the frame shown is 300 N · m. Determine (a) the tension in rods AE and FD, (b) the corresponding internal forces at Point J.

SOLUTION



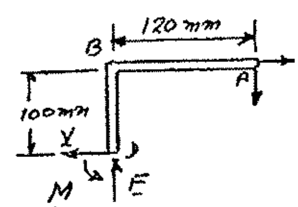
Free body: ABK



(a) $$+\circlearrowleft \Sigma M_K = 0: \quad 300\text{ N}\cdot\text{m} - \frac{8}{17}T(0.2\text{ m}) - \frac{15}{17}T(0.12\text{ m}) = 0$$ $T = 1500\text{ N}$ ◀

Free body: AJ

$$T_x = \frac{8}{17}T = \frac{8}{17}(1500\text{ N})$$
$$= 705.88\text{ N}$$
$$T_y = \frac{15}{17}T = \frac{15}{17}(1500\text{ N})$$
$$= 1323.53\text{ N}$$



Internal forces on ABJ

(b) $$\xrightarrow{+} \Sigma F_x = 0: \quad 705.88\text{ N} - V = 0$$

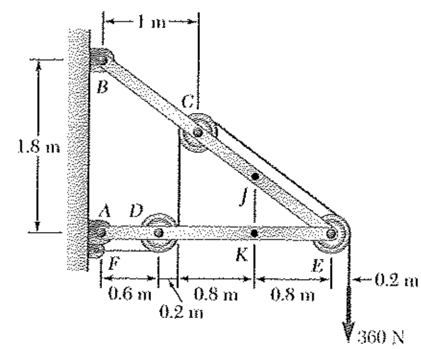
$$V = +705.88\text{ N}$$ $\mathbf{V} = 706\text{ N} \leftarrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad F - 1323.53\text{ N} = 0$$

$$F = +1323.53\text{ N}$$ $\mathbf{F} = 1324\text{ N}\uparrow$ ◀

$$+\circlearrowleft \Sigma M_J = 0: \quad M - (705.88\text{ N})(0.1\text{ m}) - (1323.53\text{ N})(0.12\text{ m}) = 0$$
$$M = +229.4\text{ N}\cdot\text{m}$$

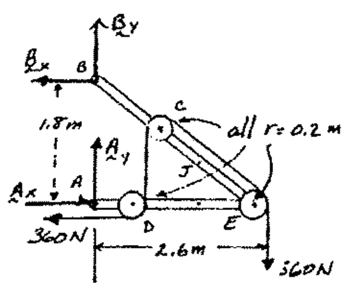
$\mathbf{M} = 229\text{ N}\cdot\text{m}\circlearrowleft$ ◀



PROBLEM 7.155

Knowing that the radius of each pulley is 200 mm and neglecting friction, determine the internal forces at Point J of the frame shown.

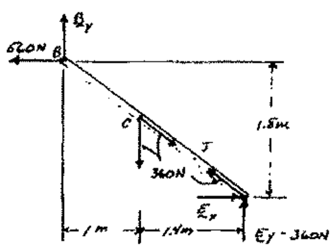
SOLUTION



FBD Frame with pulley and cord:

$$\circlearrowleft \Sigma M_A = 0: \quad (1.8\text{ m})B_x - (2.6\text{ m})(360\text{ N}) - (0.2\text{ m})(360\text{ N}) = 0$$

$$\mathbf{B}_x = 560\text{ N} \longleftarrow$$

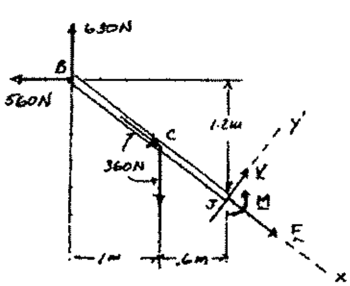


FBD BE:

Note: Cord forces have been moved to pulley hub as per Problem 6.91.

$$\circlearrowleft \Sigma M_E = 0: \quad (1.4\text{ m})(360\text{ N}) + (1.8\text{ m})(560\text{ N}) - (2.4\text{ m})B_y = 0$$

$$\mathbf{B}_y = 630\text{ N} \uparrow$$



FBD BJ:

$$\searrow \Sigma F_{x'} = 0: \quad F + 360\text{ N} - \frac{3}{5}(630\text{ N} - 360\text{ N}) - \frac{4}{5}(560\text{ N}) = 0$$

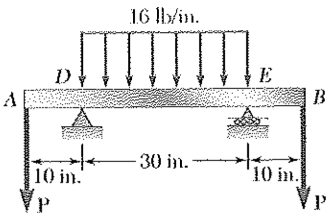
$$\mathbf{F} = 250\text{ N} \nwarrow \blacktriangleleft$$

$$\nearrow \Sigma F_{y'} = 0: \quad V + \frac{4}{5}(630\text{ N} - 360\text{ N}) - \frac{3}{5}(560\text{ N}) = 0$$

$$\mathbf{V} = 120.0\text{ N} \nearrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: \quad M + (0.6\text{ m})(360\text{ N}) + (1.2\text{ m})(560\text{ N}) - (1.6\text{ m})(630\text{ N}) = 0$$

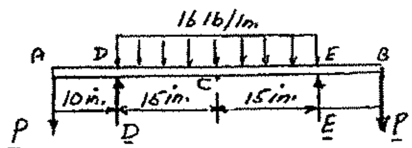
$$\mathbf{M} = 120.0\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$



PROBLEM 7.157

For the beam shown, determine (*a*) the magnitude P of the two concentrated loads for which the maximum absolute value of the bending moment is as small as possible, (*b*) the corresponding value of $|M|_{max}$.

SOLUTION

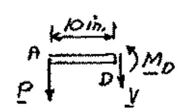


Free body: Entire beam

By symmetry

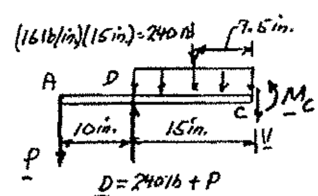
$$D = E = \frac{1}{2}(16 \text{ lb/in.})(30 \text{ in.}) + P$$

$$D = 240 \text{ lb} + P$$



Free body: Portion AD

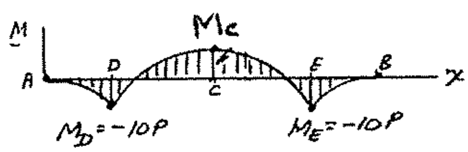
$$+\circlearrowleft \Sigma M_D = 0: \quad M_D = -10P$$



Free body: Portion ADC

$$+\circlearrowleft \Sigma M_C = 0: \quad P(25 \text{ in.}) - (240 \text{ lb} + P)(15 \text{ in.}) + (240 \text{ lb})(7.5 \text{ in.}) + M_C = 0$$

$$M_C = 1800 \text{ lb}\cdot\text{in.} - (10 \text{ in.})P$$



(*a*) We equate: $|M_D| = |M_C|$

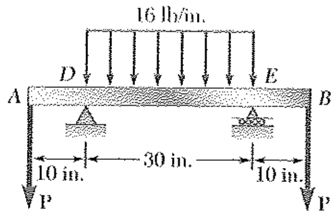
$$|-10P| = |1800 - 10P|$$

$$10P = 1800 - 10P$$

$P = 90.0$ lb ◀

(*b*) For $P = 90$ lb: $M_D = -10 \text{ lb}(90 \text{ lb})$

$|M|_{max} = 900$ lb·in. ◀



PROBLEM 7.158

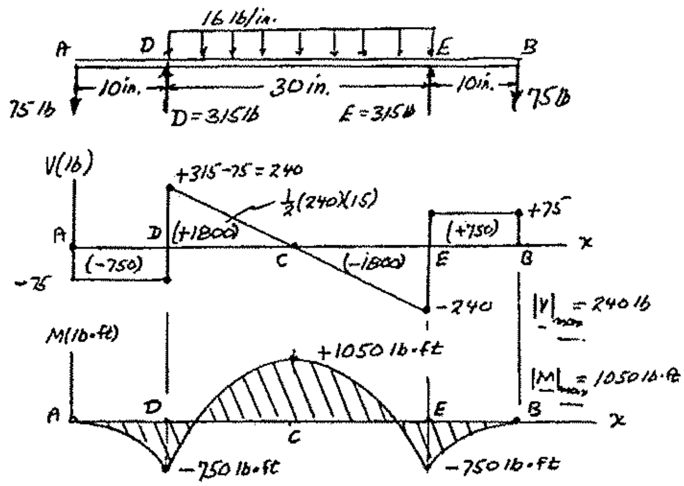
Knowing that the magnitude of the concentrated loads **P** is 75 lb, (*a*) draw the shear and bending-moment diagrams for beam *AB*, (*b*) determine the maximum absolute values of the shear and bending moment.

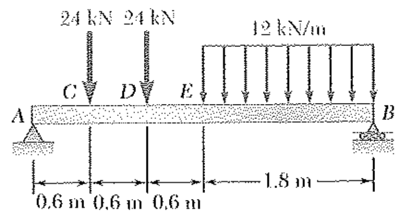
SOLUTION

By symmetry: $D = E$

$$= \frac{1}{2}(16 \text{ lb/in.})(30 \text{ in.}) + 75 \text{ lb}$$

$$= 315 \text{ lb}$$



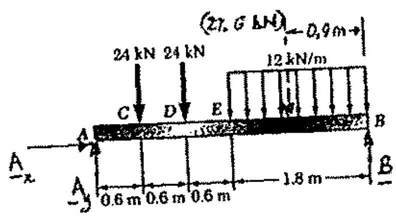


PROBLEM 7.159

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



$$+\circlearrowleft \Sigma M_B = 0: \quad (24\text{ kN})(3\text{ m})$$

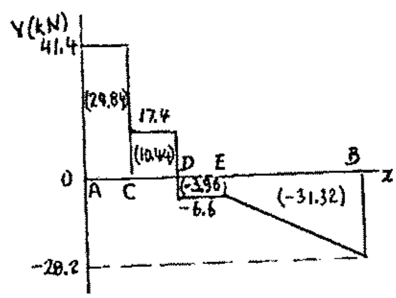
$$+(24\text{ kN})(2.4\text{ m}) + (21.6\text{ kN})(0.9\text{ m})$$

$$-A_y(3.6\text{ m}) = 0$$

$$A_y = +91.4\text{ kN}$$

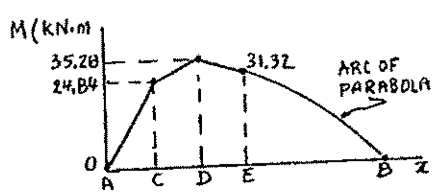
$$\Sigma F_x = 0: \quad A_x = 0$$

Shear diagram



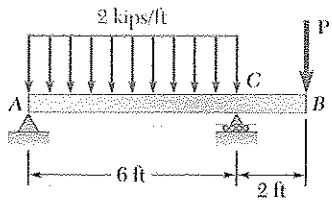
At A: $V_A = A_y = +41.4\text{ kN}$ $\qquad |V|_{max} = 41.4\text{ kN}$ ◀

Bending-moment diagram



At A: $M_A = 0$ $\qquad |M|_{max} = 35.3\text{ kN}\cdot\text{m}$ ◀

The slope of the parabola at E is the same as that of the segment DE

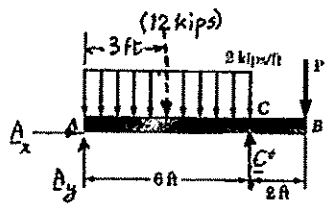


PROBLEM 7.160

For the beam shown, draw the shear and bending-moment diagrams, and determine the magnitude and location of the maximum absolute value of the bending moment, knowing that (*a*) $P = 6$ kips, (*b*) $P = 3$ kips.

SOLUTION

Free body: Beam



$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_A = 0: \quad C(6\text{ ft}) - (12\text{ kips})(3\text{ ft}) - P(8\text{ ft}) = 0$$

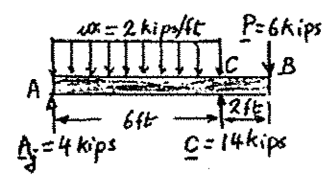
$$C = 6\text{ kips} + \frac{4}{3}P \qquad (1) \triangleleft$$

$$\Sigma F_y = 0: \quad A_y + \left(6 + \frac{4}{3}P\right) - 12 - P = 0$$

$$A_y = 6\text{ kips} - \frac{1}{3}P \qquad (2) \triangleleft$$

(*a*) $P = 6$ kips.

Load diagram

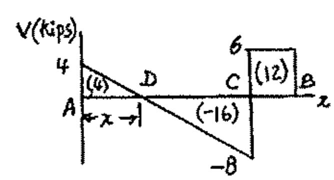


Substituting for P in Eqs. (2) and (1):

$$A_y = 6 - \frac{1}{3}(6) = 4\text{ kips}$$

$$C = 6 + \frac{4}{3}(6) = 14\text{ kips}$$

Shear diagram $\qquad V_A = A_y = +4$ kips



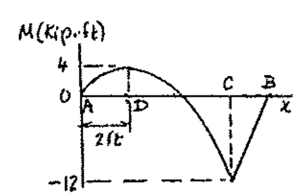
To determine Point D where $V = 0$:

$$V_D - V_A = -wx$$

$$0 - 4\text{ kips} = (2\text{ kips/ft})x \qquad x = 2\text{ ft} \triangleleft$$

We compute all areas

Bending-moment diagram

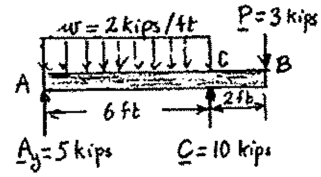


At A: $\qquad M_A = 0$

$$|M|_{max} = 12.00\text{ kip}\cdot\text{ft, at } C \blacktriangleleft$$

Parabola from A to C

(*b*) $P = 3$ kips

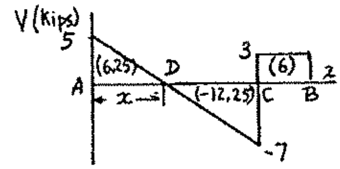


Load diagram

Substituting for P in Eqs. (2) and (1):

$$A = 6 - \frac{1}{3}(3) = 5 \text{ kips}$$

$$C = 6 + \frac{4}{3}(3) = 10 \text{ kips}$$

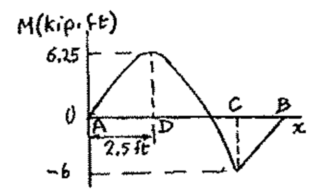


Shear diagram $\qquad V_A = A_y = +5 \text{ kips}$

To determine D where $V = 0$:

$$V_D - V_A = -wx$$
$$0 - (5 \text{ kips}) = -(2 \text{ kips/ft})x$$

$x = 2.5$ ft ◁



We compute all areas

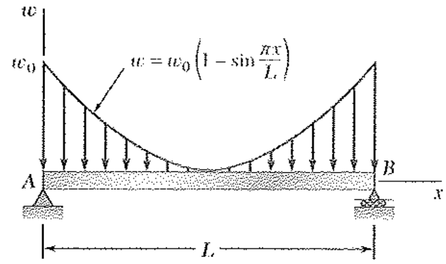
Bending-moment diagram

At A: $\qquad M_A = 0$

$|M|_{max} = 6.25$ kip·ft ◀

2.50 ft from A ◀

Parabola from A to C.



PROBLEM 7.161

For the beam and loading shown, (*a*) write the equations of the shear and bending-moment curves, (*b*) determine the maximum bending moment.

SOLUTION

(*a*) Reactions at supports: $A = B = \frac{1}{2}W$, where $\frac{W}{L}$ = Total load

$$W = \int_0^L w dx = w_0 \int_0^L \left(1 - \sin\frac{\pi x}{L}\right) dx$$

$$= w_0 \left[x + \frac{L}{x}\cos\frac{\pi x}{L} \right]_0^L$$

$$= w_0 L\left(1 - \frac{2}{\pi}\right)$$

Thus $$V_A = A = \frac{1}{2}W = \frac{1}{2}w_0 L\left(1 - \frac{2}{\pi}\right)$$

$$M_A = 0 \qquad (1)$$

Load: $$w(x) = w_0\left(1 - \sin\frac{\pi x}{L}\right)$$

Shear: From Eq. (7.2):

$$V(x) - V_A = -\int_0^x w(x) dx$$

$$= -w_0 \int_0^x \left(1 - \sin\frac{\pi x}{L}\right) dx$$

Integrating and recalling first of Eqs. (1),

$$V(x) - \frac{1}{2}w_0 L\left(1 - \frac{2}{\pi}\right) = -w_0\left[x + \frac{L}{\pi}\cos\frac{\pi x}{L}\right]_0^x$$

$$V(x) = \frac{1}{2}w_0 L\left(1 - \frac{2}{\pi}\right) - w_0\left(2 + \frac{L}{\pi}\cos\frac{\pi x}{L}\right) + w_0\frac{L}{\pi}$$

$$V(x) = w_0\left(\frac{L}{2} - x - \frac{L}{\pi}\cos\frac{\pi x}{L}\right) \qquad (2) ◄$$

Bending moment: From Eq. (7.4) and recalling that $M_A = 0$.

$$M(x) - M_A = \int_0^x V(x)dx$$

$$= w_0\left[\frac{L}{2}x - \frac{1}{2}x^2 - \left(\frac{L}{\pi}\right)^2 \sin\frac{\pi x}{L}\right]_0^x$$

$$M(x) = \frac{1}{2}w_0\left(Lx - x^2 - \frac{2L^2}{\pi^2}\sin\frac{\pi x}{L}\right) \qquad (3) ◀$$

(*b*) Maximum bending moment

$$\frac{dM}{dx} = V = 0.$$

This occurs at $x = \frac{L}{2}$ as we may check from (2):

$$V\left(\frac{L}{2}\right) = w_0\left(\frac{L}{2} - \frac{L}{2} - \frac{L}{\pi}\cos\frac{\pi}{2}\right) = 0$$

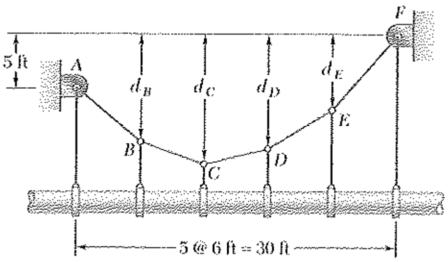
From (3):

$$M\left(\frac{L}{2}\right) = \frac{1}{2}w_0\left(\frac{L^2}{2} - \frac{L^2}{4} - \frac{2L^2}{\pi^2}\sin\frac{\pi}{2}\right)$$

$$= \frac{1}{8}w_0L^2\left(1 - \frac{8}{\pi^2}\right)$$

$$= 0.0237w_0L^2$$

$$M_{\max} = 0.0237w_0L^2, \quad \text{at} \quad x = \frac{L}{2} \qquad ◀$$

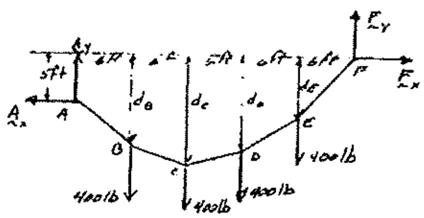


PROBLEM 7.162

An oil pipeline is supported at 6-ft intervals by vertical hangers attached to the cable shown. Due to the combined weight of the pipe and its contents the tension in each hanger is 400 lb. Knowing that $d_C = 12$ ft, determine (*a*) the maximum tension in the cable, (*b*) the distance d_D.

SOLUTION

FBD Cable: Hanger forces at A and F act on the supports, so $\mathbf{A}_y$ and $\mathbf{F}_y$ act on the cable.



$$\circlearrowleft \Sigma M_F = 0: \quad (6\text{ ft} + 12\text{ ft} + 18\text{ ft} + 24\text{ ft})(400\text{ lb})$$

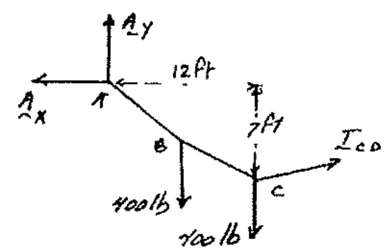
$$-(30\text{ ft})A_y - (5\text{ ft})A_x = 0$$

$$A_x + 6A_y = 4800\text{ lb} \qquad (1)$$

FBD ABC:

$$\circlearrowleft \Sigma M_C = 0: \quad (7\text{ ft})A_x - (12\text{ ft})A_y + (6\text{ ft})(400\text{ lb}) = 0 \qquad (2)$$

Solving (1) and (2)



$$\mathbf{A}_x = 800\text{ lb} \longleftarrow$$

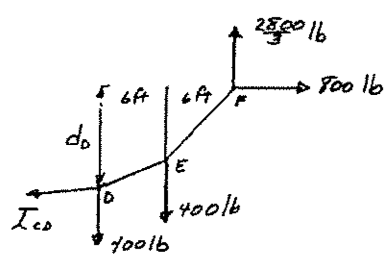
$$\mathbf{A}_y = \frac{2000}{3}\text{ lb}\uparrow$$

From FBD Cable:

$$\longrightarrow \Sigma F_x = 0: \quad -800\text{ lb} + F_x = 0$$

$$\mathbf{F}_x = 800\text{ lb} \longrightarrow$$

FBD DEF:



$$\uparrow \Sigma F_y = 0: \quad \frac{200}{3}\text{ lb} - 4(400\text{ lb}) + F_y = 0$$

$$\mathbf{F}_y = \frac{2800}{3}\text{ lb}\uparrow$$

Since $A_x = F_x$ and $F_y > A_y$,

$$T_{max} = T_{EF} = \sqrt{(800\text{ lb})^2 + \left(\frac{2800}{3}\text{ lb}\right)^2}$$

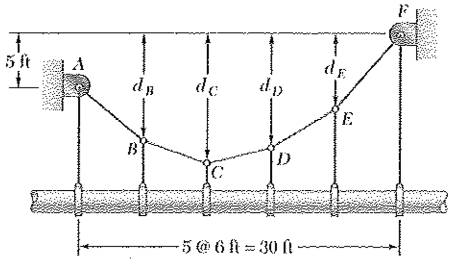
PROBLEM 7.162 (Continued)

(*a*) $T_{max} = 1229.27$ lb, $T_{max} = 1229$ lb ◀

$$\circlearrowleft \Sigma M_D = 0: \quad (12 \text{ ft})\left(\frac{2800}{3}\text{lb}\right) - d_D(800 \text{ lb}) - (6 \text{ ft})(400 \text{ lb}) = 0$$

(*b*) $d_D = 11.00$ ft ◀

PROBLEM 7.163

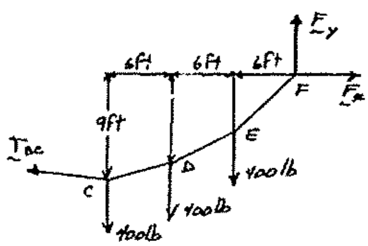


Solve Problem 7.162 assuming that $d_C = 9$ ft.

PROBLEM 7.162 An oil pipeline is supported at 6-ft intervals by vertical hangers attached to the cable shown. Due to the combined weight of the pipe and its contents the tension in each hanger is 400 lb. Knowing that $d_C = 12$ ft, determine (*a*) the maximum tension in the cable, (*b*) the distance d_D.

SOLUTION

FBD CDEF:



$\circlearrowleft \Sigma M_C = 0$: $(18\text{ ft})F_y - (9\text{ ft})F_y - (6\text{ ft} + 12\text{ ft})(400\text{ lb}) = 0$

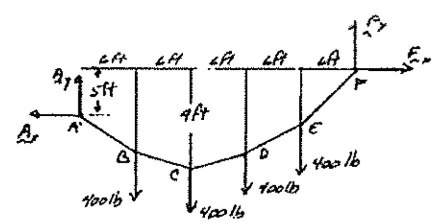
$$F_x - 2F_y = -800\text{ lb} \qquad (1)$$

FBD Cable:

$\circlearrowleft EM_A = 0$: $(30\text{ ft})F_y - (5\text{ ft})F_x$

$-(6\text{ ft})(1 + 2 + 3 + 4)(400\text{ lb}) = 0$

$$F_x - 6F_y = -4800\text{ lb} \qquad (2)$$

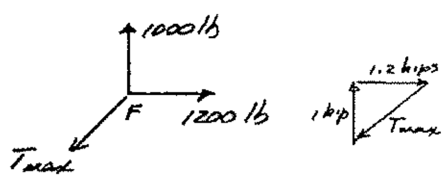


Solving (1) and (2),

$$\mathbf{F}_x = 1200\text{ lb} \longrightarrow, \quad \mathbf{F}_y = 1000\text{ lb}\uparrow$$

$$\longrightarrow \Sigma F_x = 0: \quad -A_x + 1200\text{ lb} = 0, \quad \mathbf{A}_x = 1200\text{ lb} \longrightarrow$$

Point F:



$$\uparrow \Sigma F_y = 0: \quad A_y + 1000\text{ lb} - 4(400\text{ lb}) = 0, \qquad \mathbf{A}_y = 600\text{ lb}\uparrow$$

PROBLEM 7.163 (Continued)

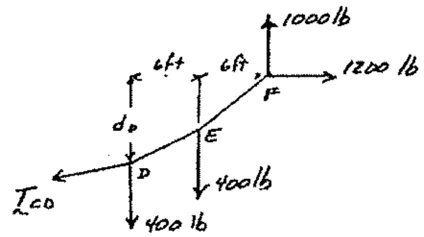
Since

$$A_x = A_y \quad \text{and} \quad F_y > A_y,\ T_{\max} = T_{EF}$$
$$T_{\max} = \sqrt{(1\text{ kip})^2 + (1.2\text{ kips})^2}$$

(*a*) $T_{\max} = 1.562$ kips ◀

FBD DEF:

$$\circlearrowleft \Sigma M_D = 0\text{:}\quad (12\text{ ft})(1000\text{ lb}) - d_D(1200\text{ lb}) - (6\text{ ft})(400\text{ lb}) = 0$$

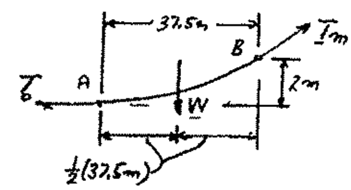


(*b*) $d_D = 8.00$ ft ◀

PROBLEM 7.164

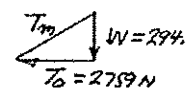
A transmission cable having a mass per unit length of 0.8 kg/m is strung between two insulators at the same elevation that are 75 m apart. Knowing that the sag of the cable is 2 m, determine (*a*) the maximum tension in the cable, (*b*) the length of the cable.

SOLUTION



$$w = (0.8\text{ kg/m})(9.81\text{ m/s}^2)$$
$$= 7.848\text{ N/m}$$
$$W = (7.848\text{ N/m})(37.5\text{ m})$$
$$W = 294.3\text{ N}$$

(*a*) $$+\circlearrowleft \Sigma M_B = 0:\quad T_0(2\text{ m}) - W\left(\frac{1}{2}37.5\text{ m}\right) = 0$$



$$T_0(2\text{ m}) - (294.3\text{ N})\frac{1}{2}(37.5\text{ m}) = 0$$

$$T_0 = 2759\text{ N}$$

$$T_m^2 = (294.3\text{ N})^2 + (2759\text{ N})^2$$

$T_m = 2770$ N ◀

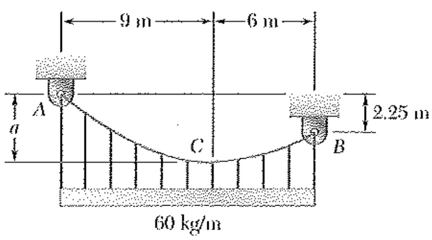
(*b*) $$s_B = x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2 + \cdots\right]$$

$$= 37.5\text{ m}\left[1 + \frac{2}{3}\left(\frac{2\text{ m}}{37.5\text{ m}}\right)^2 + \cdots\right]$$

$$= 37.57\text{ m}$$

Length $= 2s_B = 2(37.57\text{ m})$

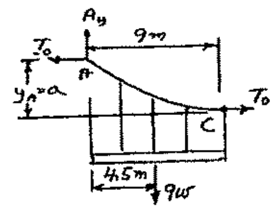
Length = 75.14 m ◀



PROBLEM 7.165

Cable ACB supports a load uniformly distributed along the horizontal as shown. The lowest Point C is located 9 m to the right of A. Determine (a) the vertical distance a, (b) the length of the cable, (c) the components of the reaction at A.

SOLUTION



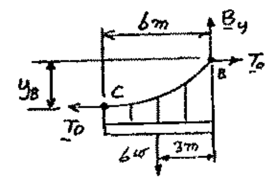
Free body: Portion AC

$$+\uparrow \Sigma F_y = 0: \quad A_y - 9w = 0$$

$$\mathbf{A}_y = 9w \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad T_0 a - (9w)(4.5 \text{ m}) = 0 \qquad (1)$$

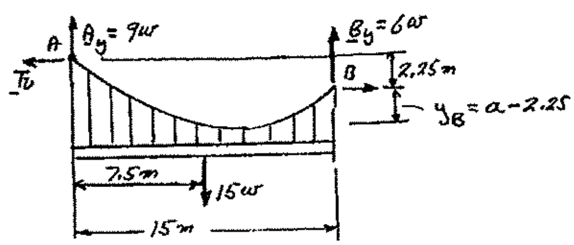
Free body: Portion CB



$$+\uparrow \Sigma F_y = 0: \quad B_y - 6w = 0$$

$$\mathbf{B}_y = 6w \uparrow$$

Free body: Entire cable



$$+\circlearrowleft \Sigma M_A = 0: \quad 15w(7.5 \text{ m}) - 6w(15 \text{ m}) - T_0(2.25 \text{ m}) = 0$$

(a) $\qquad T_0 = 10w$

Eq. (1): $\quad T_0 a - (9w)(4.5 \text{ m}) = 0$

$$10wa = (9w)(4.5) = 0 \qquad a = 4.05 \text{ m} ◀$$

(*b*) Length $= AC + CB$

Portion AC:

$$x_A = 9 \text{ m}, \quad y_A = a = 4.05 \text{ m}; \quad \frac{y_A}{x_A} = \frac{4.05}{9} = 0.45$$

$$s_{AC} = x_B \left[1 + \frac{2}{3}\left(\frac{y_A}{x_A}\right)^2 - \frac{2}{5}\left(\frac{y_B}{x_A}\right)^4 + \cdots \right]$$

$$s_{AC} = 9 \text{ m}\left(1 + \frac{2}{3}0.45^2 - \frac{2}{5}0.45^4 + \cdots\right) = 10.067 \text{ m}$$

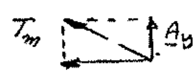
Portion CB:

$$x_B = 6 \text{ m}, \quad y_B = 4.05 - 2.25 = 1.8 \text{ m}; \quad \frac{y_B}{x_B} = 0.3$$

$$s_{CB} = 6 \text{ m}\left(1 + \frac{2}{3}0.3^2 - \frac{2}{5}0.3^4 + \cdots\right) = 6.341 \text{ m}$$

Total length $= 10.067 \text{ m} + 6.341 \text{ m}$ — Total length $= 16.41$ m ◀

(*c*) Components of reaction at A.



$$A_y = 9w = 9(60 \text{ kg/m})(9.81 \text{ m/s}^2)$$
$$= 5297.4 \text{ N}$$
$$A_x = T_0 = 10w = 10(60 \text{ kg/m})(9.81 \text{ m/s}^2)$$
$$= 5886 \text{ N}$$

$\mathbf{A}_x = 5890$ N ← ◀

$\mathbf{A}_y = 5300$ N ↑ ◀