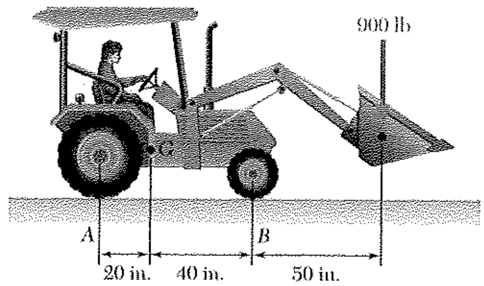
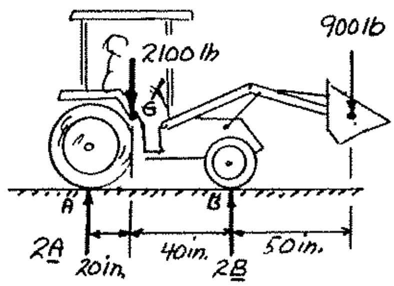
CHAPTER 4



PROBLEM 4.1

A 2100-lb tractor is used to lift 900 lb of gravel. Determine the reaction at each of the two (*a*) rear wheels *A*, (*b*) front wheels *B*.

SOLUTION



(*a*) Rear wheels

$$+\circlearrowleft \Sigma M_B = 0: \quad +(2100 \text{ lb})(40 \text{ in.}) - (900 \text{ lb})(50 \text{ in.}) + 2A(60 \text{ in.}) = 0$$

$$A = +325 \text{ lb} \qquad \mathbf{A} = 325 \text{ lb} \uparrow \blacktriangleleft$$

(*b*) Front wheels

$$+\circlearrowleft \Sigma M_A: \quad -(2100 \text{ lb})(20 \text{ in.}) - (900 \text{ lb})(110 \text{ in.}) - 2B(60 \text{ in.}) = 0$$

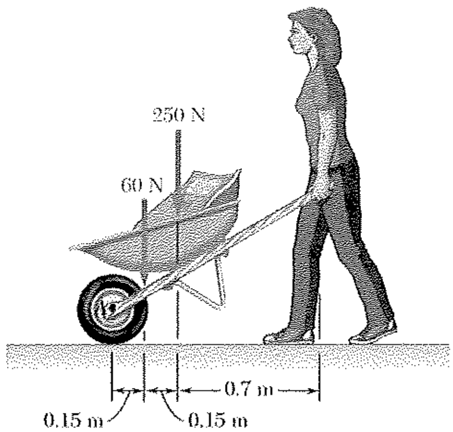
$$B = +1175 \text{ lb} \qquad \mathbf{B} = 1175 \text{ lb} \uparrow \blacktriangleleft$$

Check:

$$+\Sigma F_y = 0: \quad 2A + 2B - 2100 \text{ lb} - 900 \text{ lb} = 0$$

$$2(325 \text{ lb}) + 2(1175 \text{ lb}) - 2100 \text{ lb} - 900 = 0$$

$$0 = 0 \quad \text{(Checks)}$$

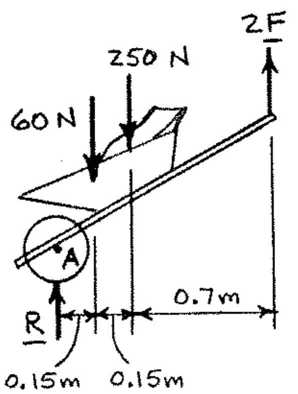


PROBLEM 4.2

A gardener uses a 60-N wheelbarrow to transport a 250-N bag of fertilizer. What force must she exert on each handle?

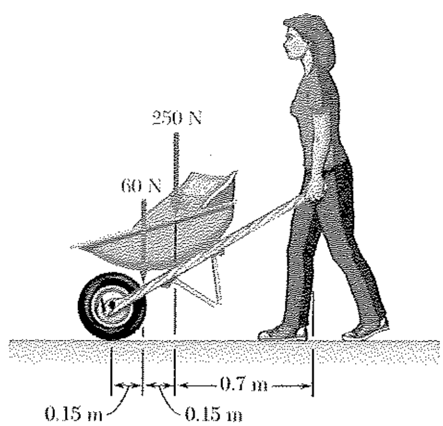
SOLUTION

Free-Body Diagram:



$$+\circlearrowleft \Sigma M_A = 0: \quad (2F)(1\text{ m}) - (60\text{ N})(0.15\text{ m}) - (250\text{ N})(0.3\text{ m}) = 0$$

$$\mathbf{F} = 42.0\text{ N}\uparrow \blacktriangleleft$$



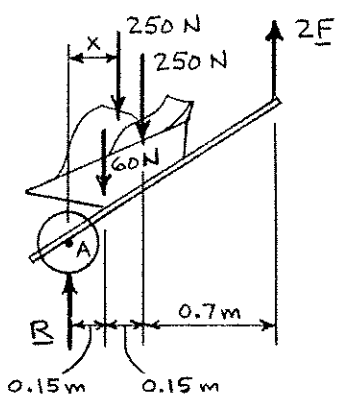
PROBLEM 4.3

The gardener of Problem 4.2 wishes to transport a second 250-N bag of fertilizer at the same time as the first one. Determine the maximum allowable horizontal distance from the axle A of the wheelbarrow to the center of gravity of the second bag if she can hold only 75 N with each arm.

PROBLEM 4.2 A gardener uses a 60-N wheelbarrow to transport a 250-N bag of fertilizer. What force must she exert on each handle?

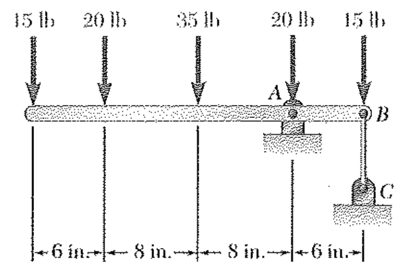
SOLUTION

Free-Body Diagram:



$$+\circlearrowleft \Sigma M_A = 0: \quad 2(75\text{ N})(1\text{ m}) - (60\text{ N})(0.15\text{ m}) - (250\text{ N})(0.3\text{ m}) - (250\text{ N})x = 0$$

$$x = 0.264\text{ m} \blacktriangleleft$$

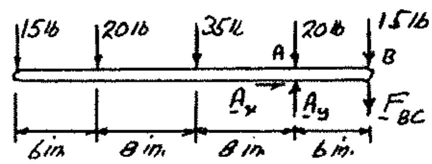


PROBLEM 4.4

For the beam and loading shown, determine (*a*) the reaction at *A*, (*b*) the tension in cable *BC*.

SOLUTION

Free-Body Diagram:



(*a*) Reaction at *A*: $\Sigma F_x = 0$: $A_x = 0$

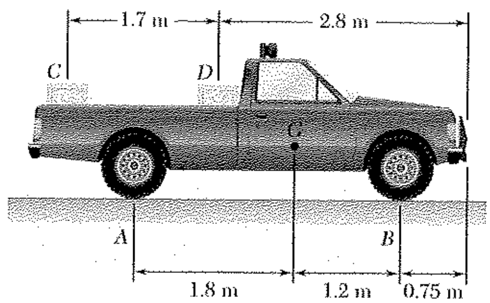
$+\circlearrowleft \Sigma M_B = 0$: $(15\text{ lb})(28\text{ in.}) + (20\text{ lb})(22\text{ in.}) + (35\text{ lb})(14\text{ in.})$
$+ (20\text{ lb})(6\text{ in.}) - A_y(6\text{ in.}) = 0$

$A_y = +245\text{ lb}$ **A** $= 245\text{ lb}\uparrow$ ◀

(*b*) Tension in *BC* $+\circlearrowleft \Sigma M_A = 0$: $(15\text{ lb})(22\text{ in.}) + (20\text{ lb})(16\text{ in.}) + (35\text{ lb})(8\text{ in.})$
$- (15\text{ lb})(6\text{ in.}) - F_{BC}(6\text{ in.}) = 0$

$F_{BC} = +140.0\text{ lb}$ $F_{BC} = 140.0\text{ lb}$ ◀

Check: $+\uparrow \Sigma F_y = 0$: $-15\text{ lb} - 20\text{ lb} = 35\text{ lb} - 20\text{ lb} + A - F_{BC} = 0$
$-105\text{ lb} + 245\text{ lb} - 140.0 = 0$
$0 = 0$ (Checks)

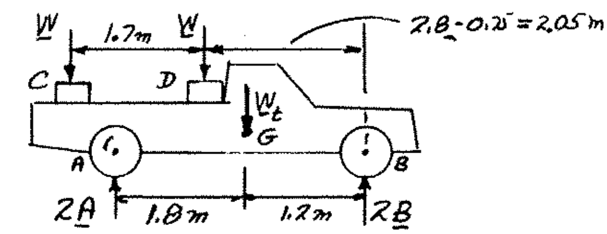


PROBLEM 4.5

Two crates, each of mass 350 kg, are placed as shown in the bed of a 1400-kg pickup truck. Determine the reactions at each of the two (*a*) rear wheels *A*, (*b*) front wheels *B*.

SOLUTION

Free-Body Diagram:



$$W = (350\text{ kg})(9.81\text{ m/s}^2) = 3.434\text{ kN}$$

$$W_t = (1400\text{ kg})(9.81\text{ m/s}^2) = 13.734\text{ kN}$$

(*a*) Rear wheels

$$+\circlearrowleft \Sigma M_B = 0: \quad W(1.7\text{ m} + 2.05\text{ m}) + W(2.05\text{ m}) + W_t(1.2\text{ m}) - 2A(3\text{ m}) = 0$$

$$(3.434\text{ kN})(3.75\text{ m}) + (3.434\text{ kN})(2.05\text{ m}) + (13.734\text{ kN})(1.2\text{ m}) - 2A(3\text{ m}) = 0$$

$$A = +6.0663\text{ kN}$$

$\mathbf{A} = 6.07\text{ kN}\uparrow$ ◀

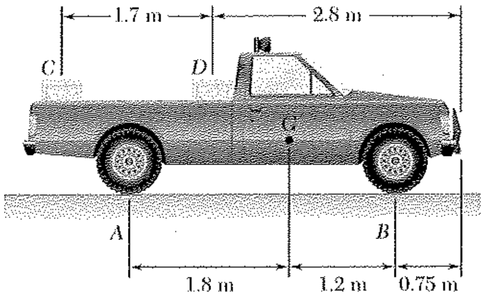
(*b*) Front wheels

$$+\uparrow \Sigma F_y = 0: \quad -W - W - W_t + 2A + 2B = 0$$

$$-3.434\text{ kN} - 3.434\text{ kN} - 13.734\text{ kN} + 2(6.0663\text{ kN}) + 2B = 0$$

$$B = +4.2347\text{ kN}$$

$\mathbf{B} = 4.23\text{ kN}\uparrow$ ◀



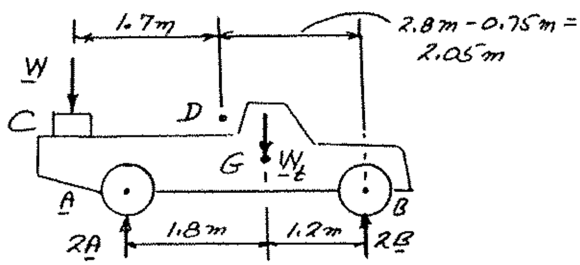
PROBLEM 4.6

Solve Problem 4.5, assuming that crate D is removed and that the position of crate C is unchanged.

PROBLEM 4.5 Two crates, each of mass 350 kg, are placed as shown in the bed of a 1400-kg pickup truck. Determine the reactions at each of the two (*a*) rear wheels A, (*b*) front wheels B.

SOLUTION

Free-Body Diagram:



$$W = (350\text{ kg})(9.81\text{ m/s}^2) = 3.434\text{ kN}$$

$$W_t = (1400\text{ kg})(9.81\text{ m/s}^2) = 13.734\text{ kN}$$

(*a*) Rear wheels

$$+\circlearrowleft \Sigma M_B = 0: \quad W(1.7\text{ m} + 2.05\text{ m}) + W_t(1.2\text{ m}) - 2A(3\text{ m}) = 0$$

$$(3.434\text{ kN})(3.75\text{ m}) + (13.734\text{ kN})(1.2\text{ m}) - 2A(3\text{ m}) = 0$$

$$A = +4.893\text{ kN}$$

$\mathbf{A} = 4.89\text{ kN}\uparrow$ ◀

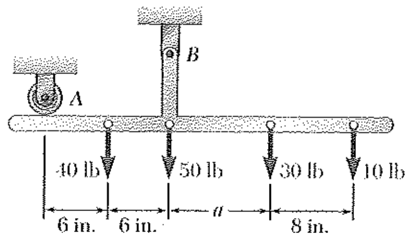
(*b*) Front wheels

$$+\uparrow \Sigma M_y = 0: \quad -W - W_t + 2A + 2B = 0$$

$$-3.434\text{ kN} - 13.734\text{ kN} + 2(4.893\text{ kN}) + 2B = 0$$

$$B = +3.691\text{ kN}$$

$\mathbf{B} = 3.69\text{ kN}\uparrow$ ◀

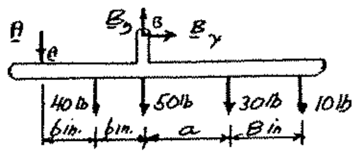


PROBLEM 4.7

A T-shaped bracket supports the four loads shown. Determine the reactions at A and B (a) if $a = 10$ in., (b) if $a = 7$ in.

SOLUTION

Free-Body Diagram:



$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (40 \text{ lb})(6 \text{ in.}) - (30 \text{ lb})a - (10 \text{ lb})(a + 8 \text{ in.}) + (12 \text{ in.})A = 0$$

$$A = \frac{(40a - 160)}{12} \qquad (1)$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -(40 \text{ lb})(6 \text{ in.}) - (50 \text{ lb})(12 \text{ in.}) - (30 \text{ lb})(a + 12 \text{ in.})$$
$$-(10 \text{ lb})(a + 20 \text{ in.}) + (12 \text{ in.})B_y = 0$$

$$B_y = \frac{(1400 + 40a)}{12}$$

Since

$$B_x = 0 \quad B = \frac{(1400 + 40a)}{12} \qquad (2)$$

(a) For $a = 10$ in.

Eq. (1):

$$A = \frac{(40 \times 10 - 160)}{12} = +20.0 \text{ lb} \qquad \mathbf{A} = 20.0 \text{ lb}\downarrow \blacktriangleleft$$

Eq. (2):

$$B = \frac{(1400 + 40 \times 10)}{12} = +150.0 \text{ lb} \qquad \mathbf{B} = 150.0 \text{ lb}\downarrow \blacktriangleleft$$

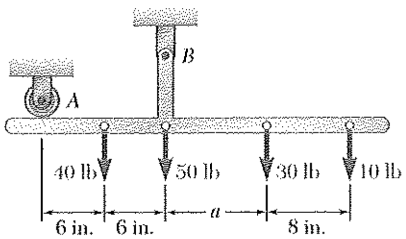
(b) For $a = 7$ in.

Eq. (1):

$$A = \frac{(40 \times 7 - 160)}{12} = +10.00 \text{ lb} \qquad \mathbf{A} = 10.00 \text{ lb}\downarrow \blacktriangleleft$$

Eq. (2):

$$B = \frac{(1400 + 40 \times 7)}{12} = +140.0 \text{ lb} \qquad \mathbf{B} = 140.0 \text{ lb}\uparrow \blacktriangleleft$$



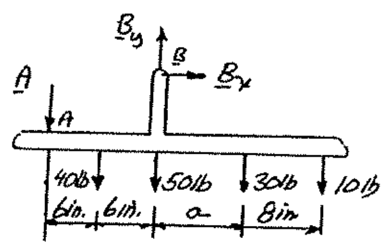
PROBLEM 4.8

For the bracket and loading of Problem 4.7, determine the smallest distance a if the bracket is not to move.

PROBLEM 4.7 A T-shaped bracket supports the four loads shown. Determine the reactions at A and B (*a*) if $a = 10$ in., (*b*) if $a = 7$ in.

SOLUTION

Free-Body Diagram:

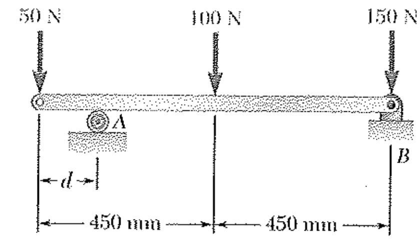


For no motion, reaction at A must be downward or zero; smallest distance a for no motion corresponds to $\mathbf{A} = 0$.

$$+\circlearrowleft \Sigma M_B = 0: \quad (40\text{ lb})(6\text{ in.}) - (30\text{ lb})a - (10\text{ lb})(a + 8\text{ in.}) + (12\text{ in.})A = 0$$

$$A = \frac{(40a - 160)}{12}$$

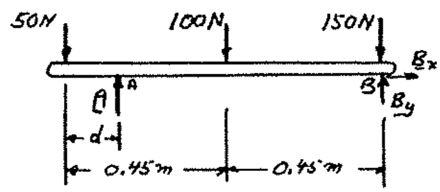
$$A = 0: \quad (40a - 160) = 0 \qquad a = 4.00\text{ in.} \blacktriangleleft$$



PROBLEM 4.9

The maximum allowable value of each of the reactions is 180 N. Neglecting the weight of the beam, determine the range of the distance d for which the beam is safe.

SOLUTION



$$\Sigma F_x = 0: \quad B_x = 0$$

$$B = B_y$$

$$+\circlearrowleft \Sigma M_A = 0: \quad (50 \text{ N})d - (100 \text{ N})(0.45 \text{ m} - d) - (150 \text{ N})(0.9 \text{ m} - d) + B(0.9 \text{ m} - d) = 0$$

$$50d - 45 + 100d - 135 + 150d + 0.9B - Bd$$

$$d = \frac{180 \text{ N}\cdot\text{m} - (0.9 \text{ m})B}{300A - B} \qquad (1)$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (50 \text{ N})(0.9 \text{ m}) - A(0.9 \text{ m} - d) + (100 \text{ N})(0.45 \text{ m}) = 0$$

$$45 - 0.9A + Ad + 45 = 0$$

$$d = \frac{(0.9 \text{ m})A - 90 \text{ N}\cdot\text{m}}{A} \qquad (2)$$

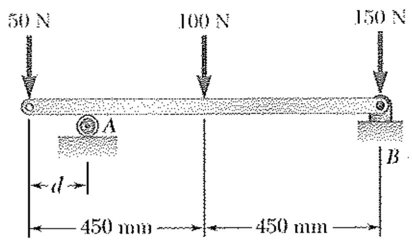
Since $B \leq 180$ N, Eq. (1) yields.

$$d \geq \frac{180 - (0.9)180}{300 - 180} = \frac{18}{120} = 0.15 \text{ m} \qquad d \geq 150.0 \text{ mm} \triangleleft$$

Since $A \leq 180$ N, Eq. (2) yields.

$$d \leq \frac{(0.9)180 - 90}{180} = \frac{72}{180} = 0.40 \text{ m} \qquad d \leq 400 \text{ mm} \triangleleft$$

Range: $150.0 \text{ mm} \leq d \leq 400 \text{ mm}$ ◀

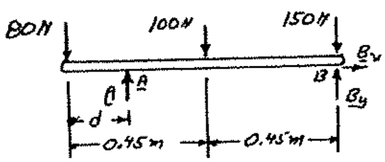


PROBLEM 4.10

Solve Problem 4.9 if the 50-N load is replaced by an 80-N load.

PROBLEM 4.9 The maximum allowable value of each of the reactions is 180 N. Neglecting the weight of the beam, determine the range of the distance d for which the beam is safe.

SOLUTION



$$\Sigma F_x = 0: \quad B_x = 0$$

$$B = B_y$$

$$+\circlearrowleft \Sigma M_A = 0: \quad (80\text{ N})d - (100\text{ N})(0.45\text{ m} - d) - (150\text{ N})(0.9\text{ m} - d) + B(0.9\text{ m} - d) = 0$$

$$80d - 45 + 100d - 135 + 150d + 0.9B - Bd = 0$$

$$d = \frac{180\text{ N}\cdot\text{m} - 0.9B}{330\text{N} - B} \qquad (1)$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (80\text{ N})(0.9\text{ m}) - A(0.9\text{ m} - d) + (100\text{ N})(0.45\text{ m}) = 0$$

$$d = \frac{0.9A - 117}{A} \qquad (2)$$

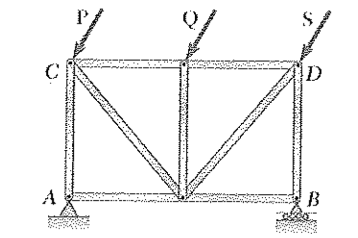
Since $B \leq 180$ N, Eq. (1) yields.

$$d \geq (180 - 0.9\times180)/(330 - 180) = \frac{18}{150} = 0.12\text{ m} \qquad d = 120.0\text{ mm} \triangleleft$$

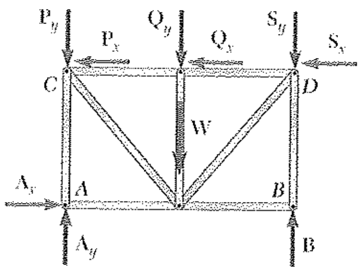
Since $A \leq 180$ N, Eq. (2) yields.

$$d \leq (0.9\times180 - 112)/180 = \frac{45}{180} = 0.25\text{ m} \qquad d = 250\text{ mm} \triangleleft$$

Range: $120.0\text{ mm} \leq d \leq 250\text{ mm}$ ◀



(a)

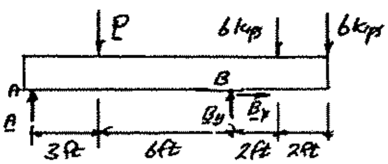


(b)

PROBLEM 4.11

For the beam of Sample Problem 4.2, determine the range of values of P for which the beam will be safe, knowing that the maximum allowable value of each of the reactions is 30 kips and that the reaction at A must be directed upward.

SOLUTION



$$\Sigma F_x = 0: \quad B_x = 0$$

$$\mathbf{B} = B_y \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -P(3\text{ ft}) + B(9\text{ ft}) - (6\text{ kips})(11\text{ ft}) - (6\text{ kips})(13\text{ ft}) = 0$$

$$P = 3B - 48\text{ kips} \qquad (1)$$

$$+\circlearrowleft \Sigma M_B = 0: \quad -A(9\text{ ft}) + P(6\text{ ft}) - (6\text{ kips})(2\text{ ft}) - (6\text{ kips})(4\text{ ft}) = 0$$

$$P = 1.5A + 6\text{ kips} \qquad (2)$$

Since $B \leq 30$ kips, Eq. (1) yields.

$$P \leq (3)(30\text{ kips}) - 48\text{ kips} \qquad P \leq 42.0\text{ kips} \triangleleft$$

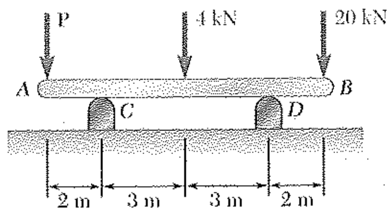
Since $0 \leq A \leq 30$ kips. Eq. (2) yields.

$$0 + 6\text{ kips} \leq P \leq (1.5)(30\text{ kips})1.6\text{ kips}$$

$$6.00\text{ kips} \leq P \leq 51.0\text{ kips} \qquad \triangleleft$$

Range of values of P for which beam will be safe:

$$6.00\text{ kips} \leq P \leq 42.0\text{ kips} \qquad \blacktriangleleft$$

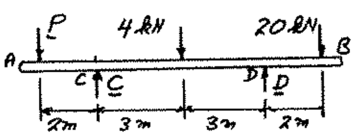


PROBLEM 4.12

The 10-m beam AB rests upon, but is not attached to, supports at C and D. Neglecting the weight of the beam, determine the range of values of P for which the beam will remain in equilibrium.

SOLUTION

Free-Body Diagram:



$$+\circlearrowleft \Sigma M_C = 0: \quad P(2\text{ m}) - (4\text{ kN})(3\text{ m}) - (20\text{ kN})(8\text{ m}) + D(6\text{ m}) = 0$$

$$P = 86\text{ kN} - 3D \qquad (1)$$

$$+\circlearrowleft \Sigma M_D = 0: \quad P(8\text{ m}) + (4\text{ kN})(3\text{ m}) - (20\text{ kN})(2\text{ m}) - C(6\text{ m}) = 0$$

$$P = 3.5\text{ kN} + 0.75C \qquad (2)$$

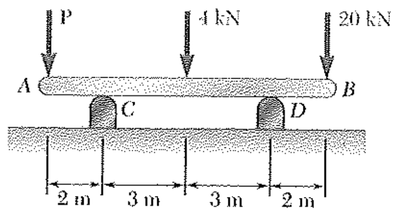
For no motion $C \geq 0$ and $D \geq 0$

For $C \geq 0$ from (2) $P \leq 3.50$ kN

For $D \geq 0$ from (1) $P \leq 86.0$ kN

Range of P for no motion:

$3.50\text{ kN} \leq P \leq 86.0\text{ kN}$ ◀

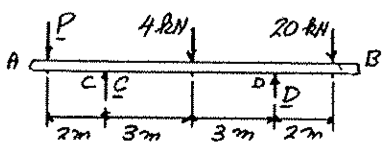


PROBLEM 4.13

The maximum allowable value of each of the reactions is 50 kN, and each reaction must be directed upward. Neglecting the weight of the beam, determine the range of values of P for which the beam is safe.

SOLUTION

Free-Body Diagram:



$$+\circlearrowleft \Sigma M_C = 0: \quad P(2\text{ m}) - (4\text{ kN})(3\text{ m}) - (20\text{ kN})(8\text{ m}) + D(6\text{ m}) = 0$$

$$P = 86\text{ kN} - 3D \qquad (1)$$

$$+\circlearrowleft \Sigma M_D = 0: \quad P(8\text{ m}) + (4\text{ kN})(3\text{ m}) - (20\text{ kN})(2\text{ m}) - C(6\text{ m}) = 0$$

$$P = 3.5\text{ kN} + 0.75C \qquad (2)$$

For $C \geq 0$, from (2): $\quad P \geq 3.50\text{ kN}$ ◁

For $D \geq 0$, from (1): $\quad P \leq 86.0\text{ kN}$ ◁

For $C \leq 50\text{ kN}$, from (2):

$$P \leq 3.5\text{ kN} + 0.75(50\text{ kN})$$

$$P \leq 41.0\text{ kN}$$ ◁

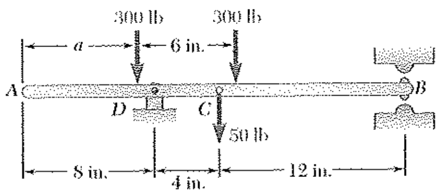
For $D \leq 50\text{ kN}$, from (1):

$$P \geq 86\text{ kN} - 3(50\text{ kN})$$

$$P \geq -64.0\text{ kN}$$ ◁

Comparing the four criteria, we find

$$3.50\text{ kN} \leq P \leq 41.0\text{ kN}$$ ◀

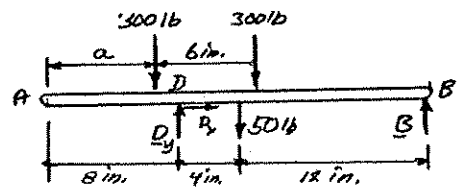


PROBLEM 4.14

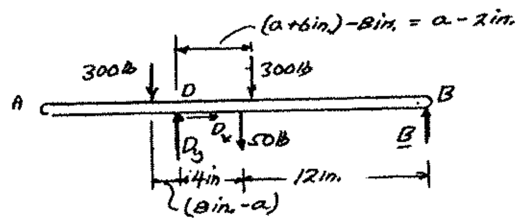
For the beam and loading shown, determine the range of the distance a for which the reaction at B does not exceed 100 lb downward or 200 lb upward.

SOLUTION

Assume B is positive when directed ↑



Sketch showing distance from D to forces.



$$+\circlearrowleft \Sigma M_D = 0: \quad (300\text{ lb})(8\text{ in.} - a) - (300\text{ lb})(a - 2\text{ in.}) - (50\text{ lb})(4\text{ in.}) + 16B = 0$$

$$-600a + 2800 + 16B = 0$$

$$a = \frac{(2800 + 16B)}{600} \qquad (1)$$

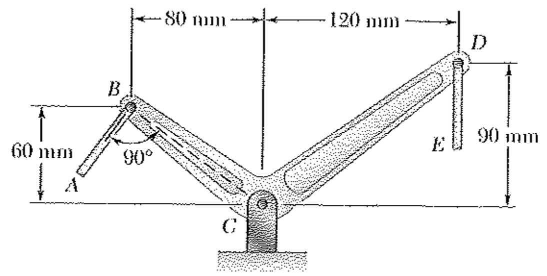
For $B = 100\text{ lb}\downarrow = -100\text{ lb}$, Eq. (1) yields:

$$a \geq \frac{[2800 + 16(-100)]}{600} = \frac{1200}{600} = 2\text{ in.} \qquad a \geq 2.00\text{ in.} \ \triangleleft$$

For $B = 200\uparrow = +200\text{ lb}$, Eq. (1) yields:

$$a \leq \frac{[2800 + 16(200)]}{600} = \frac{6000}{600} = 10\text{ in.} \qquad a \leq 10.00\text{ in.} \ \triangleleft$$

Required range: $2.00\text{ in.} \leq a \leq 10.00\text{ in.}$ ◀

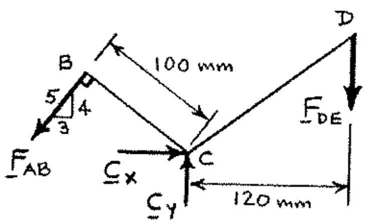


PROBLEM 4.15

Two links *AB* and *DE* are connected by a bell crank as shown. Knowing that the tension in link *AB* is 720 N, determine (*a*) the tension in link *DE*, (*b*) the reaction at *C*.

SOLUTION

Free-Body Diagram:



$$+\circlearrowleft \Sigma M_C = 0: \quad F_{AB}(100\text{ mm}) - F_{DE}(120\text{ mm}) = 0$$

$$F_{DE} = \frac{5}{6}F_{AB} \qquad (1)$$

(*a*) For

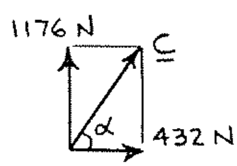
$$F_{AB} = 720\text{ N}$$

$$F_{DE} = \frac{5}{6}(720\text{ N}) \qquad F_{DE} = 600\text{ N} \blacktriangleleft$$

(*b*)

$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{3}{5}(720\text{ N}) + C_x = 0$$

$$C_x = +432\text{ N}$$



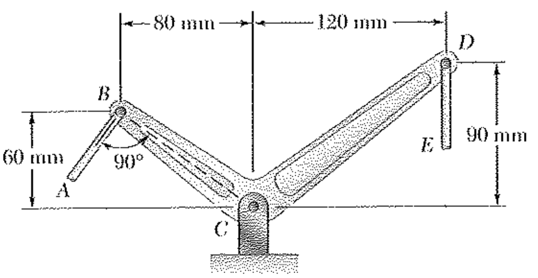
$$+\uparrow \Sigma F_y = 0: \quad -\frac{4}{5}(720\text{ N}) + C_y - 600\text{ N} = 0$$

$$C_y = +1176\text{ N}$$

$$C = 1252.84\text{ N}$$

$$\alpha = 69.829^\circ$$

$$\mathbf{C} = 1253\text{ N} \measuredangle 69.8^\circ \blacktriangleleft$$



PROBLEM 4.16

Two links AB and DE are connected by a bell crank as shown. Determine the maximum force that may be safely exerted by link AB on the bell crank if the maximum allowable value for the reaction at C is 1600 N.

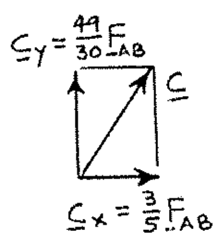
SOLUTION

See solution to Problem 4.15 for F. B. D. and derivation of Eq. (1)

$$F_{DE} = \frac{5}{6} F_{AB} \quad (1)$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{3}{5} F_{AB} + C_x = 0 \quad C_x = \frac{3}{5} F_{AB}$$

$$+\uparrow \Sigma F_y = 0: \quad -\frac{4}{5} F_{AB} + C_y - F_{DE} = 0$$



$$-\frac{4}{5} F_{AB} + C_y - \frac{5}{6} F_{AB} = 0$$

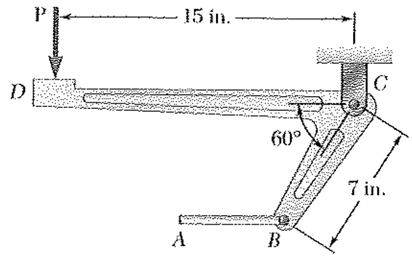
$$C_y = \frac{49}{30} F_{AB}$$

$$C = \sqrt{C_x^2 + C_y^2}$$

$$= \frac{1}{30}\sqrt{(49)^2 + (18)^2}\, F_{AB}$$

$$C = 1.74005 F_{AB}$$

For $C = 1600$ N, $1600 \text{ N} = 1.74005 F_{AB}$ $\qquad F_{AB} = 920$ N ◀

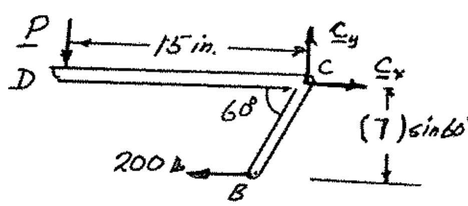


PROBLEM 4.17

The required tension in cable AB is 200 lb. Determine (*a*) the vertical force **P** that must be applied to the pedal, (*b*) the corresponding reaction at C.

SOLUTION

Free-Body Diagram:



$BC = 7$ in.

(*a*)

$$+\circlearrowleft \Sigma M_C = 0: \quad P(15 \text{ in.}) - (200 \text{ lb})(6.062 \text{ in.}) = 0$$

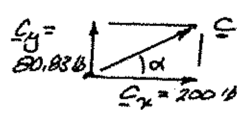
$$P = 80.83 \text{ lb}$$

$\mathbf{P} = 80.8 \text{ lb} \downarrow$ ◀

(*b*)

$$\xrightarrow{+} \Sigma F_y = 0: \quad C_x - 200 \text{ lb} = 0$$

$\mathbf{C}_x = 200 \text{ lb} \longrightarrow$



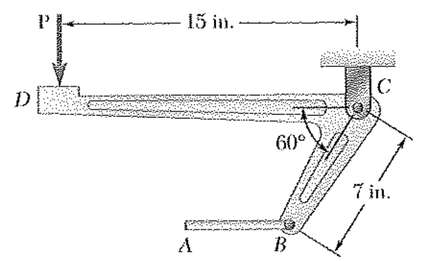
$$+\uparrow \Sigma F_y = 0: \quad C_y - P = 0 \quad C_y - 80.83 \text{ lb} = 0$$

$\mathbf{C}_y = 80.83 \text{ lb} \uparrow$

$$\alpha = 22.0°$$

$$C = 215.7 \text{ lb}$$

$\mathbf{C} = 216 \text{ lb} \measuredangle 22.0°$ ◀

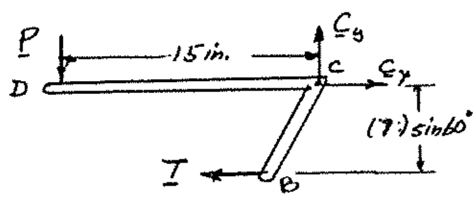


PROBLEM 4.18

Determine the maximum tension that can be developed in cable AB if the maximum allowable value of the reaction at C is 250 lb.

SOLUTION

Free-Body Diagram:



$BC = 7$ in.

$+\circlearrowleft \Sigma M_C = 0$: $P(15 \text{ in.}) - T(6.062 \text{ in.}) = 0 \quad P = 0.40415T$

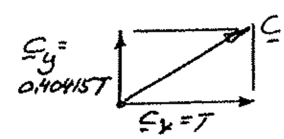
$+\uparrow \Sigma F_y = 0$: $-P + C_y = 0 \quad -0.40415P + C_y = 0$

$$C_y = 0.40415T$$

$\xrightarrow{+} \Sigma F_x = 0$: $-T + C_x = 0 \qquad C_x = T$

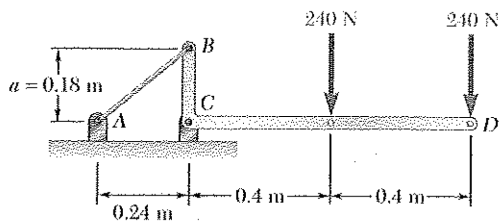
$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{T^2 + (0.40415T)^2}$$

$$C = 1.0786T$$



For $\quad C = 250$ lb

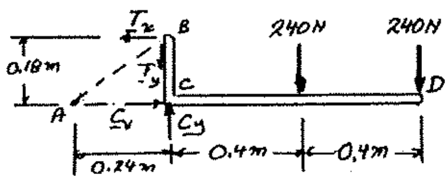
$250 \text{ lb} = 1.0786T \qquad T = 232 \text{ lb}$ ◀



PROBLEM 4.19

The bracket *BCD* is hinged at *C* and attached to a control cable at *B*. For the loading shown, determine (*a*) the tension in the cable, (*b*) the reaction at *C*.

SOLUTION



At *B*:

$$\frac{T_y}{T_x} = \frac{0.18 \text{ m}}{0.24 \text{ m}}$$

$$T_y = \frac{3}{4} T_x \qquad (1)$$

(*a*)

$$+\circlearrowleft \Sigma M_C = 0: \quad T_x(0.18 \text{ m}) - (240 \text{ N})(0.4 \text{ m}) - (240 \text{ N})(0.8 \text{ m}) = 0$$

$$T_x = +1600 \text{ N}$$

Eq. (1)

$$T_y = \frac{3}{4}(1600 \text{ N}) = 1200 \text{ N}$$

$$T = \sqrt{T_x^2 + T_y^2} = \sqrt{1600^2 + 1200^2} = 2000 \text{ N} \qquad T = 2.00 \text{ kN} \blacktriangleleft$$

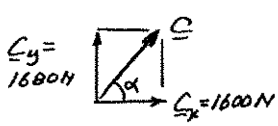
(*b*)

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - T_x = 0$$

$$C_x - 1600 \text{ N} = 0 \quad C_x = +1600 \text{ N} \qquad \mathbf{C}_x = 1600 \text{ N} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - T_y - 240 \text{ N} - 240 \text{ N} = 0$$

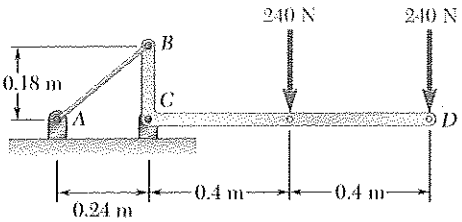
$$C_y - 1200 \text{ N} - 480 \text{ N} = 0$$



$$C_y = +1680 \text{ N} \qquad \mathbf{C}_y = 1680 \text{ N} \uparrow$$

$$\alpha = 46.4°$$

$$C = 2320 \text{ N} \qquad \mathbf{C} = 2.32 \text{ kN} \measuredangle 46.4° \blacktriangleleft$$

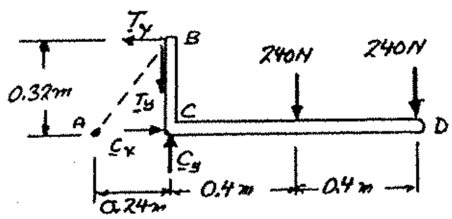


PROBLEM 4.20

Solve Problem 4.19, assuming that $a = 0.32$ m.

PROBLEM 4.19 The bracket *BCD* is hinged at *C* and attached to a control cable at *B*. For the loading shown, determine (*a*) the tension in the cable, (*b*) the reaction at *C*.

SOLUTION



At B:

$$\frac{T_y}{T_x} = \frac{0.32 \text{ m}}{0.24 \text{ m}}$$

$$T_y = \frac{4}{3} T_x$$

$$+\circlearrowleft \Sigma M_C = 0: \quad T_x(0.32 \text{ m}) - (240 \text{ N})(0.4 \text{ m}) - (240 \text{ N})(0.8 \text{ m}) = 0$$

$$T_x = 900 \text{ N}$$

Eq. (1)

$$T_y = \frac{4}{3}(900 \text{ N}) = 1200 \text{ N}$$

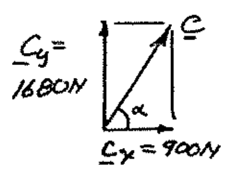
$$T = \sqrt{T_x^2 + T_y^2} = \sqrt{900^2 + 1200^2} = 1500 \text{ N} \qquad T = 1.500 \text{ kN} \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - T_x = 0$$

$$C_x - 900 \text{ N} = 0 \quad C_x = +900 \text{ N} \qquad \mathbf{C}_x = 900 \text{ N} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - T_y - 240 \text{ N} - 240 \text{ N} = 0$$

$$C_y - 1200 \text{ N} - 480 \text{ N} = 0$$

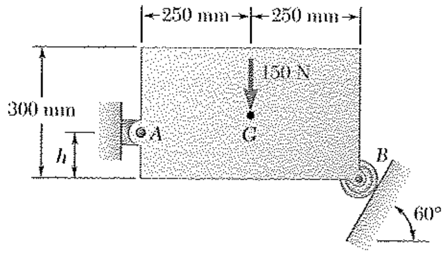


$$C_y = +1680 \text{ N} \qquad \mathbf{C}_y = 1680 \text{ N} \uparrow$$

$$\alpha = 61.8°$$

$$C = 1906 \text{ N} \qquad \mathbf{C} = 1.906 \text{ kN} \measuredangle 61.8° \blacktriangleleft$$

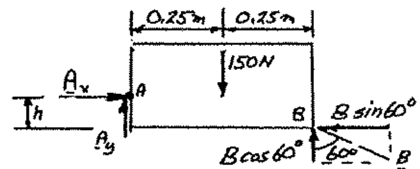
PROBLEM 4.21



Determine the reactions at A and B when (*a*) $h = 0$, (*b*) $h = 200$ mm.

SOLUTION

Free-Body Diagram:



$$+\circlearrowleft \Sigma M_A = 0: \quad (B\cos 60°)(0.5 \text{ m}) - (B\sin 60°)h - (150 \text{ N})(0.25 \text{ m}) = 0$$

$$B = \frac{37.5}{0.25 - 0.866h} \qquad (1)$$

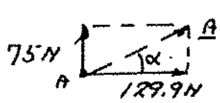
(*a*) When $h = 0$:

Eq. (1): $B = \frac{37.5}{0.25} = 150 \text{ N}$ **B** $= 150.0$ N $\searrow$ 30.0° ◀

$\xrightarrow{+} \Sigma F_y = 0: \quad A_x - B\sin 60° = 0$

$A_x = (150)\sin 60° = 129.9$ N **A**$_x = 129.9$ N $\longrightarrow$

$+\uparrow \Sigma F_y = 0: \quad A_y - 150 + B\cos 60° = 0$



$A_y = 150 - (150)\cos 60° = 75$ N **A**$_y = 75$ N $\uparrow$

$\alpha = 30°$

$A = 150.0$ N **A** $= 150.0$ N $\measuredangle$ 30.0° ◀

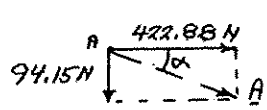
(*b*) <u>When $h = 200 \text{ mm} = 0.2 \text{ m}$</u>

Eq. (1): $B = \frac{37.5}{0.25 - 0.866(0.2)} = 488.3 \text{ N}$ **B** $= 488$ N $\searrow$ 30.0° ◀

$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - B\sin 60° = 0$

$A_x = (488.3)\sin 60° = 422.88$ N **A**$_x = 422.88$ N $\longrightarrow$

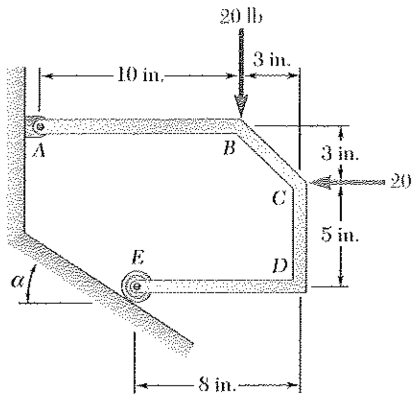
$+\uparrow \Sigma F_y = 0: \quad A_y - 150 + B\cos 60° = 0$



$A_y = 150 - (488.3)\cos 60° = -94.15$ N **A**$_y = 94.15$ N $\downarrow$

$\alpha = 12.55°$

$A = 433.2$ N **A** $= 433$ N $\nwarrow$ 12.6° ◀

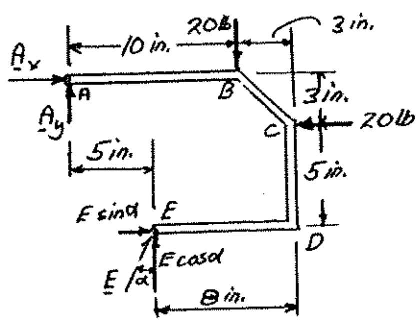


PROBLEM 4.22

For the frame and loading shown, determine the reactions at A and E when (*a*) $\alpha = 30°$, (*b*) $\alpha = 45°$.

SOLUTION

Free-Body Diagram:



$$+\circlearrowleft \Sigma M_A = 0: \quad (E\sin\alpha)(8\text{ in.}) + (E\cos\alpha)(5\text{ in.}) - (20\text{ lb})(10\text{ in.}) - (20\text{ lb})(3\text{ in.}) = 0$$

$$E = \frac{260}{8\sin\alpha + 5\cos\alpha}$$

(*a*) When $\alpha = 30°$:

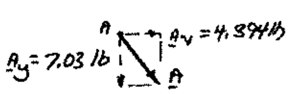
$$E = \frac{260}{8\sin 30° + 5\cos 30°} = 31.212\text{ lb}$$

$\mathbf{E} = 31.2\text{ lb}\ \angle 60.0°$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - 20\text{ lb} + (31.212\text{ lb})\sin 30° = 0$$

$$A_x = +4.394\text{ lb}$$

$\mathbf{A}_x = 4.394\text{ lb} \longrightarrow$



$$+\uparrow \Sigma F_y = 0: \quad A_y - 20° + (31.212\text{ lb})\cos 30° = 0$$

$$A_y = -7.03\text{ lb}$$

$\mathbf{A}_y = 7.03\text{ lb} \downarrow$

$\mathbf{A} = 8.29\text{ lb}\ \nwarrow 58.0°$ ◀

PROBLEM 4.22 (Continued)

(*b*) When $\alpha = 45°$:

$$E = \frac{260}{8\sin 45° + 5\cos\alpha} = 28.28 \text{ lb}$$ $\mathbf{E} = 28.3 \text{ lb} \measuredangle 45.0°$ ◀

$\underrightarrow{+} \Sigma F_x = 0$: $A_x - 20 \text{ lb} + (28.28 \text{ lb})\sin 45° = 0$

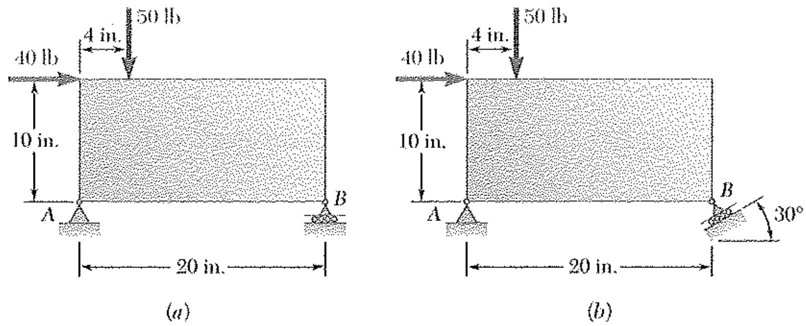
$A_x = 0$ $\mathbf{A}_x = 0$

$+\uparrow \Sigma F_y = 0$: $A_y - 20 \text{ lb} + (28.28 \text{ lb})\cos 45° = 0$

$A_y = 0$ $A_y = 0$ $\mathbf{A} = 0$ ◀

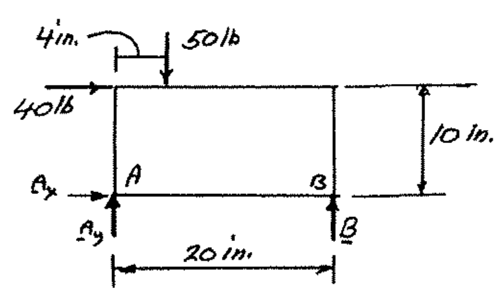
PROBLEM 4.23

For each of the plates and loadings shown, determine the reactions at A and B.



SOLUTION

(*a*) **Free-Body Diagram:**



$$+\circlearrowleft \Sigma M_A = 0: \quad B(20\text{ in.}) - (50\text{ lb})(4\text{ in.}) - (40\text{ lb})(10\text{ in.}) = 0$$

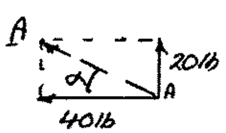
$B = +30$ lb $\qquad$ $\mathbf{B} = 30.0\text{ lb}\uparrow$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + 40\text{ lb} = 0$$

$A_x = -40$ lb $\qquad$ $\mathbf{A}_x = 40.0\text{ lb}\longleftarrow$

$$+\uparrow \Sigma F_y = 0: \quad A_y + B - 50\text{ lb} = 0$$

$$A_y + 30\text{ lb} - 50\text{ lb} = 0$$



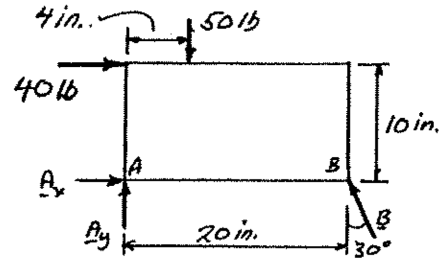
$A_y = +20$ lb $\qquad$ $\mathbf{A}_y = 20.0\text{ lb}\uparrow$

$\alpha = 26.56°$

$A = 44.72$ lb $\qquad$ $\mathbf{A} = 44.7\text{ lb} \searrow 26.6°$ ◀

PROBLEM 4.23 (Continued)

(*b*) **Free-Body Diagram:**



$+\circlearrowleft \Sigma M_A = 0$: $(B\cos 30°)(20 \text{ in.}) - (40 \text{ lb})(10 \text{ in.}) - (50 \text{ lb})(4 \text{ in.}) = 0$

$B = 34.64 \text{ lb}$ $\qquad$ $\mathbf{B} = 34.6 \text{ lb} \searrow 60.0°$ ◀

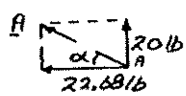
$\xrightarrow{+} \Sigma F_x = 0$: $A_x - B\sin 30° + 40 \text{ lb}$

$A_x - (34.64 \text{ lb})\sin 30° + 40 \text{ lb} = 0$

$A_x = -22.68 \text{ lb}$ $\qquad$ $\mathbf{A}_x = 22.68 \text{ lb} \longleftarrow$

$+\uparrow \Sigma F_y = 0$: $A_y + B\cos 30° - 50 \text{ lb} = 0$

$A_y + (34.64 \text{ lb})\cos 30° - 50 \text{ lb} = 0$



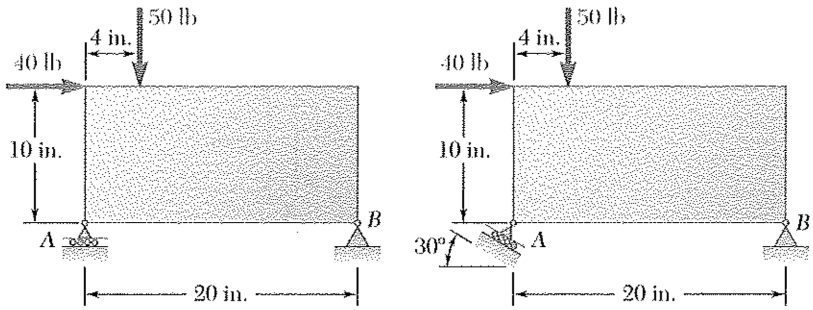
$A_y = +20 \text{ lb}$ $\qquad$ $\mathbf{A}_y = 20.0 \text{ lb} \uparrow$

$\alpha = 41.4°$

$A = 30.24 \text{ lb}$ $\qquad$ $\mathbf{A} = 30.2 \text{ lb} \searrow 41.4°$ ◀

PROBLEM 4.24

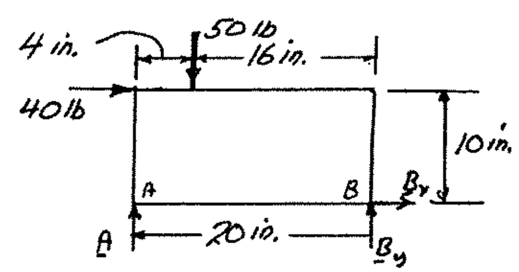
For each of the plates and loadings shown, determine the reactions at A and B.



(*a*) (*b*)

SOLUTION

(*a*) **Free-Body Diagram:**



$$+\circlearrowleft \Sigma M_B = 0: \quad A(20 \text{ in.}) + (50 \text{ lb})(16 \text{ in.}) - (40 \text{ lb})(10 \text{ in.}) = 0$$

$$A = +20 \text{ lb}$$

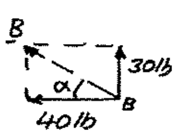
$\mathbf{A} = 20.0 \text{ lb} \uparrow$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad 40 \text{ lb} + B_x = 0$$

$$B_x = -40 \text{ lb}$$

$\mathbf{B}_x = 40 \text{ lb} \longleftarrow$

$$+\uparrow \Sigma F_y = 0: \quad A + B_y - 50 \text{ lb} = 0$$



$$20 \text{ lb} + B_y - 50 \text{ lb} = 0$$

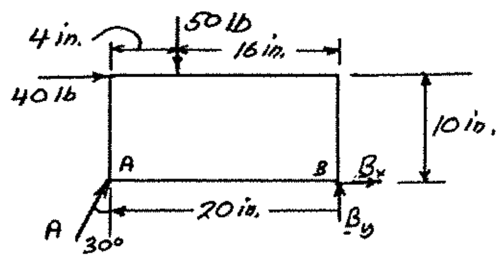
$$B_y = +30 \text{ lb}$$

$\mathbf{B}_y = 30 \text{ lb} \uparrow$

$\alpha = 36.87°$ $\qquad B = 50 \text{ lb}$

$\mathbf{B} = 50.0 \text{ lb} \nwarrow 36.9°$ ◀

(*b*)



$+\circlearrowleft \Sigma M_A = 0: \quad -(A\cos 30°)(20 \text{ in.}) - (40 \text{ lb})(10 \text{ in.}) + (50 \text{ lb})(16 \text{ in.}) = 0$

$A = 23.09 \text{ lb}$ $\qquad$ $\mathbf{A} = 23.1 \text{ lb} \angle 60.0°$ ◀

$\xrightarrow{+} \Sigma F_x = 0: \quad A\sin 30° + 40 \text{ lb} + B_x = 0$

$(23.09 \text{ lb})\sin 30° + 40 \text{ lb} + 8_x = 0$

$B_x = -51.55 \text{ lb}$ $\qquad$ $\mathbf{B}_x = 51.55 \text{ lb} \longleftarrow$

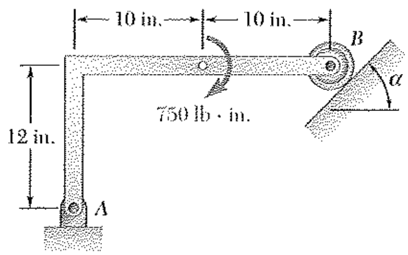
$+\uparrow \Sigma F_y = 0: \quad A\cos 30° + B_y - 50 \text{ lb} = 0$

$(23.09 \text{ lb})\cos 30° + B_y - 50 \text{ lb} = 0$

$B_y = +30 \text{ lb}$ $\qquad$ $\mathbf{B}_y = 30 \text{ lb} \uparrow$

$\alpha = 30.2°$

$B = 59.64 \text{ lb}$ $\qquad$ $\mathbf{B} = 59.6 \text{ lb} \searrow 30.2°$ ◀

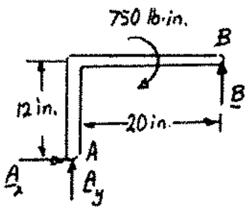


PROBLEM 4.25

Determine the reactions at A and B when (*a*) $\alpha = 0$, (*b*) $\alpha = 90°$, (*c*) $\alpha = 30°$.

SOLUTION

(*a*) $\alpha = 0$



$$+\circlearrowleft \Sigma M_A = 0: \quad B(20 \text{ in.}) - 750 \text{ lb} \cdot \text{in.} = 0$$

$$B = 37.5 \text{ lb}$$

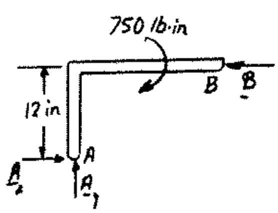
$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + 37.5 \text{ lb} = 0$$

$$A_y = -37.5 \text{ lb}$$

$\mathbf{A} = 37.5 \text{ lb} \downarrow$ $\qquad$ $\mathbf{B} = 37.5 \text{ lb} \uparrow$ ◀

(*b*) $\alpha = 90°$



$$+\circlearrowleft \Sigma M_A = 0: \quad B(12 \text{ in.}) - 750 \text{ lb} \cdot \text{in.} = 0$$

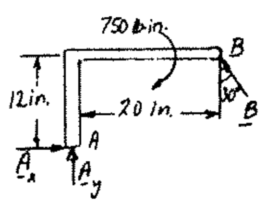
$$B = 62.5 \text{ lb}$$

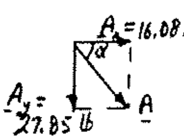
$$\xrightarrow{+} \Sigma F_A = 0: \quad A_x - 62.5 \text{ lb} = 0, \quad A_x = 62.5 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad A_y = 0$$

$\mathbf{A} = 62.5 \text{ lb} \longrightarrow$ $\qquad$ $\mathbf{B} = 62.5 \text{ lb} \longleftarrow$ ◀

(*c*) $\alpha = 30°$





$$+\circlearrowleft \Sigma M_A = 0: \quad (B\cos 30°)(20 \text{ in.}) + (B\sin 30°)(12 \text{ in.}) - 750 \text{ lb} \cdot \text{in.} = 0$$

$$B = 32.16 \text{ lb}$$

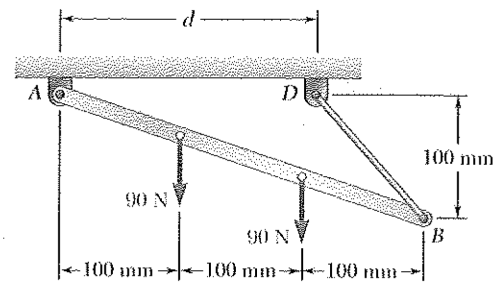
$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - (32.16 \text{ lb})\sin 30° = 0$$

$$A_x = 16.08 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + (32.16 \text{ lb})\cos 30° = 0 \quad A_y = -27.85 \text{ lb}$$

$$A = 32.16 \text{ lb} \quad \alpha = 60.0°$$

$\mathbf{A} = 32.2 \text{ lb} \measuredangle 60.0°$ $\qquad$ $\mathbf{B} = 32.2 \text{ lb} \measuredangle 60.0°$ ◀

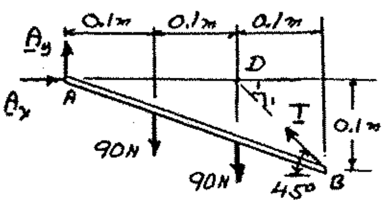


PROBLEM 4.26

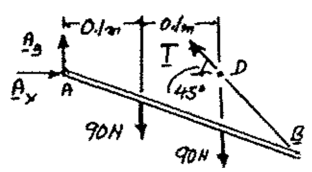
A rod AB hinged at A and attached at B to cable BD supports the loads shown. Knowing that $d = 200$ mm, determine (*a*) the tension in cable BD, (*b*) the reaction at A.

SOLUTION

Free-Body Diagram:



(*a*) Move **T** along BD until it acts at Point D.



$$+\circlearrowleft \Sigma M_A = 0: \quad (T\sin 45°)(0.2 \text{ m}) + (90 \text{ N})(0.1 \text{ m}) + (90 \text{ N})(0.2 \text{ m}) = 0$$

$T = 190.92$ N $\qquad T = 190.9$ N ◀

(*b*)

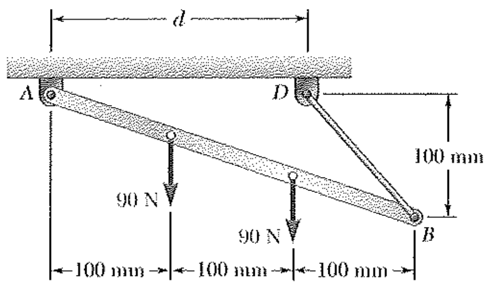
$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - (190.92 \text{ N})\cos 45° = 0$$

$A_x = +135$ N $\qquad \mathbf{A}_x = 135.0$ N ⟶

$$+\uparrow \Sigma F_y = 0: \quad A_y - 90 \text{ N} - 90 \text{ N} + (190.92 \text{ N})\sin 45° = 0$$

$A_y = +45$ N $\qquad \mathbf{A}_y = 45.0$ N ↑

$\mathbf{A} = 142.3$ N ∠ 18.43° ◀

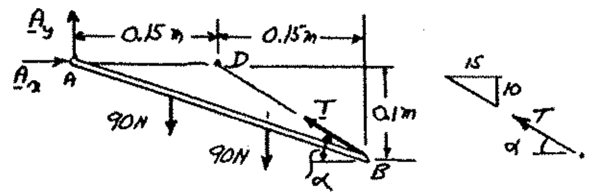


PROBLEM 4.27

A rod AB hinged at A and attached at B to cable BD supports the loads shown. Knowing that $d = 150$ mm, determine (*a*) the tension in cable BD, (*b*) the reaction at A.

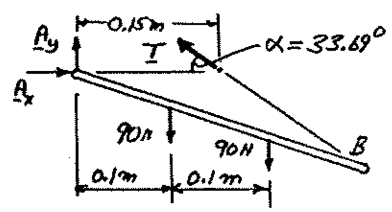
SOLUTION

Free-Body Diagram:



$$\tan\alpha = \frac{10}{15}; \quad \alpha = 33.69°$$

(*a*) Move **T** along BD until it acts at Point D.



$$+\circlearrowleft \Sigma M_A = 0: \quad (T\sin 33.69°)(0.15 \text{ m}) - (90 \text{ N})(0.1 \text{ m}) - (90 \text{ N})(0.2 \text{ m}) = 0$$

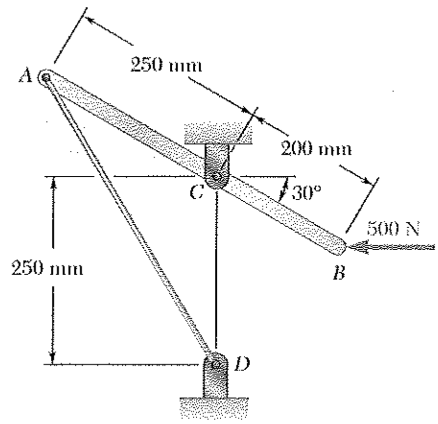
$T = 324.5$ N $\qquad T = 324$ N ◀

(*b*) $\xrightarrow{+} \Sigma F_x = 0: \quad A_x - (324.99 \text{ N})\cos 33.69° = 0$

$A_x = +270$ N $\qquad \mathbf{A}_x = 270$ N ⟶

$+\uparrow \Sigma F_y = 0: \quad A_y - 90 \text{ N} - 90 \text{ N} + (324.5 \text{ N})\sin 33.69° = 0$

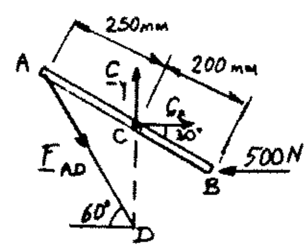
$A_y = 0 \qquad \mathbf{A}_y = 0 \qquad \mathbf{A} = 270$ N ⟶ ◀



PROBLEM 4.28

A lever AB is hinged at C and attached to a control cable at A. If the lever is subjected to a 500-N horizontal force at B, determine (a) the tension in the cable, (b) the reaction at C.

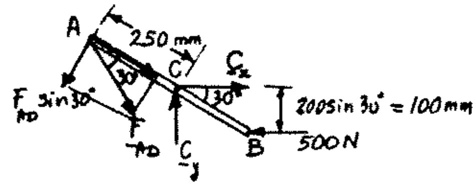
SOLUTION



Triangle ACD is isosceles with $\sphericalangle C = 90° + 30° = 120°$ $\sphericalangle A = \sphericalangle D = \frac{1}{2}(180° - 120°) = 30°$

Thus DA forms angle of 60° with horizontal.

(a) We resolve $\mathbf{F}_{AD}$ into components along AB and perpendicular to AB.



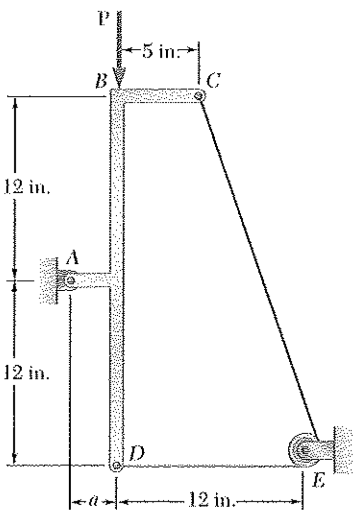
$$+\circlearrowleft \Sigma M_C = 0: \quad (F_{AD} \sin 30°)(250 \text{ mm}) - (500 \text{ N})(100 \text{ mm}) = 0 \qquad F_{AD} = 400 \text{ N} \blacktriangleleft$$

(b)

$$\xrightarrow{+} \Sigma F_x = 0: \quad -(400 \text{ N})\cos 60° + C_x - 500 \text{ N} = 0 \qquad C_x = +300 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad -(400 \text{ N})\sin 60° + C_y = 0 \qquad C_y = +346.4 \text{ N}$$

$$\mathbf{C} = 458 \text{ N} \measuredangle 49.1° \blacktriangleleft$$

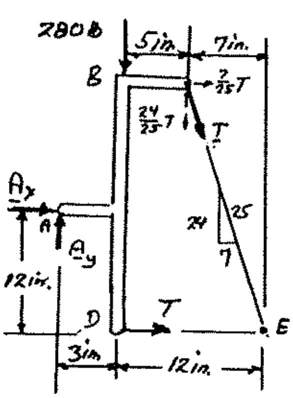


PROBLEM 4.29

A force **P** of magnitude 280 lb is applied to member *ABCD*, which is supported by a frictionless pin at *A* and by the cable *CED*. Since the cable passes over a small pulley at *E*, the tension may be assumed to be the same in portions *CE* and *ED* of the cable. For the case when $a = 3$ in., determine (*a*) the tension in the cable, (*b*) the reaction at *A*.

SOLUTION

Free-Body Diagram:



(*a*) $+\circlearrowleft \Sigma M_A = 0: \quad -(280 \text{ lb})(8 \text{ in.})$

$$T(12 \text{ in.}) - \frac{7}{25}T(12 \text{ in.})$$

$$-\frac{24}{25}T(8 \text{ in.}) = 0$$

$$(12 - 11.04)T = 840$$

$T = 875$ lb ◀

(*b*) $\xrightarrow{+} \Sigma F_x = 0: \quad \frac{7}{25}(875 \text{ lb}) + 875 \text{ lb} + A_x = 0$

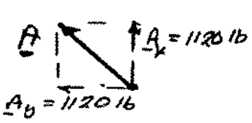
$$A_x = -1120$$

$\mathbf{A}_x = 1120 \text{ lb} \longleftarrow$

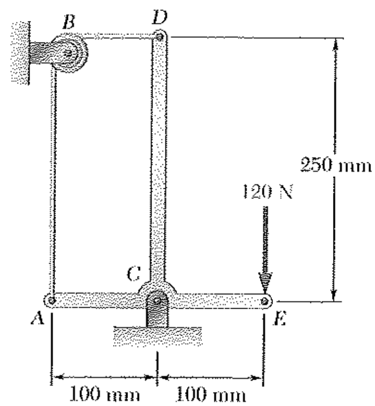
$+\uparrow \Sigma F_y = 0: \quad A_y - 280 \text{ lb} - \frac{24}{25}(875 \text{ lb}) = 0$

$$A_y = +1120$$

$\mathbf{A}_y = 1120 \text{ lb} \uparrow$



$\mathbf{A} = 1584$ lb ⦣ 45.0° ◀

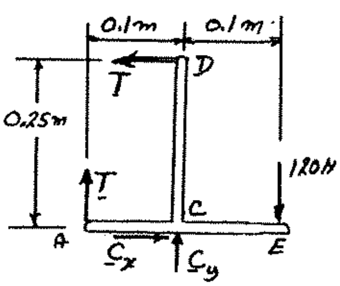


PROBLEM 4.30

Neglecting friction, determine the tension in cable ABD and the reaction at support C.

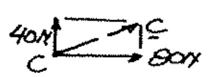
SOLUTION

Free-Body Diagram:



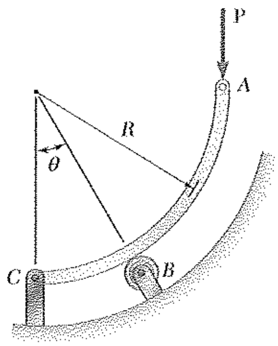
$+\circlearrowleft \Sigma M_C = 0$: $T(0.25\text{ m}) - T(0.1\text{ m}) - (120\text{ N})(0.1\text{ m}) = 0$ $\qquad T = 80.0\text{ N}$ ◀

$\xrightarrow{+} \Sigma F_x = 0$: $C_x - 80\text{ N} = 0 \quad C_x = +80\text{ N}$ $\qquad \mathbf{C}_x = 80.0\text{ N} \longrightarrow$



$+\uparrow \Sigma F_y = 0$: $C_y - 120\text{ N} + 80\text{ N} = 0 \quad C_y = +40\text{ N}$ $\qquad \mathbf{C}_y = 40.0\text{ N}\uparrow$

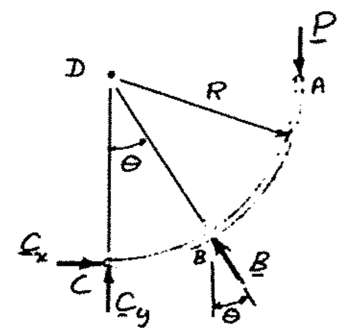
$\mathbf{C} = 89.4\text{ N} \measuredangle 26.6°$ ◀



PROBLEM 4.31

Rod *ABC* is bent in the shape of an arc of circle of radius *R*. Knowing the $\theta = 30°$, determine the reaction (*a*) at *B*, (*b*) at *C*.

SOLUTION



Free-Body Diagram: $+\circlearrowleft \Sigma M_D = 0$: $C_x(R) - P(R) = 0$

$$C_x = +P$$

$\xrightarrow{+} \Sigma F_x = 0$: $C_x - B\sin\theta = 0$

$$P - B\sin\theta = 0$$

$$B = P/\sin\theta$$

$$\mathbf{B} = \frac{P}{\sin\theta} \searrow \theta$$

$+\uparrow \Sigma F_y = 0$: $C_y + B\cos\theta - P = 0$

$$C_y + (P/\sin\theta)\cos\theta - P = 0$$

$$C_y = P\left(1 - \frac{1}{\tan\theta}\right)$$

For $\theta = 30°$:

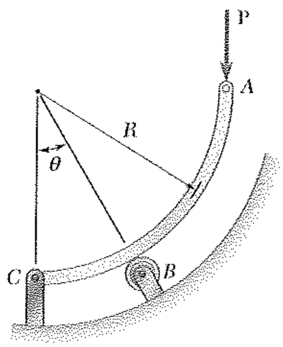
(*a*) $B = P/\sin 30° = 2P$ $\qquad \mathbf{B} = 2P \searrow 60.0°$ ◀

(*b*) $C_x = +P \quad C_x = P \longrightarrow$

$C_y = P(1 - 1/\tan 30°) = -0.7321P$ $\qquad C_y = 0.7321P \downarrow$

$\mathbf{C} = 1.239P \nwarrow 36.2°$ ◀

PROBLEM 4.32



Rod ABC is bent in the shape of an arc of circle of radius R. Knowing the $\theta = 60°$, determine the reaction (*a*) at B, (*b*) at C.

SOLUTION

See the solution to Problem 4.31 for the free-body diagram and analysis leading to the following expressions:

$$C_x = +P$$

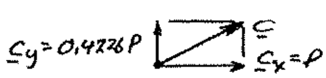
$$C_y = P\left(1 - \frac{1}{\tan\theta}\right)$$

$$B = \frac{P}{\sin\theta}$$

For $\theta = 60°$:

(*a*) $B = P/\sin 60° = 1.1547P$ **B** $= 1.155P$ ⦣ $30.0°$ ◀

(*b*) $C_x = +P \quad C_x = P \longrightarrow$

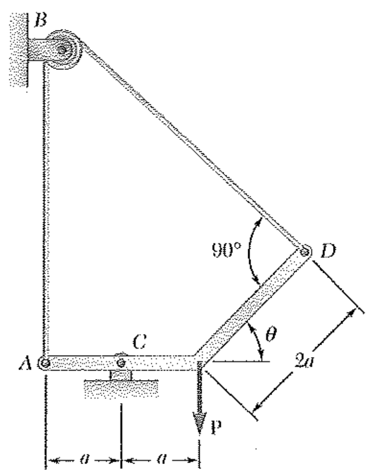


$C_y = P(1 - 1/\tan 60°) = +0.4226P$ $\mathbf{C}_y = 0.4226P \downarrow$

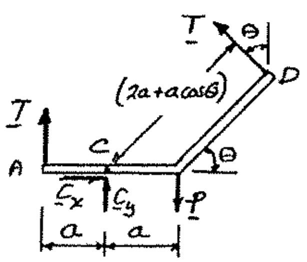
C $= 1.086P$ ⦣ $22.9°$ ◀

PROBLEM 4.33

Neglecting friction, determine the tension in cable ABD and the reaction at C when $\theta = 60°$.



SOLUTION



$+\circlearrowleft \Sigma M_C = 0: \quad T(2a + a\cos\theta) - Ta + Pa = 0$

$$T = \frac{P}{1+\cos\theta} \qquad (1)$$

$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - T\sin\theta = 0$

$$C_x = T\sin\theta = \frac{P\sin\theta}{1+\cos\theta}$$

$+\uparrow \Sigma F_y = 0: \quad C_y + T + T\cos\theta - P = 0$

$$C_y = P - T(1+\cos\theta) = P - P\frac{1+\cos\theta}{1+\cos\theta}$$

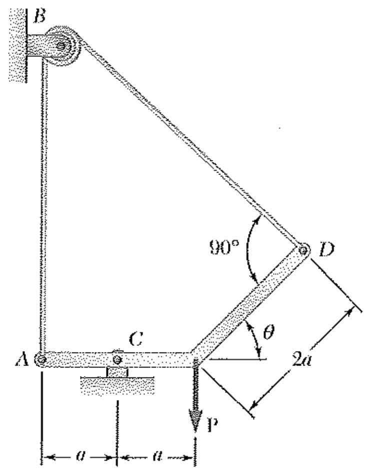
$$C_y = 0$$

Since $\quad C_y = 0, \quad C = C_x \qquad \mathbf{C} = P\dfrac{\sin\theta}{1+\cos\theta} \longrightarrow \quad (2)$

For $\theta = 60°$:

Eq. (1): $\quad T = \dfrac{P}{1+\cos 60°} = \dfrac{P}{1+\frac{1}{2}} \qquad T = \dfrac{2}{3}P$ ◀

Eq. (2): $\quad C = P\dfrac{\sin 60°}{1+\cos 60°} = P\dfrac{0.866}{1+\frac{1}{2}} \qquad \mathbf{C} = 0.577P \longrightarrow$ ◀

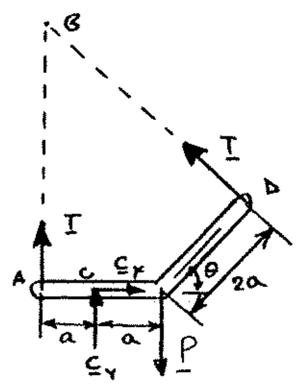


PROBLEM 4.34

Neglecting friction, determine the tension in cable ABD and the reaction at C when $\theta = 45°$.

SOLUTION

Free-Body Diagram:



Equilibrium for bracket:

$$+\circlearrowleft \Sigma M_C = 0: \quad -T(a) - P(a) + (T\sin 45°)(2a\sin 45°)$$
$$+ (T\cos 45°)(a + 2a\cos 45°) = 0$$

$$T = 0.58579 \qquad \text{or } T = 0.586P \blacktriangleleft$$

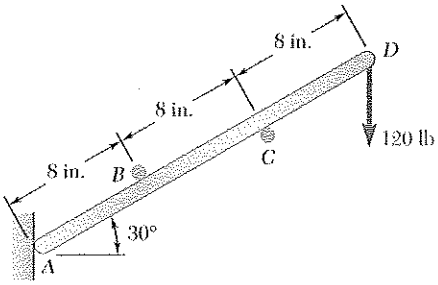
$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + (0.58579P)\sin 45° = 0$$

$$C_x = 0.41422P$$

$$+\uparrow \Sigma F_y = 0: \quad C_y + 0.58579P - P + (0.58579P)\cos 45° = 0$$

$$C_y = 0 \qquad \text{or } \mathbf{C} = 0.414P \longrightarrow \blacktriangleleft$$

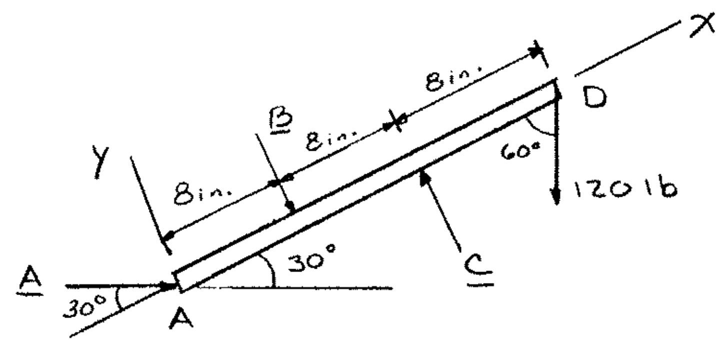
PROBLEM 4.35



A light rod AD is supported by frictionless pegs at B and C and rests against a frictionless wall at A. A vertical 120-lb force is applied at D. Determine the reactions at A, B, and C.

SOLUTION

Free-Body Diagram:



$\nearrow \Sigma F_x = 0$: $A\cos 30° - (120 \text{ lb})\cos 60° = 0$

$A = 69.28 \text{ lb}$ **A** $= 69.3 \text{ lb} \longrightarrow$ ◀

$+\circlearrowleft \Sigma M_B = 0$: $C(8 \text{ in.}) - (120 \text{ lb})(16 \text{ in.})\cos 30°$
$+ (69.28 \text{ lb})(8 \text{ in.})\sin 30° = 0$

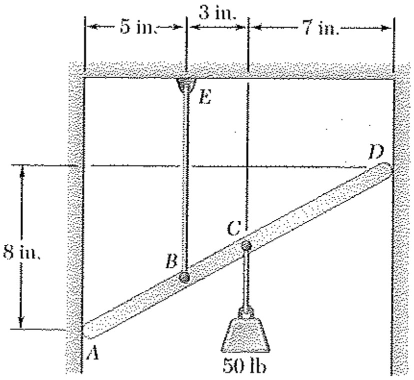
$C = 173.2 \text{ lb}$ **C** $= 173.2 \text{ lb} \searrow 60.0°$ ◀

$+\circlearrowleft \Sigma M_C = 0$: $B(8 \text{ in.}) - (120 \text{ lb})(8 \text{ in.})\cos 30°$
$+ (69.28 \text{ lb})(16 \text{ in.})\sin 30° = 0$

$B = 34.6 \text{ lb}$ **B** $= 34.6 \text{ lb} \nwarrow 60.0°$ ◀

Check: $\nwarrow \Sigma F_y = 0$: $173.2 - 34.6 - (69.28)\sin 30° - (120)\sin 60° = 0$

$0 = 0 \text{(check)}$

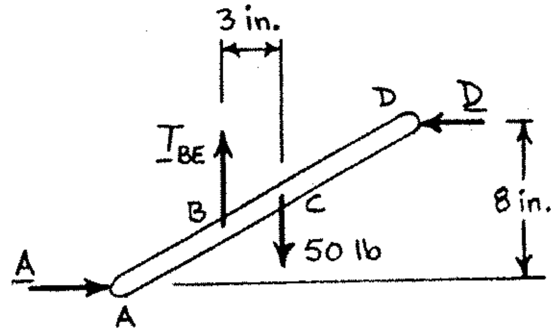


PROBLEM 4.36

A light bar AD is suspended from a cable BE and supports a 50-lb block at C. The ends A and D of the bar are in contact with frictionless vertical walls. Determine the tension in cable BE and the reactions at A and D.

SOLUTION

Free-Body Diagram:



$\Sigma F_x = 0$: $A = D$

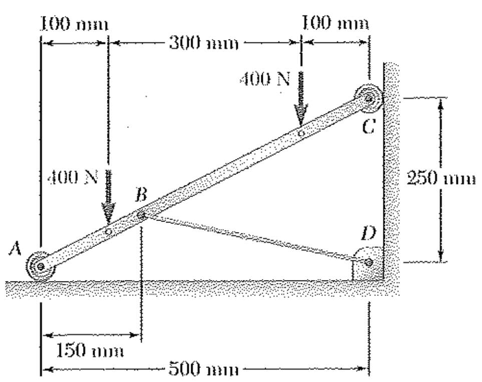
$\Sigma F_y = 0$: $T_{BE} = 50.0 \text{ lb}$ ◀

We note that the forces shown form two couples.

$+\circlearrowleft \Sigma M = 0$: $A(8 \text{ in.}) - (50 \text{ lb})(3 \text{ in.}) = 0$

$A = 18.75 \text{ lb}$

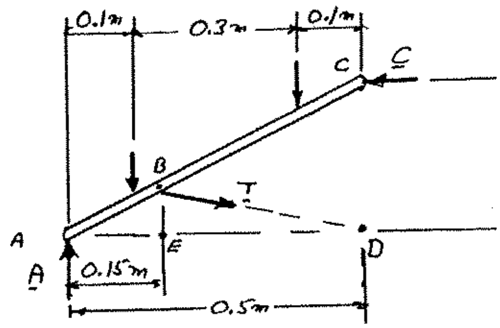
$\mathbf{A} = 18.75 \text{ lb} \longrightarrow$ $\mathbf{D} = 18.75 \text{ lb} \longleftarrow$ ◀



PROBLEM 4.37

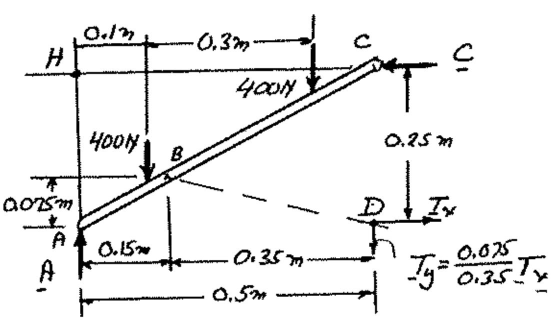
Bar AC supports two 400-N loads as shown. Rollers at A and C rest against frictionless surfaces and a cable BD is attached at B. Determine (*a*) the tension in cable BD, (*b*) the reaction at A, (*c*) the reaction at C.

SOLUTION



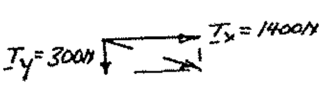
Similar triangles: ABE and ACD

$$\frac{AE}{AD}=\frac{BE}{CD};\quad \frac{0.15\text{ m}}{0.5\text{ m}}=\frac{BE}{0.25\text{ m}};\quad BE=0.075\text{ m}$$



(*a*) $$+\circlearrowleft\Sigma M_A=0:\quad T_x(0.25\text{ m})-\left(\frac{0.075}{0.35}T_x\right)(0.5\text{ m})-(400\text{ N})(0.1\text{ m})-(400\text{ N})(0.4\text{ m})=0$$

$$T_x=1400\text{ N}$$



$$T_y=\frac{0.075}{0.35}(1400\text{ N})$$
$$=300\text{ N}$$

$$T=1432\text{ lb} ◀$$

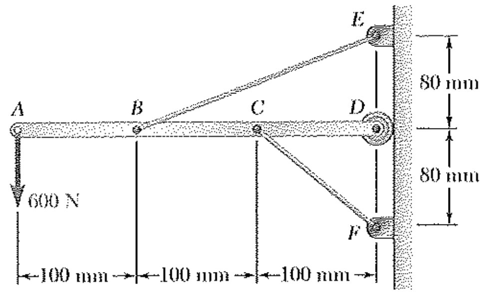
PROBLEM 4.37 (Continued)

(*b*) $+\uparrow \Sigma F_y = 0$: $A - 300\text{ N} - 400\text{ N} - 400\text{ N} = 0$

$A = +1100\text{ N}$ $\mathbf{A} = 1100\text{ N}\uparrow$ ◀

(*c*) $\xrightarrow{+} \Sigma F_x = 0$: $-C + 1400\text{ N} = 0$

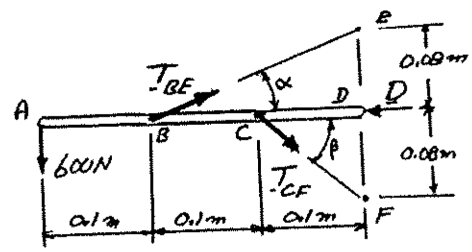
$C = +1400\text{ N}$ $\mathbf{C} = 1400\text{ N} \longleftarrow$ ◀



PROBLEM 4.38

Determine the tension in each cable and the reaction at D.

SOLUTION



$$\tan\alpha = \frac{0.08\text{ m}}{0.2\text{ m}}$$

$$\alpha = 21.80°$$

$$\tan\beta = \frac{0.08\text{ m}}{0.1\text{ m}}$$

$$\beta = 38.66°$$

$+\circlearrowleft \Sigma M_B = 0$: $(600\text{ N})(0.1\text{ m}) - (T_{CF}\sin 38.66°)(0.1\text{ m}) = 0$

$T_{CF} = 960.47\text{ N}$ $\qquad$ $T_{CF} = 96.0\text{ N}$ ◀

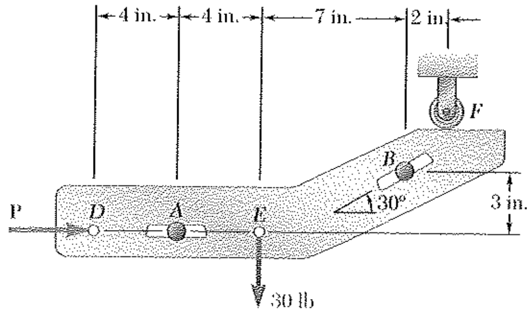
$+\circlearrowleft \Sigma M_C = 0$: $(600\text{ N})(0.2\text{ m}) - (T_{BE}\sin 21.80°)(0.1\text{ m}) = 0$

$T_{BE} = 3231.1\text{ N}$ $\qquad$ $T_{BE} = 3230\text{ N}$ ◀

$\xrightarrow{+} \Sigma F_y = 0$: $T_{BE}\cos\alpha + T_{CF}\cos\beta - D = 0$

$$(3231.1\text{ N})\cos 21.80° + (960.47\text{ N})\cos 38.66° - D = 0$$

$D = 3750.03\text{ N}$ $\qquad$ $\mathbf{D} = 3750\text{ N} \longleftarrow$ ◀

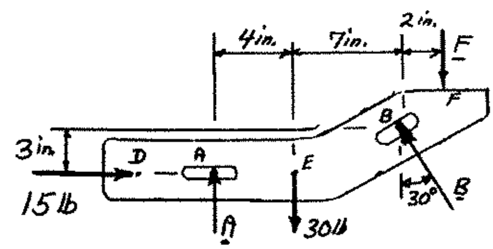


PROBLEM 4.39

Two slots have been cut in plate *DEF*, and the plate has been placed so that the slots fit two fixed, frictionless pins *A* and *B*. Knowing that $P = 15$ lb, determine (*a*) the force each pin exerts on the plate, (*b*) the reaction at *F*.

SOLUTION

Free-Body Diagram:



$\xrightarrow{+} \Sigma F_x = 0$: $15 \text{ lb} - B\sin 30° = 0$ **B** = 30.0 lb ⦪ 60.0° ◀

$+\circlearrowleft \Sigma M_A = 0$: $-(30 \text{ lb})(4 \text{ in.}) + B\sin 30°(3 \text{ in.}) + B\cos 30°(11 \text{ in.}) - F(13 \text{ in.}) = 0$

$$-120 \text{ lb}\cdot\text{in.} + (30 \text{ lb})\sin 30°(3 \text{ in.}) + (30 \text{ lb})\cos 30°(11 \text{ in.}) - F(13 \text{ in.}) = 0$$

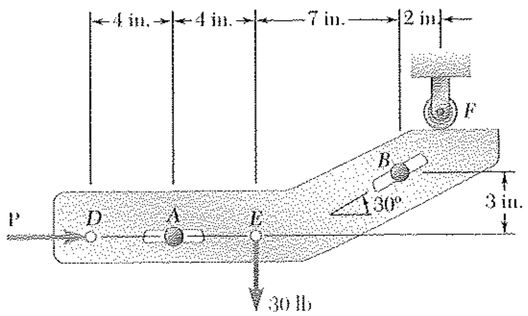
$$F = +16.2145 \text{ lb}$$ **F** = 16.21 lb↓ ◀

$+\uparrow \Sigma F_y = 0$: $A - 30 \text{ lb} + B\cos 30° - F = 0$

$$A - 30 \text{ lb} + (30 \text{ lb})\cos 30° - 16.2145 \text{ lb} = 0$$

$$A = +20.23 \text{ lb}$$ **A** = 20.2 lb↑ ◀

PROBLEM 4.40

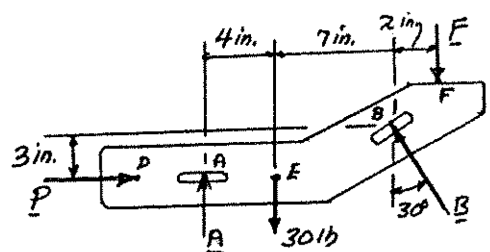


For the plate of Problem 4.39 the reaction at F must be directed downward, and its maximum allowable value is 20 lb. Neglecting friction at the pins, determine the required range of values of P.

PROBLEM 4.39 Two slots have been cut in plate DEF, and the plate has been placed so that the slots fit two fixed, frictionless pins A and B. Knowing that $P = 15$ lb, determine (a) the force each pin exerts on the plate, (b) the reaction at F.

SOLUTION

Free-Body Diagram:



$\xrightarrow{+} \Sigma F_x = 0$: $P - B\sin 30° = 0$ **B** $= 2P \measuredangle 60°$

$+\circlearrowleft \Sigma M_A = 0$: $-(30 \text{ lb})(4 \text{ in.}) + B\sin 30°(3 \text{ in.}) + B\cos 30°(11 \text{ in.}) - F(13 \text{ in.}) = 0$

$-120 \text{ lb}\cdot\text{in.} + 2P\sin 30°(3 \text{ in.}) + 2P\cos 30°(11 \text{ in.}) - F(13 \text{ in.}) = 0$

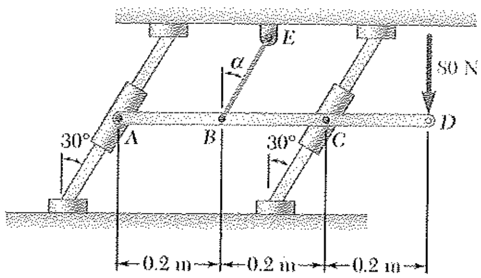
$-120 + 3P + 19.0525P - 13F = 0$

$$P = \frac{13E + 120}{22.0525} \quad (1)$$

For $F = 0$: $P = \dfrac{13(0) + 120}{22.0525} = 5.442 \text{ lb}$

For $P = 20$ lb: $P = \dfrac{13(20) + 120}{22.0525} = 17.232 \text{ lb}$

For $0 \le F \le 20$ lb: $5.44 \text{ lb} \le P \le 17.231 \text{ lb}$ ◀

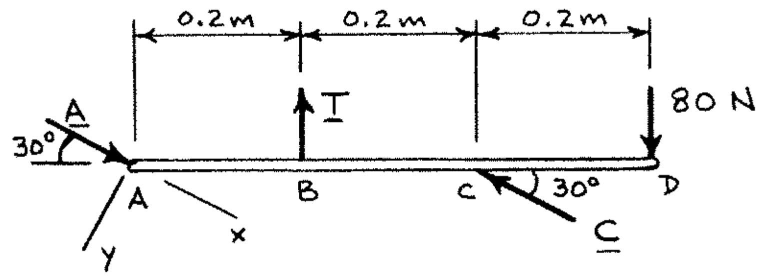


PROBLEM 4.41

Bar AD is attached at A and C to collars that can move freely on the rods shown. If the cord BE is vertical ($\alpha = 0$), determine the tension in the cord and the reactions at A and C.

SOLUTION

Free-Body Diagram:



$+\swarrow \Sigma F_y = 0$: $\quad -T\cos 30° + (80\text{ N})\cos 30° = 0$

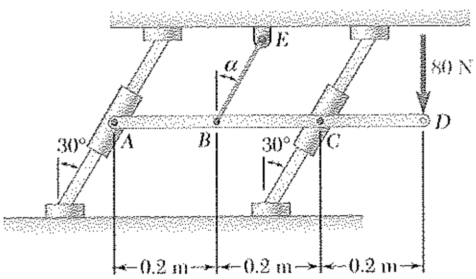
$T = 80\text{ N}$ $\qquad T = 80.0\text{ N}$ ◀

$+\circlearrowleft \Sigma M_C = 0$: $\quad (A\sin 30°)(0.4\text{ m}) - (80\text{ N})(0.2\text{ m}) - (80\text{ N})(0.2\text{ m}) = 0$

$A = +160\text{ N}$ $\qquad \mathbf{A} = 160.0\text{ N}$ ⦪ $30.0°$ ◀

$+\circlearrowleft \Sigma M_A = 0$: $\quad (80\text{ N})(0.2\text{ m}) - (80\text{ N})(0.6\text{ m}) + (C\sin 30°)(0.4\text{ m}) = 0$

$C = +160\text{ N}$ $\qquad \mathbf{C} = 160.0\text{ N}$ ⦪ $30.0°$ ◀



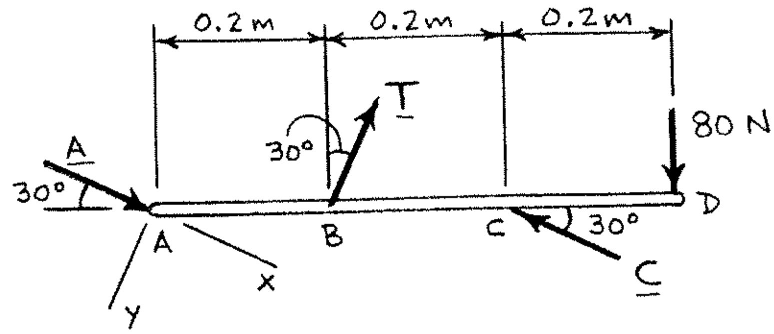
PROBLEM 4.42

Solve Problem 4.41 if the cord BE is parallel to the rods ($\alpha = 30°$).

PROBLEM 4.41 Bar *AD* is attached at *A* and *C* to collars that can move freely on the rods shown. If the cord *BE* is vertical ($\alpha = 0$), determine the tension in the cord and the reactions at *A* and *C*.

SOLUTION

Free-Body Diagram:



$$+\!\!\!\diagup\ \Sigma F_y = 0: \quad -T + (80\text{ N})\cos 30° = 0$$

$$T = 69.282\text{ N} \qquad\qquad T = 69.3\text{ N} \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_C = 0: \quad -(69.282\text{ N})\cos 30°(0.2\text{ m}) - (80\text{ N})(0.2\text{ m}) + (A\sin 30°)(0.4\text{ m}) = 0$$

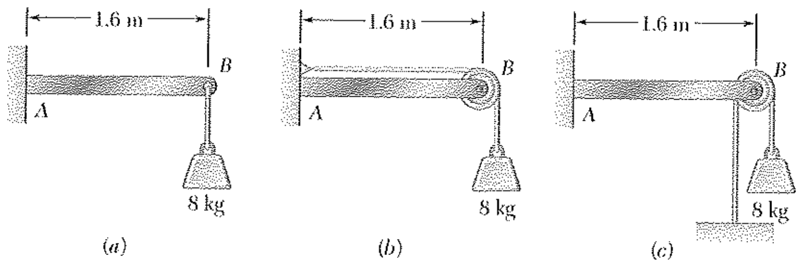
$$A = +140.000\text{ N} \qquad \mathbf{A} = 140.0\text{ N} \measuredangle 30.0° \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0: \quad +(69.282\text{ N})\cos 30°(0.2\text{ m}) - (80\text{ N})(0.6\text{ m}) + (C\sin 30°)(0.4\text{ m}) = 0$$

$$C = +180.000\text{ N} \qquad \mathbf{C} = 180.0\text{ N} \measuredangle 30.0° \blacktriangleleft$$

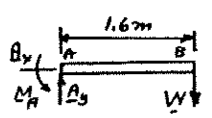
PROBLEM 4.43

An 8-kg mass can be supported in the three different ways shown. Knowing that the pulleys have a 100-mm radius, determine the reaction at A in each case.



SOLUTION

$$W = mg = (8\text{ kg})(9.81\text{ m/s}^2) = 78.48\text{ N}$$



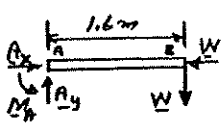
(a) $\Sigma F_x = 0$: $A_x = 0$

$+\uparrow \Sigma F_y = 0$: $A_y - W = 0$ $\qquad \mathbf{A}_y = 78.48\text{ N}\uparrow$

$+\circlearrowleft \Sigma M_A = 0$: $M_A - W(1.6\text{ m}) = 0$

$M_A = +(78.48\text{ N})(1.6\text{ m})$ $\qquad \mathbf{M}_A = 125.56\text{ N}\cdot\text{m}\circlearrowleft$

$\mathbf{A} = 78.5\text{ N}\uparrow$ $\qquad \mathbf{M}_A = 125.6\text{ N}\cdot\text{m}\circlearrowleft$ ◀



(b) $\overset{+}{\rightarrow} \Sigma F_x = 0$: $A_x - W = 0$ $\qquad \mathbf{A}_x = 78.48\uparrow$

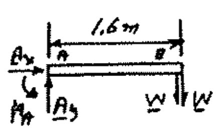
$+\uparrow \Sigma F_y = 0$: $A_y - W = 0$ $\qquad \mathbf{A}_y = 78.48\leftarrow$

$$\mathbf{A} = (78.48\text{ N})\sqrt{2} = 110.99\text{ N} \measuredangle 45°$$

$+\circlearrowleft \Sigma M_A = 0$: $M_A - W(1.6\text{ m}) = 0$

$M_A = +(78.48\text{ N})(1.6\text{ m})$ $\qquad \mathbf{M}_A = 125.56\text{ N}\cdot\text{m}\circlearrowleft$

$\mathbf{A} = 111.0\text{ N} \measuredangle 45°$ $\qquad \mathbf{M}_A = 125.6\text{ N}\cdot\text{m}\circlearrowleft$ ◀



(c) $\Sigma F_x = 0$: $A_x = 0$

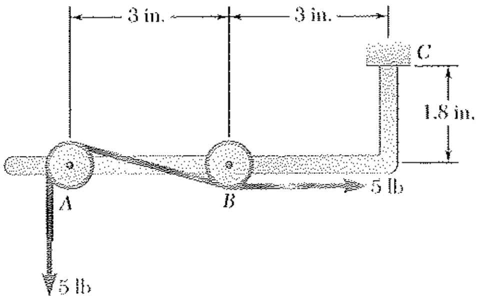
$+\uparrow \Sigma F_y = 0$: $A_y - 2W = 0$

$$A_y = 2W = 2(78.48\text{ N}) = 156.96\text{ N}\uparrow$$

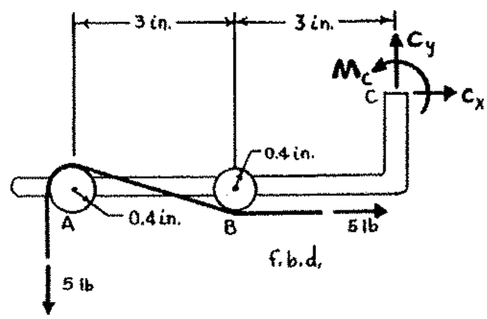
$+\circlearrowleft \Sigma M_A = 0$: $M_A - 2W(1.6\text{ m}) = 0$

$M_A = +2(78.48\text{ N})(1.6\text{ m})$ $\qquad \mathbf{M}_A = 125.1\text{ N}\cdot\text{m}\circlearrowleft$

$\mathbf{A} = 157.0\text{ N}\uparrow$ $\qquad \mathbf{M}_A = 125\text{ N}\cdot\text{m}\circlearrowleft$ ◀

	**PROBLEM 4.44** A tension of 5 lb is maintained in a tape as it passes through the support system shown. Knowing that the radius of each pulley is 0.4 in., determine the reaction at C.

SOLUTION



From f.b.d. of system

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + (5 \text{ lb}) = 0$$

$$C_x = -5 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - (5 \text{ lb}) = 0$$

$$C_y = 5 \text{ lb}$$

Then

$$C = \sqrt{(C_x)^2 + (C_y)^2}$$
$$= \sqrt{(5)^2 + (5)^2}$$
$$= 7.0711 \text{ lb}$$

and

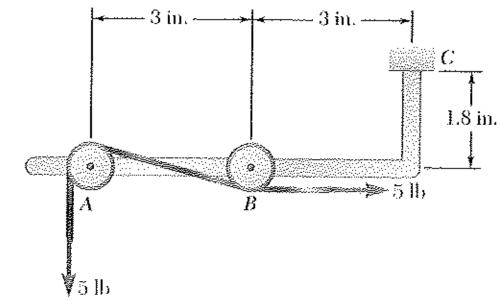
$$\theta = \tan^{-1}\left(\frac{+5}{-5}\right) = -45°$$

or $\mathbf{C} = 7.07 \text{ lb} \measuredangle 45.0°$ ◀

$$+\circlearrowleft \Sigma M_C = 0: \quad M_C + (5 \text{ lb})(6.4 \text{ in.}) + (5 \text{ lb})(2.2 \text{ in.}) = 0$$

$$M_C = -43.0 \text{ lb} \cdot \text{in}$$

or $\mathbf{M}_C = 43.0 \text{ lb} \cdot \text{in.} \circlearrowright$ ◀



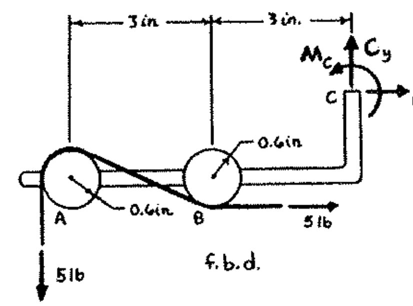
PROBLEM 4.45

Solve Problem 4.44, assuming that 0.6-in.-radius pulleys are used.

PROBLEM 4.44 A tension of 5 lb is maintained in a tape as it passes through the support system shown. Knowing that the radius of each pulley is 0.4 in., determine the reaction at *C*.

SOLUTION

From f.b.d. of system



$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + (5\text{ lb}) = 0$$

$$C_x = -5\text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - (5\text{ lb}) = 0$$

$$C_y = 5\text{ lb}$$

Then

$$C = \sqrt{(C_x)^2 + (C_y)^2}$$
$$= \sqrt{(5)^2 + (5)^2}$$
$$= 7.0711\text{ lb}$$

and

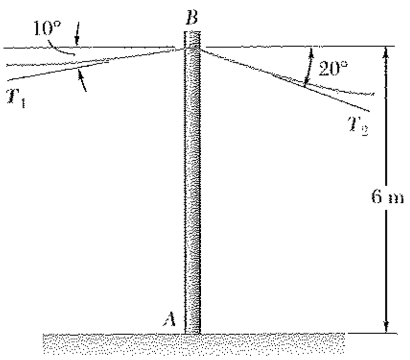
$$\theta = \tan^{-1}\left(\frac{5}{-5}\right) = -45.0^\circ$$

or $\mathbf{C} = 7.07\text{ lb} \measuredangle 45.0^\circ$ ◀

$$+\circlearrowleft \Sigma M_C = 0: \quad M_C + (5\text{ lb})(6.6\text{ in.}) + (5\text{ lb})(2.4\text{ in.}) = 0$$

$$M_C = -45.0\text{ lb}\cdot\text{in.}$$

or $\mathbf{M}_C = 45.0\text{ lb}\cdot\text{in.} \circlearrowright$ ◀

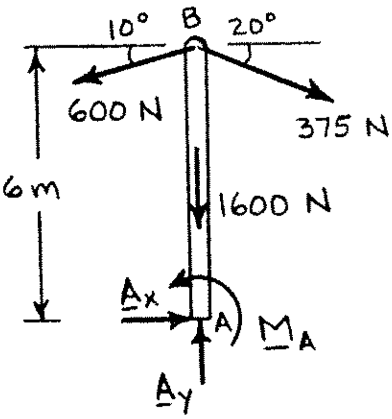


PROBLEM 4.46

A 6-m telephone pole weighing 1600 N is used to support the ends of two wires. The wires form the angles shown with the horizontal and the tensions in the wires are, respectively, $T_1 = 600$ N and $T_2 = 375$ N. Determine the reaction at the fixed end A.

SOLUTION

Free-Body Diagram:



$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + (375 \text{ N})\cos 20° - (600 \text{ N})\cos 10° = 0$$

$$A_x = +238.50 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 1600 \text{ N} - (600 \text{ N})\sin 10° - (375 \text{ N})\sin 20° = 0$$

$$A_y = +1832.45 \text{ N}$$

$$A = \sqrt{238.50^2 + 1832.45^2}$$

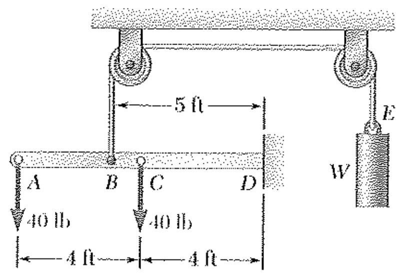
$$\theta = \tan^{-1}\frac{1832.45}{238.50}$$

$\mathbf{A} = 1848 \text{ N} \angle 82.6°$ ◀

$$+\circlearrowleft \Sigma M_A = 0: \quad M_A + (600 \text{ N})\cos 10°(6 \text{ m}) - (375 \text{ N})\cos 20°(6 \text{ m}) = 0$$

$$M_A = -1431.00 \text{ N}\cdot\text{m}$$

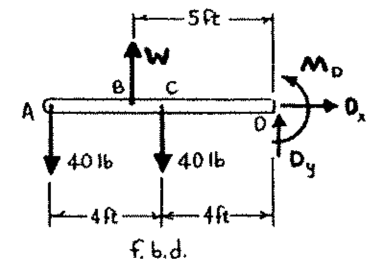
$\mathbf{M}_A = 1431 \text{ N}\cdot\text{m} \circlearrowright$ ◀



PROBLEM 4.47

Beam AD carries the two 40-lb loads shown. The beam is held by a fixed support at D and by the cable BE that is attached to the counterweight W. Determine the reaction at D when (*a*) $W = 100$ lb, (*b*) $W = 90$ lb.

SOLUTION



(*a*) $W = 100$ lb

From f.b.d. of beam AD

$$\xrightarrow{+} \Sigma F_x = 0: \quad D_x = 0$$

$$+\!\uparrow \Sigma F_y = 0: \quad D_y - 40\text{ lb} - 40\text{ lb} + 100\text{ lb} = 0$$

$$D_y = -20.0\text{ lb}$$

or $\mathbf{D} = 20.0\text{ lb}\downarrow$ ◀

$$+\circlearrowleft \Sigma M_D = 0: \quad M_D - (100\text{ lb})(5\text{ ft}) + (40\text{ lb})(8\text{ ft}) + (40\text{ lb})(4\text{ ft}) = 0$$

$$M_D = 20.0\text{ lb}\cdot\text{ft}$$

or $\mathbf{M}_D = 20.0\text{ lb}\cdot\text{ft}\circlearrowleft$ ◀

(*b*) $W = 90$ lb

From f.b.d. of beam AD

$$\xrightarrow{+} \Sigma F_x = 0: \quad D_x = 0$$

$$+\!\uparrow \Sigma F_y = 0: \quad D_y + 90\text{ lb} - 40\text{ lb} - 40\text{ lb} = 0$$

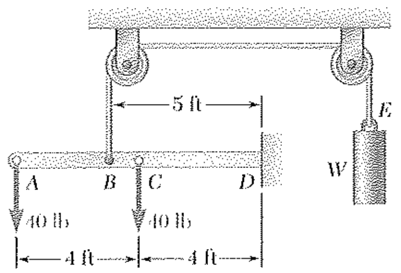
$$D_y = -10.00\text{ lb}$$

or $\mathbf{D} = 10.00\text{ lb}\downarrow$ ◀

$$+\circlearrowleft \Sigma M_D = 0: \quad M_D - (90\text{ lb})(5\text{ ft}) + (40\text{ lb})(8\text{ ft}) + (40\text{ lb})(4\text{ ft}) = 0$$

$$M_D = -30.0\text{ lb}\cdot\text{ft}$$

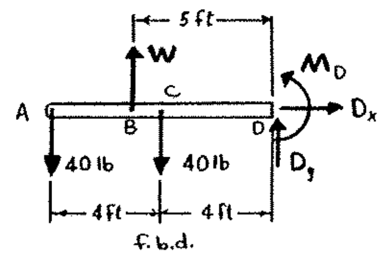
or $\mathbf{M}_D = -30.0\text{ lb}\cdot\text{ft}\circlearrowright$ ◀



PROBLEM 4.48

For the beam and loading shown, determine the range of values of W for which the magnitude of the couple at D does not exceed 40 lb · ft.

SOLUTION



For W_{min}, $\quad M_D = -40 \text{ lb} \cdot \text{ft}$

From f.b.d. of beam AD $\quad +\circlearrowleft \Sigma M_D = 0$: $(40 \text{ lb})(8 \text{ ft}) - W_{min}(5 \text{ ft}) + (40 \text{ lb})(4 \text{ ft}) - 40 \text{ lb} \cdot \text{ft} = 0$

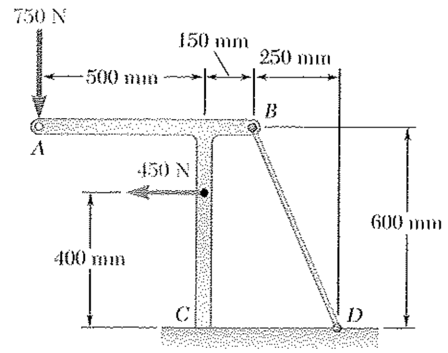
$$W_{min} = 88.0 \text{ lb}$$

For W_{max}, $\quad M_D = 40 \text{ lb} \cdot \text{ft}$

From f.b.d. of beam AD $\quad +\circlearrowleft \Sigma M_D = 0$: $(40 \text{ lb})(8 \text{ ft}) - W_{max}(5 \text{ ft}) + (40 \text{ lb})(4 \text{ ft}) + 40 \text{ lb} \cdot \text{ft} = 0$

$$W_{max} = 104.0 \text{ lb}$$

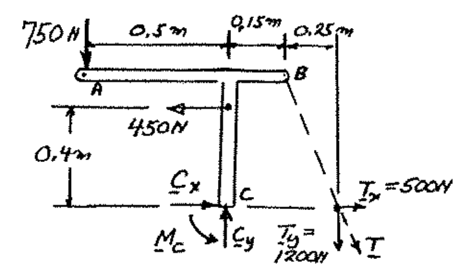
or $88.0 \text{ lb} \le W \le 104.0 \text{ lb}$ ◀



PROBLEM 4.49

Knowing that the tension in wire BD is 1300 N, determine the reaction at the fixed support C of the frame shown.

SOLUTION



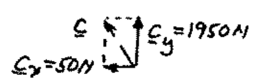
$$T = 1300 \text{ N}$$

$$T_x = \frac{5}{13}T = 500 \text{ N}$$

$$T_y = \frac{12}{13}T = 1200 \text{ N}$$

$\xrightarrow{+} \Sigma M_x = 0$: $C_x - 450 \text{ N} + 500 \text{ N} = 0 \quad C_x = -50 \text{ N}$ $\qquad$ $\mathbf{C}_x = 50 \text{ N} \longleftarrow$

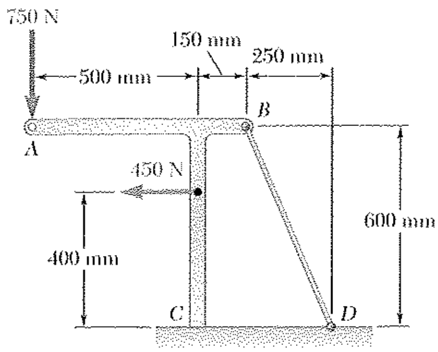
$+\uparrow \Sigma F_y = 0$: $C_y - 750 \text{ N} - 1200 \text{ N} = 0 \quad C_y = +1950 \text{ N}$ $\qquad$ $\mathbf{C}_y = 1950 \text{ N} \uparrow$



$\mathbf{C} = 1951 \text{ N} \measuredangle 88.5°$ ◀

$+\circlearrowleft \Sigma M_C = 0$: $M_C + (750 \text{ N})(0.5 \text{ m}) + (4.50 \text{ N})(0.4 \text{ m}) - (1200 \text{ N})(0.4 \text{ m}) = 0$

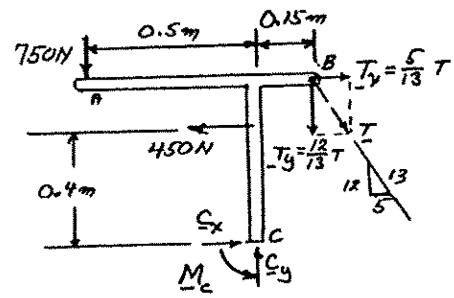
$M_C = -75.0 \text{ N} \cdot \text{m}$ $\qquad$ $\mathbf{M}_C = 75.0 \text{ N} \cdot \text{m} \circlearrowright$ ◀



PROBLEM 4.50

Determine the range of allowable values of the tension in wire BD if the magnitude of the couple at the fixed support C is not to exceed 100 N · m.

SOLUTION



$$+\circlearrowleft \Sigma M_C = 0: \quad (750\text{ N})(0.5\text{ m}) + (450\text{ N})(0.4\text{ m}) - \left(\frac{5}{13}T\right)(0.6\text{ m})$$

$$-\left(\frac{12}{13}T\right)(0.15\text{ m}) + M_C = 0$$

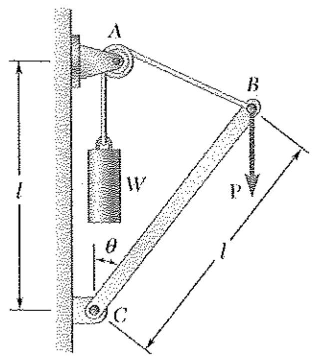
$$375\text{ N}\cdot\text{m} + 180\text{ N}\cdot\text{m} - \left(\frac{4.8}{13}\text{m}\right)T + M_C = 0$$

$$T = \frac{13}{4.8}(555 + M_C)$$

For $\quad M_C = -100\text{ N}\cdot\text{m}: \quad T = \frac{13}{4.8}(555 - 100) = 1232\text{ N}$

For $\quad M_C = +100\text{ N}\cdot\text{m}: \quad T = \frac{13}{4.8}(555 + 100) = 1774\text{ N}$

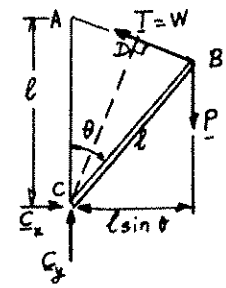
For $\quad |M_C| \le 100\text{ N}\cdot\text{m}: \quad 1.232\text{ kN} \le T \le 1.774\text{ kN}$ ◀



PROBLEM 4.51

A vertical load **P** is applied at end B of rod BC. (a) Neglecting the weight of the rod, express the angle θ corresponding to the equilibrium position in terms of P, l, and the counterweight W. (b) Determine the value of θ corresponding to equilibrium if $P = 2W$.

SOLUTION



(a) Triangle ABC is isosceles.

We have $$CD = (BC)\cos\frac{\theta}{2} = l\cos\frac{\theta}{2}$$

$$+\circlearrowleft \Sigma M_C = 0: \quad W\left(l\cos\frac{\theta}{2}\right) - P(l\sin\theta) = 0$$

Setting $$\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}: \quad Wl\cos\frac{\theta}{2} - 2Pl\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 0$$

$$W - 2P\sin\frac{\theta}{2} = 0$$

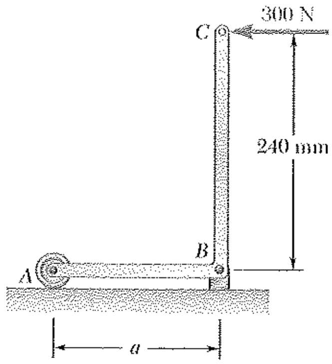
$$\theta = 2\sin^{-1}\left(\frac{W}{2P}\right) \blacktriangleleft$$

(b) For $P = 2W$: $$\sin\frac{\theta}{2} = \frac{W}{2P} = \frac{W}{4W} = 0.25$$

$$\frac{\theta}{2} = 14.5^\circ$$

$$\theta = 29.0^\circ \blacktriangleleft$$

or $$\frac{\theta}{2} = 165.5^\circ \quad \theta = 331^\circ\text{(discard)}$$



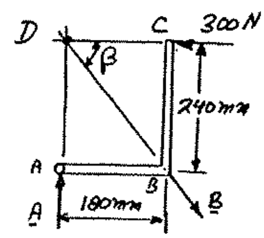
PROBLEM 4.61

Determine the reactions at A and B when $a = 180$ mm.

SOLUTION

Reaction at B must pass through D where A and 300-N load intersect.

Free-Body Diagram:
(Three-force member)

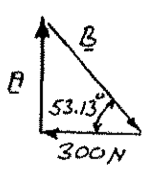


<u>ΔBCD:</u>

$$\tan\beta = \frac{240}{180}$$

$$\beta = 53.13^\circ$$

<u>Force triangle</u>

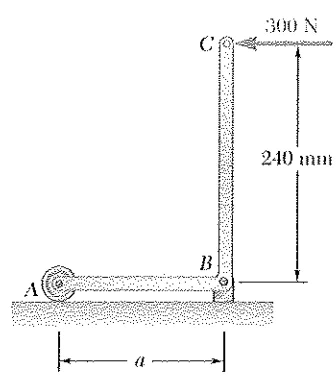


$$A = (300\text{ N})\tan 53.13^\circ$$
$$= 400\text{ N}$$

A = 400 N ↑ ◀

$$B = \frac{300\text{ N}}{\cos 53.13^\circ}$$
$$= 500\text{ N}$$

B = 500 N ⦪ 53.1° ◀



PROBLEM 4.62

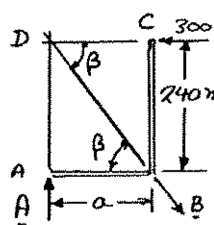
For the bracket and loading shown, determine the range of values of the distance a for which the magnitude of the reaction at B does not exceed 600 N.

SOLUTION

Reaction at B must pass through D where A and 300-N load intersect.

Free-Body Diagram:

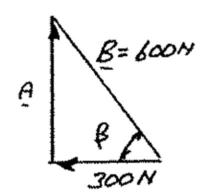
(Three-force member)



$$a = \frac{240 \text{ mm}}{\tan \beta} \qquad (1)$$

Force Triangle

(with $B = 600$ N)



$$\cos \beta = \frac{300 \text{ N}}{600 \text{ N}} = 0.5$$

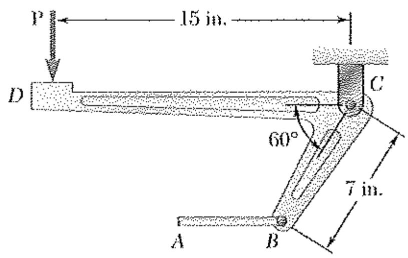
$$\beta = 60.0°$$

Eq. (1)

$$a = \frac{240 \text{ mm}}{\tan 60.0°}$$

$$= 138.56 \text{ mm}$$

For $B \leq 600$ N $\quad a \geq 138.6$ mm ◄



PROBLEM 4.63

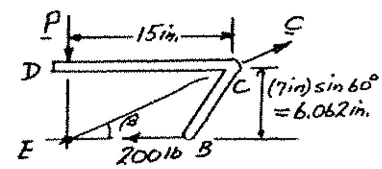
Using the method of Section 4.7, solve Problem 4.17.

PROBLEM 4.17 The required tension in cable *AB* is 200 lb. Determine (*a*) the vertical force **P** that must be applied to the pedal, (*b*) the corresponding reaction at *C*.

SOLUTION

Free-Body Diagram:

(Three-Force body)

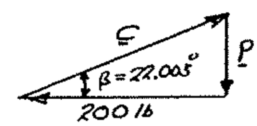


Reaction at *C* must pass through *E*, where *D* and 200-lb force intersect.

$$\tan\beta = \frac{6.062 \text{ in.}}{15 \text{ in.}}$$

$$\beta = 22.005°$$

Force triangle



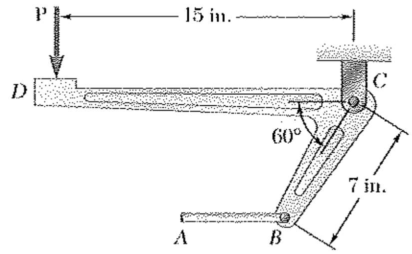
(*a*) $P = (200 \text{ lb})\tan 22.005°$

$P = 80.83 \text{ lb}$

$\mathbf{P} = 80.8 \text{ lb}\downarrow$ ◀

(*b*) $C = \dfrac{200 \text{ lb}}{\cos 22.005°} = 215.7 \text{ lb}$

$\mathbf{C} = 216 \text{ lb} \measuredangle 22.0°$ ◀



PROBLEM 4.64

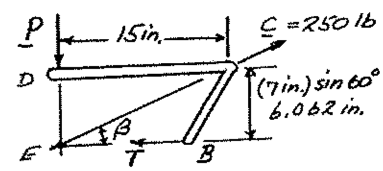
Using the method of Section 4.7, solve Problem 4.18.

PROBLEM 4.18 Determine the maximum tension that can be developed in cable *AB* if the maximum allowable value of the reaction at *C* is 250 lb.

SOLUTION

Free-Body Diagram:

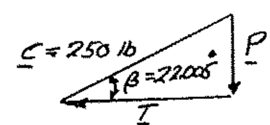
(Three -Force body)



Reaction at *C* must pass through *E*, where *D* and the force **T** intersect.

$$\tan \beta = \frac{6.062 \text{ in.}}{15 \text{ in.}}$$

$$\beta = 22.005°$$

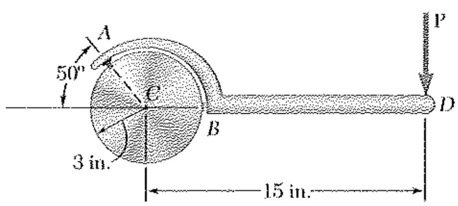


Force triangle

$$T = (250 \text{ lb}) \cos 22.005°$$

$$T = 231.8 \text{ lb}$$

$T = 232$ lb ◀



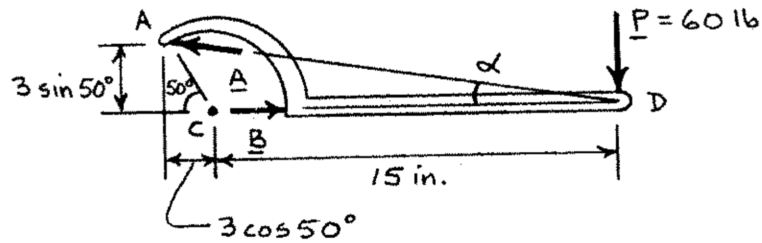
PROBLEM 4.65

The spanner shown is used to rotate a shaft. A pin fits in a hole at A, while a flat, frictionless surface rests against the shaft at B. If a 60-lb force **P** is exerted on the spanner at D, find the reactions at A and B.

SOLUTION

Free-Body Diagram:

(Three-Force body)

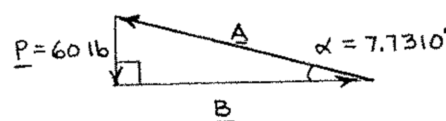


The line of action of **A** must pass through D, where **B** and **P** intersect.

$$\tan\alpha = \frac{3\sin 50°}{3\cos 50° + 15}$$
$$= 0.135756$$
$$\alpha = 7.7310°$$

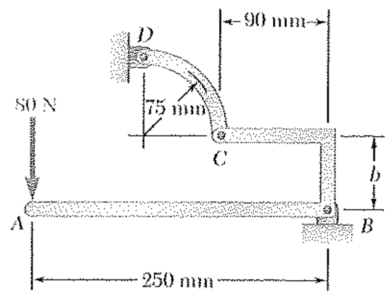
Force triangle

$$A = \frac{60 \text{ lb}}{\sin 7.7310°}$$
$$= 446.02 \text{ lb}$$
$$B = \frac{60 \text{ lb}}{\tan 7.7310°}$$
$$= 441.97 \text{ lb}$$



A = 446 lb ⦫ 7.73° ◀

B = 442 lb ⟶ ◀



PROBLEM 4.66

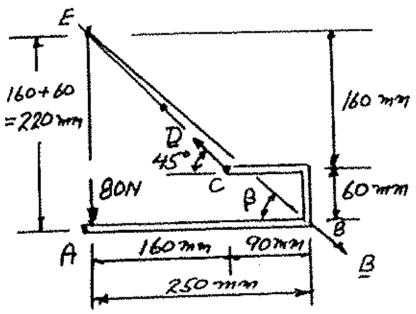
Determine the reactions at B and D when $b = 60$ mm.

SOLUTION

Since CD is a two-force member, the line of action of reaction at D must pass through Points C and D. 45°

Free-Body Diagram:

(Three-Force body)

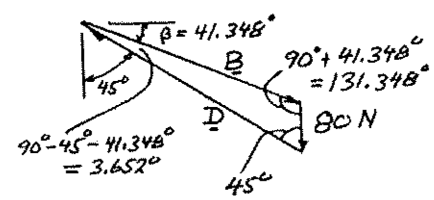


Reaction at B must pass through E, where the reaction at D and 80-N force intersect.

$$\tan\beta = \frac{220 \text{ mm}}{250 \text{ mm}}$$

$$\beta = 41.348°$$

Force triangle



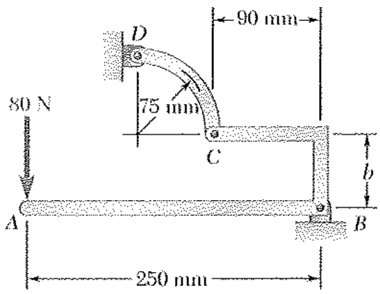
Law of sines

$$\frac{80 \text{ N}}{\sin 3.652°} = \frac{B}{\sin 45°} = \frac{D}{\sin 131.348°}$$

$$B = 888.0 \text{ N}$$

$$D = 942.8 \text{ N}$$

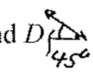
B $= 888$ N ⦪ $41.3°$ **D** $= 943$ N ⦫ $45.0°$ ◀



PROBLEM 4.67

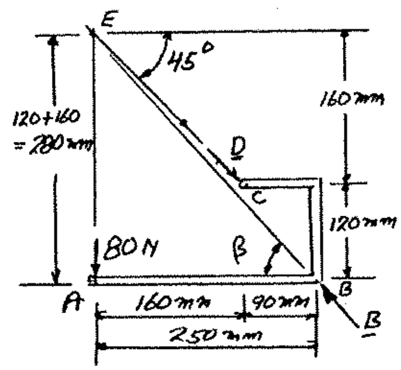
Determine the reactions at B and D when $b = 120$ mm.

SOLUTION

Since CD is a two-force member, line of action of reaction at D must pass through C and D .

Free-Body Diagram:

(Three-Force body)

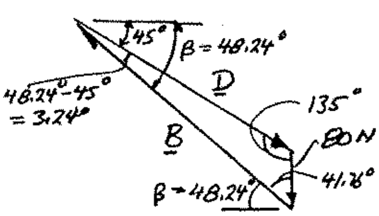


Reaction at B must pass through E, where the reaction at D and 80-N force intersect.

$$\tan\beta = \frac{280 \text{ mm}}{250 \text{ mm}}$$

$$\beta = 48.24°$$

Force triangle



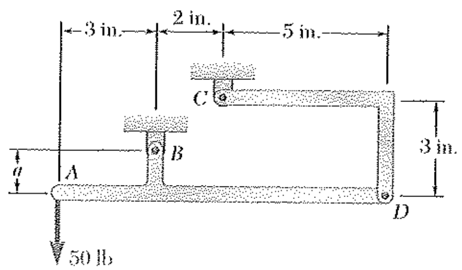
Law of sines

$$\frac{80 \text{ N}}{\sin 3.24°} = \frac{B}{\sin 135°} = \frac{D}{\sin 41.76°}$$

$$B = 1000.9 \text{ N}$$

$$D = 942.8 \text{ N}$$

B = 1001 N ⦬ 48.2°   **D** = 943 N ⦪ 45.0° ◀



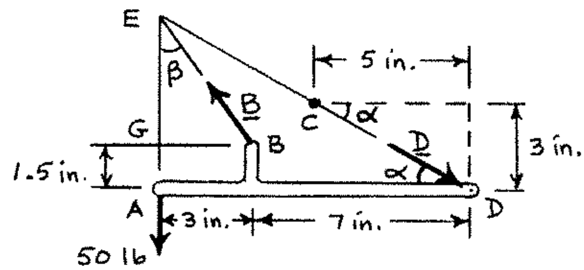
PROBLEM 4.68

Determine the reactions at B and C when $a = 1.5$ in.

SOLUTION

Since CD is a two-force member, the force it exerts on member ABD is directed along DC.

Free-Body Diagram of ABD: (Three-Force member)



The reaction at B must pass through E, where **D** and the 50-lb load intersect.

Triangle CFD: $\tan\alpha = \frac{3}{5} = 0.6$

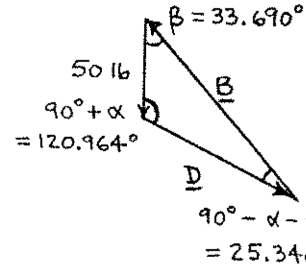
$$\alpha = 30.964^\circ$$

Triangle EAD: $AE = 10\tan\alpha = 6$ in.

$$GE = AE - AG = 6 - 1.5 = 4.5 \text{ in.}$$

Triangle EGB: $\tan\beta = \frac{GB}{GE} = \frac{3}{4.5}$

$$\beta = 33.690^\circ$$



Force triangle

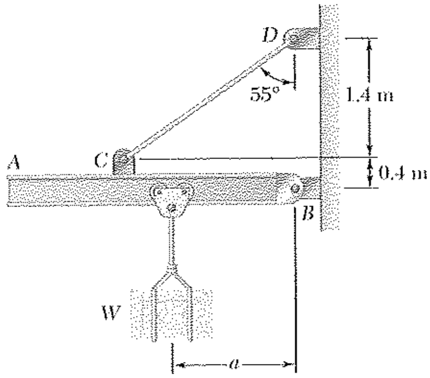
$$\frac{B}{\sin 120.964^\circ} = \frac{D}{\sin 33.690^\circ} = \frac{50 \text{ lb}}{\sin 25.346^\circ}$$

$$B = 100.155 \text{ lb}$$

$$D = 64.789 \text{ lb}$$

B $= 100.2$ lb ⦨ 56.3° ◀

C $=$ **D** $= 64.8$ lb ⦩ 31.0° ◀

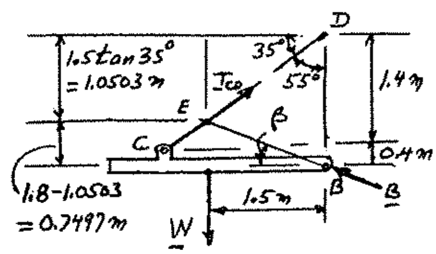


PROBLEM 4.69

A 50-kg crate is attached to the trolley-beam system shown. Knowing that $a = 1.5$ m, determine (a) the tension in cable CD, (b) the reaction at B.

SOLUTION

Free-Body Diagram:

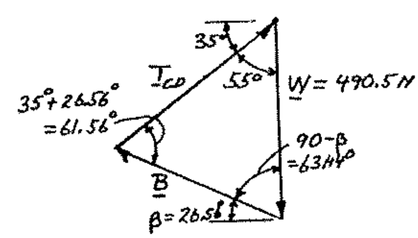


Three-Force body: **W** and $\mathbf{T}_{CD}$ intersect at E.

$$\tan\beta = \frac{0.7497\text{ m}}{1.5\text{ m}}$$

$$\beta = 26.56°$$

Force triangle 3 forces intersect at E.



$$W = (50\text{ kg})\ 9.81\text{ m/s}^2$$

$$= 490.5\text{ N}$$

Law of sines

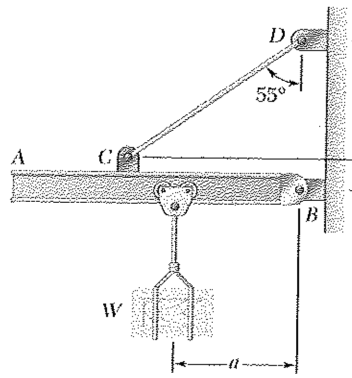
$$\frac{490.5\text{ N}}{\sin 61.56°} = \frac{T_{CD}}{\sin 63.44°} = \frac{B}{\sin 55°}$$

$$T_{CD} = 498.9\text{ N}$$

$$B = 456.9\text{ N}$$

(a) $T_{CD} = 499\text{ N}$ ◀

(b) $\mathbf{B} = 457\text{ N} \measuredangle 26.6°$ ◀



PROBLEM 4.70

Solve Problem 4.69, assuming that $a = 3$ m.

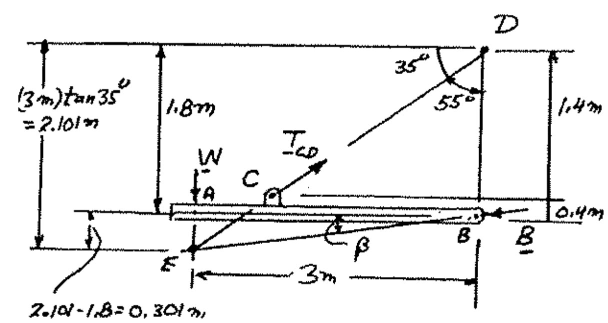
PROBLEM 4.69 A 50-kg crate is attached to the trolley-beam system shown. Knowing that $a = 1.5$ m, determine (*a*) the tension in cable *CD*, (*b*) the reaction at *B*.

SOLUTION

W and $\mathbf{T}_{CD}$ intersect at *E*

Free-Body Diagram:

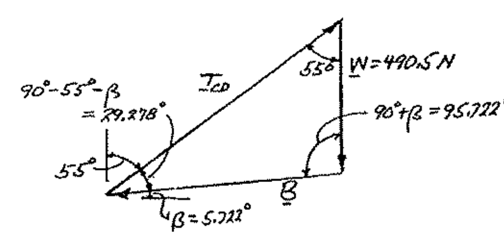
Three-Force body:



$$\tan \beta = \frac{AE}{AB} = \frac{0.301 \text{ m}}{3 \text{ m}}$$

$$\beta = 5.722°$$

Force Triangle (Three forces intersect at *E*.)



$$W = (50 \text{ kg})\, 9.81 \text{ m/s}^2$$
$$= 490.5 \text{ N}$$

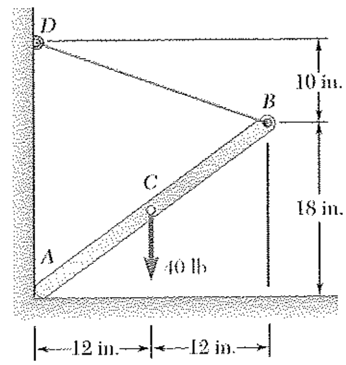
Law of sines

$$\frac{490.5 \text{ N}}{\sin 29.278°} = \frac{T_{CD}}{\sin 95.722°} = \frac{B}{\sin 55°}$$

$$T_{CD} = 997.99 \text{ N}$$
$$B = 821.59 \text{ N}$$

(*a*) $T_{CD} = 998$ N ◀

(*b*) $\mathbf{B} = 822$ N ↗ 5.72° ◀

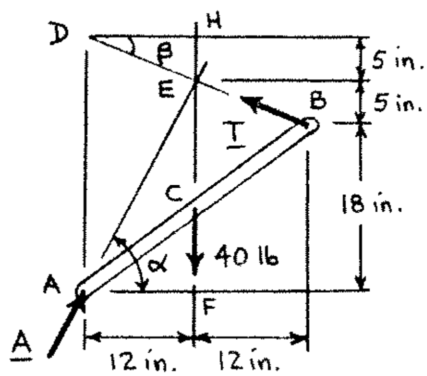


PROBLEM 4.71

One end of rod AB rests in the corner A and the other end is attached to cord BD. If the rod supports a 40-lb load at its midpoint C, find the reaction at A and the tension in the cord.

SOLUTION

Free-Body Diagram: (Three-Force body)



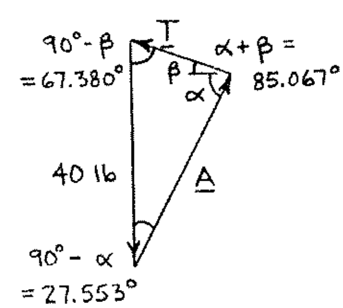
The line of action of reaction at A must pass through E, where **T** and the 40-lb load intersect.

$$\tan\alpha = \frac{EF}{AF} = \frac{23}{12}$$

$$\alpha = 62.447°$$

$$\tan\beta = \frac{EH}{DH} = \frac{5}{12}$$

$$\beta = 22.620°$$



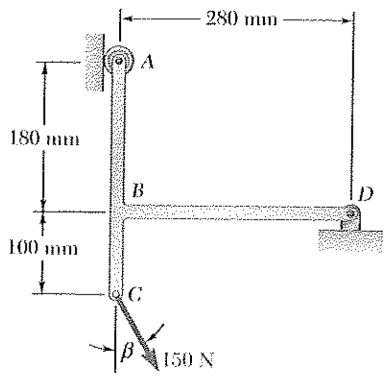
Force triangle

$$\frac{A}{\sin 67.380°} = \frac{T}{\sin 27.553°} = \frac{40 \text{ lb}}{\sin 85.067°}$$

$\mathbf{A} = 37.1 \text{ lb} \measuredangle 62.4°$ ◀

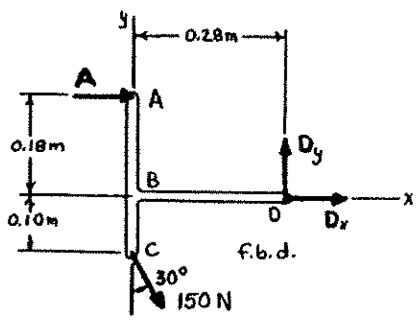
$T = 18.57 \text{ lb}$ ◀

PROBLEM 4.72



Determine the reactions at A and D when $\beta = 30°$.

SOLUTION



From f.b.d. of frame $ABCD$

$$+\circlearrowleft \Sigma M_D = 0: \quad -A(0.18 \text{ m}) + [(150 \text{ N})\sin 30°](0.10 \text{ m})$$
$$+ [(150 \text{ N})\cos 30°](0.28 \text{ m}) = 0$$

$$A = 243.74 \text{ N}$$

or $\mathbf{A} = 244 \text{ N} \longrightarrow$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad (243.74 \text{ N}) + (150 \text{ N})\sin 30° + D_x = 0$$

$$D_x = -318.74 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad D_y - (150 \text{ N})\cos 30° = 0$$

$$D_y = 129.904 \text{ N}$$

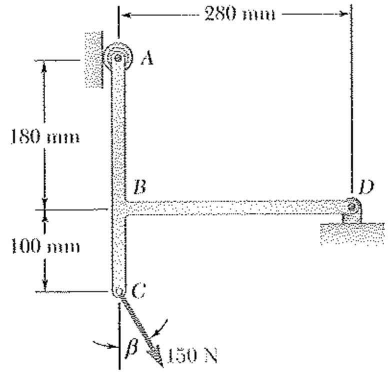
Then

$$D = \sqrt{(D_x)^2 + D_x^2}$$
$$= \sqrt{(318.74)^2 + (129.904)^2}$$
$$= 344.19 \text{ N}$$

and

$$\theta = \tan^{-1}\left(\frac{D_y}{D_x}\right)$$
$$= \tan^{-1}\left(\frac{129.904}{-318.74}\right)$$
$$= -22.174°$$

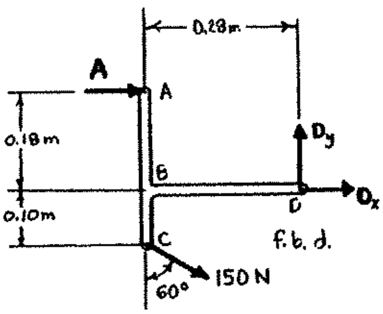
or $\mathbf{D} = 344 \text{ N} \searrow 22.2°$ ◀



PROBLEM 4.73

Determine the reactions at A and D when $\beta = 60°$.

SOLUTION



From f.b.d. of frame $ABCD$

$$+\circlearrowleft \Sigma M_D = 0: \quad -A(0.18 \text{ m}) + [(150 \text{ N})\sin 60°](0.10 \text{ m}) + [(150 \text{ N})\cos 60°](0.28 \text{ m}) = 0$$

$$A = 188.835 \text{ N}$$

or $\mathbf{A} = 188.8 \text{ N} \longrightarrow$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad (188.835 \text{ N}) + (150 \text{ N})\sin 60° + D_x = 0$$

$$D_x = -318.74 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad D_y - (150 \text{ N})\cos 60° = 0$$

$$D_y = 75.0 \text{ N}$$

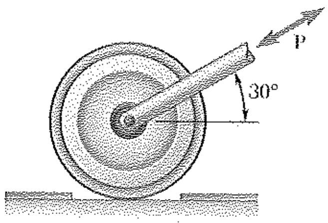
Then

$$D = \sqrt{(D_x)^2 + (D_y)^2} = \sqrt{(318.74)^2 + (75.0)^2} = 327.44 \text{ N}$$

and

$$\theta = \tan^{-1}\left(\frac{D_y}{D_x}\right) = \tan^{-1}\left(\frac{75.0}{-318.74}\right) = -13.2409°$$

or $\mathbf{D} = 327 \text{ N} \measuredangle 13.24°$ ◀



PROBLEM 4.74

A 40-lb roller, of diameter 8 in., which is to be used on a tile floor, is resting directly on the subflooring as shown. Knowing that the thickness of each tile is 0.3 in., determine the force **P** required to move the roller onto the tiles if the roller is (*a*) pushed to the left, (*b*) pulled to the right.

SOLUTION

See solution to Problem 4.73 for free-body diagram and analysis leading to the following equations:

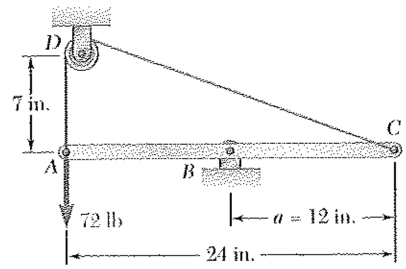
$$T = \frac{P}{1+\cos\theta} \quad (1)$$

$$\mathbf{C} = P\frac{\sin\theta}{1+\cos\theta} \quad (2)$$

For $\theta = 45°$

Eq. (1): $$T = \frac{P}{1+\cos 45°} = \frac{P}{1.7071}$$ $T = 0.586P$ ◀

Eq. (2): $$C = P\frac{\sin 45°}{1+\cos 45°} = P\frac{0.7071}{1.7071}$$ $\mathbf{C} = 0.444P \longrightarrow$ ◀

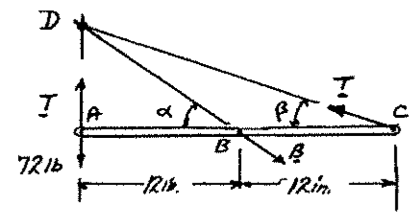


PROBLEM 4.75

Member ABC is supported by a pin and bracket at B and by an inextensible cord attached at A and C and passing over a frictionless pulley at D. The tension may be assumed to be the same in portions AD and CD of the cord. For the loading shown and neglecting the size of the pulley, determine the tension in the cord and the reaction at B.

SOLUTION

Free-Body Diagram:



Reaction at B must pass through D.

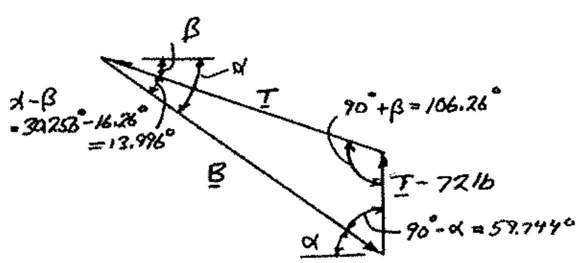
$$\tan\alpha = \frac{7\text{ in.}}{12\text{ in.}}$$

$$\alpha = 30.256°$$

$$\tan\beta = \frac{7\text{ in.}}{24\text{ in.}}$$

$$\beta = 16.26°$$

Force triangle



Law of sines

$$\frac{T}{\sin 59.744°} = \frac{T - 72\text{ lb}}{\sin 13.996°} = \frac{B}{\sin 106.26}$$

$$T(\sin 13.996°) = (T - 72\text{ lb})(\sin 59.744°)$$

$$T(0.24185) = (T - 72)(0.86378)$$

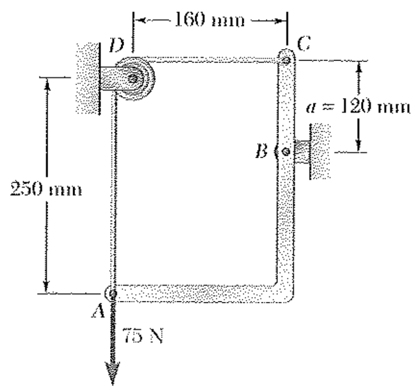
$$T = 100.00\text{ lb}$$

$T = 100.0$ lb ◀

$$B = (100\text{ lb})\frac{\sin 106.26°}{\sin 59.744}$$

$$= 111.14\text{ lb}$$

B $= 111.1$ lb ⦪ $30.3°$ ◀

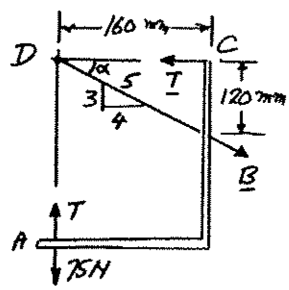


PROBLEM 4.76

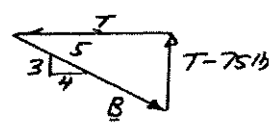
Member ABC is supported by a pin and bracket at B and by an inextensible cord attached at A and C and passing over a frictionless pulley at D. The tension may be assumed to be the same in portions AD and CD of the cord. For the loading shown and neglecting the size of the pulley, determine the tension in the cord and the reaction at B.

SOLUTION

Free-Body Diagram:



Force triangle



Reaction at B must pass through D.

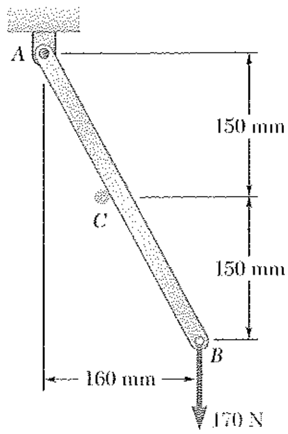
$$\tan\alpha = \frac{120}{160}; \quad \alpha = 36.9°$$

$$\frac{T}{4} = \frac{T - 75\text{ lb}}{3} = \frac{B}{5}$$

$$3T = 4T - 300; \quad T = 300\text{ lb}$$

$$B = \frac{5}{4}T = \frac{5}{4}(300\text{ lb}) = 375\text{ lb}$$

$\mathbf{B} = 375\text{ lb} \measuredangle 36.9°$

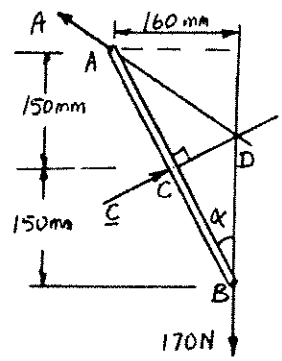


PROBLEM 4.77

Rod AB is supported by a pin and bracket at A and rests against a frictionless peg at C. Determine the reactions at A and C when a 170-N vertical force is applied at B.

SOLUTION

Free-Body Diagram:
(Three-Force body)



The reaction at A must pass through D where **C** and 170-N force intersect.

$$\tan\alpha = \frac{160\text{ mm}}{300\text{ mm}}$$

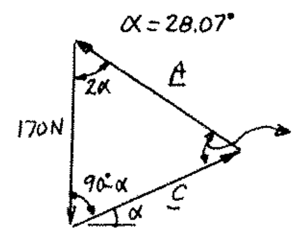
$$\alpha = 28.07^\circ$$

We note that triangle ABD is isosceles (since $AC = BC$) and, therefore

$$\sphericalangle CAD = \alpha = 28.07^\circ$$

Also, since $CD \perp CB$, reaction **C** forms angle $\alpha = 28.07^\circ$ with horizontal.

Force triangle



We note that **A** forms angle 2α with vertical. Thus **A** and **C** form angle

$$180^\circ - (90^\circ - \alpha) - 2\alpha = 90^\circ - \alpha$$

Force triangle is isosceles and we have

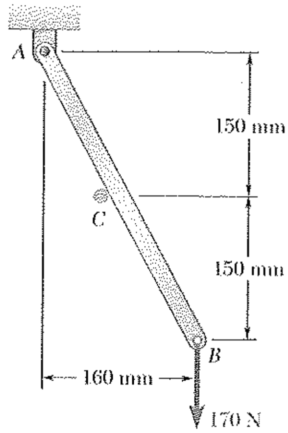
$$A = 170\text{ N}$$

$$C = 2(170\text{ N})\sin\alpha$$

$$= 160.0\text{ N}$$

$\mathbf{A} = 170.0\text{ N} \searrow 33.9^\circ \quad \mathbf{C} = 160.0\text{ N} \measuredangle\ 28.1^\circ$ ◀

PROBLEM 4.78

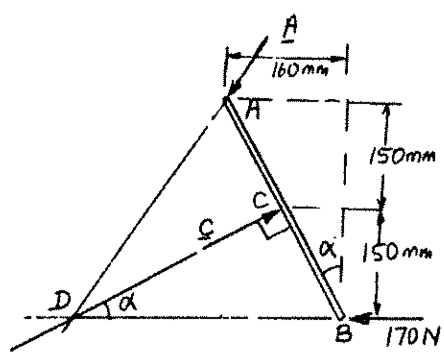


Solve Problem 4.77, assuming that the 170-N force applied at B is horizontal and directed to the left.

PROBLEM 4.77 Rod AB is supported by a pin and bracket at A and rests against a frictionless peg at C. Determine the reactions at A and C when a 170-N vertical force is applied at B.

SOLUTION

Free-Body Diagram: (Three-Force body)



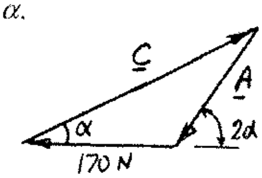
The reaction at A must pass through D, where **C** and the 170-N force intersect.

$$\tan\alpha = \frac{160 \text{ mm}}{300 \text{ mm}}$$

$$\alpha = 28.07°$$

We note that triangle ADB is isosceles (since $AC = BC$). Therefore $\sphericalangle A = \sphericalangle B = 90° - \alpha$.

Also $\sphericalangle ADB = 2\alpha$



Force triangle

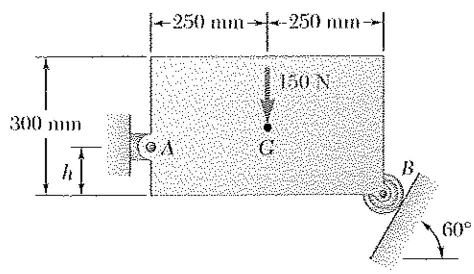
The angle between **A** and **C** must be $2\alpha - \alpha = \alpha$

Thus, force triangle is isosceles and $\alpha = 28.07°$

$$A = 170.0 \text{ N}$$

$$C = 2(170 \text{ N})\cos\alpha = 300 \text{ N}$$

$\mathbf{A} = 170.0 \text{ N} \measuredangle 56.1° \quad \mathbf{C} = 300 \text{ N} \measuredangle 28.1°$ ◀



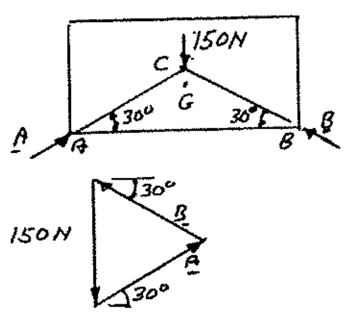
PROBLEM 4.79

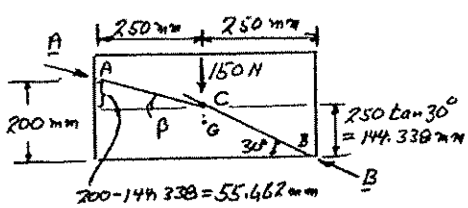
Using the method of Section 4.7, solve Problem 4.21.

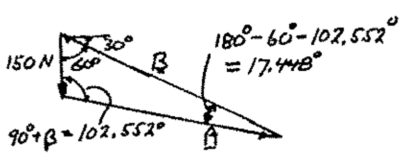
PROBLEM 4.21 Determine the reactions at A and B when (a) $h = 0$, (b) $h = 200$ mm.

SOLUTION

Free-Body Diagram:







(a) $\underline{h = 0}$

Reaction **A** must pass through C where 150-N weight and **B** interect.

Force triangle is equilateral

$\mathbf{A} = 150.0 \text{ N} \measuredangle 30.0°$ ◀

$\mathbf{B} = 150.0 \text{ N} \searrow 30.0°$ ◀

(b) $\underline{h = 200 \text{ mm}}$

$$\tan\beta = \frac{55.662}{250}$$

$$\beta = 12.552°$$

Force triangle

Law of sines

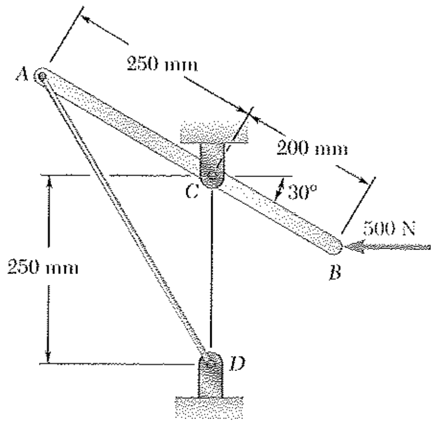
$$\frac{150 \text{ N}}{\sin 17.448°} = \frac{A}{\sin 60°} = \frac{B}{\sin 102.552°}$$

$$A = 433.247 \text{ N}$$

$$B = 488.31 \text{ N}$$

$\mathbf{A} = 433 \text{ N} \nwarrow 12.55°$ ◀

$\mathbf{B} = 488 \text{ N} \searrow 30.0°$ ◀



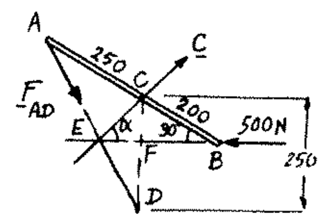
PROBLEM 4.80

Using the method of Section 4.7, solve Problem 4.28.

PROBLEM 4.28 A lever AB is hinged at C and attached to a control cable at A. If the lever is subjected to a 500-N horizontal force at B, determine (a) the tension in the cable, (b) the reaction at C.

SOLUTION

Free-Body Diagram:
(Three-Force body)



Dimensions in mm

Reaction at C must pass through E, where $\mathbf{F}_{AD}$ and 500-N force intersect.

Since $AC = CD = 250$ mm, triangle ACD is isosceles.

We have $$\measuredangle C = 90° + 30° = 120°$$

and $$\measuredangle A = \measuredangle D = \frac{1}{2}(180° - 120°) = 30°$$

On the other hand, from triangle BCF:

$$CF = (BC)\sin 30° = 200 \sin 30° = 100 \text{ mm}$$
$$FD = CD - CF = 250 - 100 = 150 \text{ mm}$$

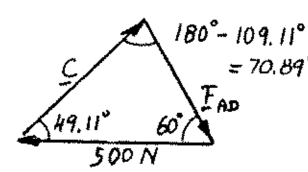
From triangle EFD, and since $\measuredangle D = 30°$:

$$EF = (FD)\tan 30° = 150 \tan 30° = 86.60 \text{ mm}$$

From triangle EFC: $$\tan\alpha = \frac{CF}{EF} = \frac{100 \text{ mm}}{86.60 \text{ mm}}$$

$$\alpha = 49.11°$$

Force triangle

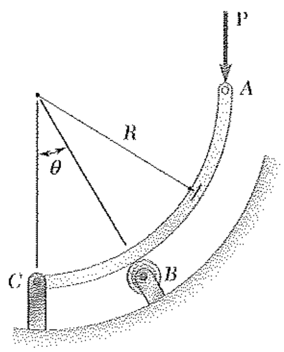


Law of sines $$\frac{F_{AD}}{\sin 49.11°} = \frac{C}{\sin 60°} = \frac{500 \text{ N}}{\sin 70.89°}$$

$$F_{AD} = 400 \text{ N}, \quad C = 458 \text{ N}$$

(a) $F_{AD} = 400$ N ◀

(b) $\mathbf{C} = 458$ N ∠ 49.1° ◀

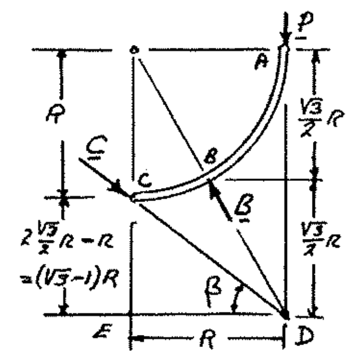


PROBLEM 4.81

Knowing that $\theta = 30°$, determine the reaction (*a*) at *B*, (*b*) at *C*.

SOLUTION

Free-Body Diagram:
(Three-Force body)

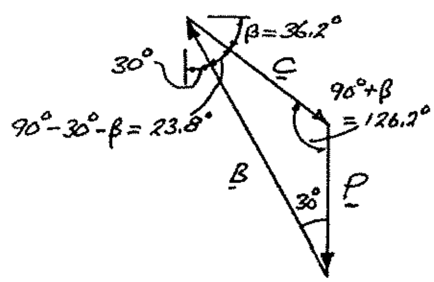


Reaction at *C* must pass through *D* where force **P** and reaction at *B* intersect.

In Δ *CDE*:

$$\tan\beta = \frac{(\sqrt{3}-1)R}{R}$$
$$= \sqrt{3}-1$$
$$\beta = 36.2°$$

Force triangle

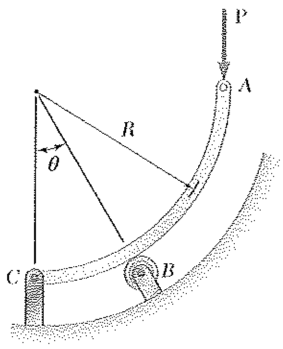


Law of sines

$$\frac{P}{\sin 23.8°} = \frac{B}{\sin 126.2°} = \frac{C}{\sin 30°}$$
$$B = 2.00P$$
$$C = 1.239P$$

(*a*) **B** $= 2P$ ⦪ 60.0° ◀

(*b*) **C** $= 1.239P$ ⦩ 36.2° ◀



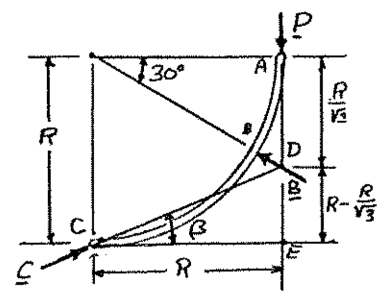
PROBLEM 4.82

Knowing that $\theta = 60°$, determine the reaction (*a*) at B, (*b*) at C.

SOLUTION

Reaction at C must pass through D where force **P** and reaction at B intersect.

Free-Body Diagram:
(Three-Force body)



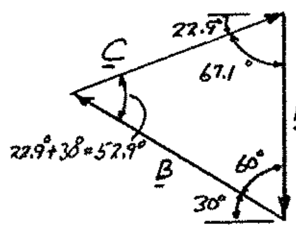
In ΔCDE:

$$\tan\beta = \frac{R - \frac{R}{\sqrt{3}}}{R}$$

$$= 1 - \frac{1}{\sqrt{3}}$$

$$\beta = 22.9°$$

Force triangle



Law of sines

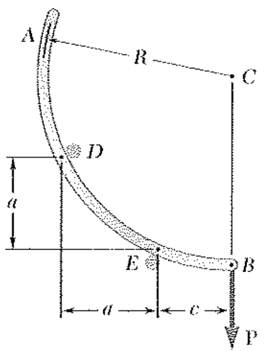
$$\frac{P}{\sin 52.9°} = \frac{B}{\sin 67.1°} = \frac{C}{\sin 60°}$$

$$B = 1.155P$$

$$C = 1.086P$$

(*a*) **B** $= 1.155P$ ⦬ $30.0°$ ◀

(*b*) **C** $= 1.086P$ ∠ $22.9°$ ◀

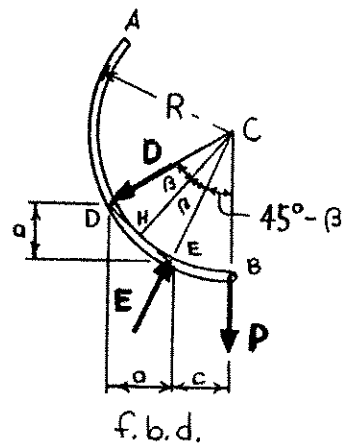


PROBLEM 4.83

Rod AB is bent into the shape of an arc of circle and is lodged between two pegs D and E. It supports a load **P** at end B. Neglecting friction and the weight of the rod, determine the distance c corresponding to equilibrium when $a = 20$ mm and $R = 100$ mm.

SOLUTION

Free-Body Diagram:



Since $y_{ED} = x_{ED} = a,$

Slope of ED is ⦣ 45°

slope of HC is ∡ 45°

Also $DE = \sqrt{2}a$

and $DH = HE = \left(\frac{1}{2}\right)DE = \frac{a}{\sqrt{2}}$

For triangles DHC and EHC $\sin\beta = \frac{\frac{a}{\sqrt{2}}}{R} = \frac{a}{\sqrt{2}R}$

Now $c = R\sin(45° - \beta)$

For $a = 20 \text{ mm} \quad \text{and} \quad R = 100 \text{ mm}$

$$\sin\beta = \frac{20 \text{ mm}}{\sqrt{2}(100 \text{ mm})}$$

$$= 0.141421$$

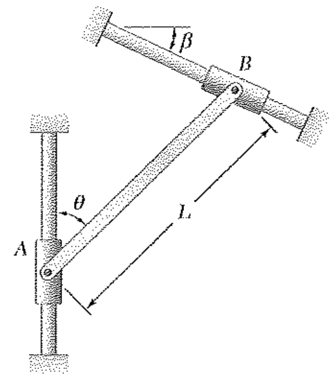
$$\beta = 8.1301°$$

and $c = (100 \text{ mm})\sin(45° - 8.1301°)$

$= 60.00$ mm

or $c = 60.0$ mm ◀

PROBLEM 4.84

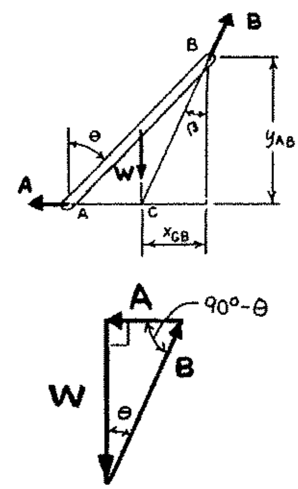


A slender rod of length L is attached to collars that can slide freely along the guides shown. Knowing that the rod is in equilibrium, derive an expression for the angle θ in terms of the angle β.

SOLUTION

As shown in the free-body diagram of the slender rod AB, the three forces intersect at C. From the force geometry

Free-Body Diagram:



$$\tan\beta = \frac{x_{GB}}{y_{AB}}$$

where

$$y_{AB} = L\cos\theta$$

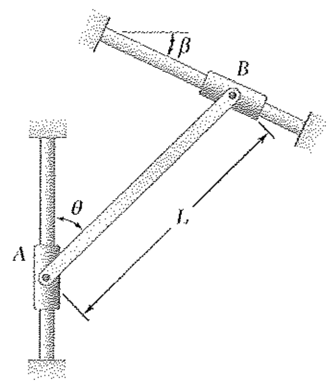
and

$$x_{GB} = \frac{1}{2}L\sin\theta$$

$$\tan\beta = \frac{\frac{1}{2}L\sin\theta}{L\cos\theta}$$

$$= \frac{1}{2}\tan\theta$$

or $\tan\theta = 2\tan\beta$ ◀



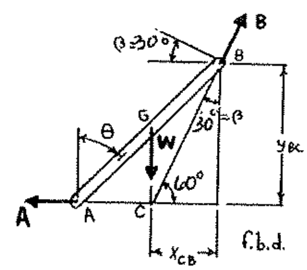
PROBLEM 4.85

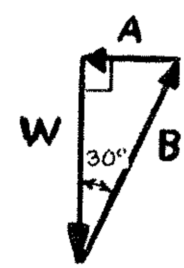
An 8-kg slender rod of length L is attached to collars that can slide freely along the guides shown. Knowing that the rod is in equilibrium and that $\beta = 30°$, determine (*a*) the angle θ that the rod forms with the vertical, (*b*) the reactions at A and B.

SOLUTION

(*a*) As shown in the free-body diagram of the slender rod AB, the three forces intersect at C. From the geometry of the forces

Free-Body Diagram:





$$\tan\beta = \frac{x_{CB}}{y_{BC}}$$

where

$$x_{CB} = \frac{1}{2}L\sin\theta$$

and

$$y_{BC} = L\cos\theta$$

$$\tan\beta = \frac{1}{2}\tan\theta$$

or $\tan\theta = 2\tan\beta$

For $\beta = 30°$

$$\tan\theta = 2\tan 30° = 1.15470$$

$$\theta = 49.107°$$

or $\theta = 49.1°$ ◀

(*b*) $W = mg = (8 \text{ kg})(9.81 \text{ m/s}^2) = 78.480 \text{ N}$

From force triangle

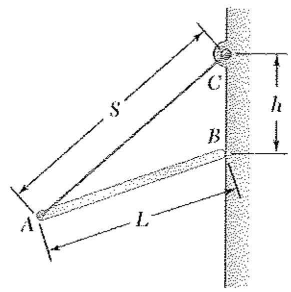
$$A = W\tan\beta = (78.480 \text{ N})\tan 30° = 45.310 \text{ N}$$

or $\mathbf{A} = 45.3 \text{ N} \longleftarrow$ ◀

and

$$B = \frac{W}{\cos\beta} = \frac{78.480 \text{ N}}{\cos 30°} = 90.621 \text{ N}$$

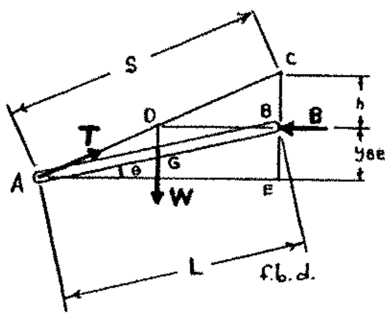
or $\mathbf{B} = 90.6 \text{ N} \measuredangle\ 60.0°$ ◀



PROBLEM 4.86

A slender uniform rod of length L is held in equilibrium as shown, with one end against a frictionless wall and the other end attached to a cord of length S. Derive an expression for the distance h in terms of L and S. Show that this position of equilibrium does not exist if $S > 2L$.

SOLUTION



From the f.b.d. of the three-force member AB, forces must intersect at D. Since the force T intersects Point D, directly above G,

$$y_{BE} = h$$

For triangle ACE: $$S^2 = (AE)^2 + (2h)^2 \qquad (1)$$

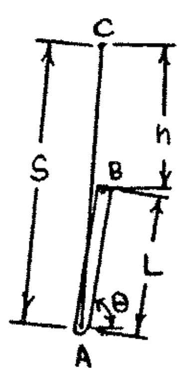
For triangle ABE: $$L^2 = (AE)^2 + (h)^2 \qquad (2)$$

Subtracting Equation (2) from Equation (1)

$$S^2 - L^2 = 3h^2 \qquad (3)$$

or $$h = \sqrt{\frac{S^2 - L^2}{3}} \blacktriangleleft$$

As length S increases relative to length L, angle θ increases until rod AB is vertical. At this vertical position:



$$h + L = S \quad \text{or} \quad h = S - L$$

Therefore, for all positions of AB

$$h \geq S - L \qquad (4)$$

or $$\sqrt{\frac{S^2 - L^2}{3}} \geq S - L$$

or $$S^2 - L^2 \geq 3(S - L)^2$$
$$= 3(S^2 - 2SL + L^2)$$
$$= 3S^2 - 6SL + 3L^2$$

or $$0 \geq 2S^2 - 6SL + 4L^2$$

and $$0 \geq S^2 - 3SL + 2L^2 = (S - L)(S - 2L)$$

PROBLEM 4.86 (Continued)

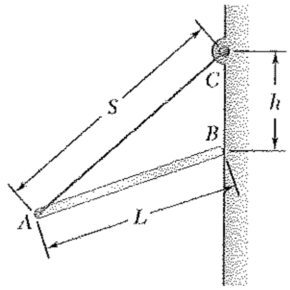
For $S - L = 0 \quad S = L$

Minimum value of S is L

For $S - 2L = 0 \quad S = 2L$

Maximum value of S is $2L$

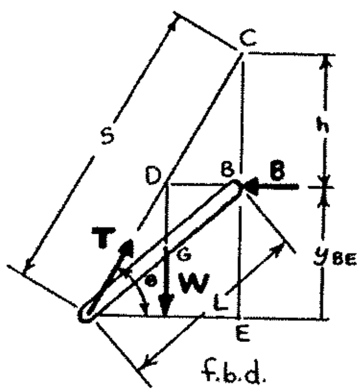
Therefore, equilibrium does not exist if $S > 2L$ ◀



PROBLEM 4.87

A slender uniform rod of length $L = 20$ in. is held in equilibrium as shown, with one end against a frictionless wall and the other end attached to a cord of length $S = 30$ in. Knowing that the weight of the rod is 10 lb, determine (*a*) the distance h, (*b*) the tension in the cord, (*c*) the reaction at B.

SOLUTION



From the f.b.d. of the three-force member AB, forces must intersect at D. Since the force T intersects Point D, directly above G,

$$y_{BE} = h$$

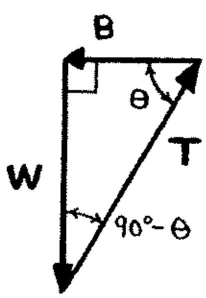
For triangle ACE: $$S^2 = (AE)^2 + (2h)^2 \quad (1)$$

For triangle ABE: $$L^2 = (AE)^2 + (h)^2 \quad (2)$$

Subtracting Equation (2) from Equation (1)

$$S^2 - L^2 = 3h^2$$

or $$h = \sqrt{\frac{S^2 - L^2}{3}}$$



(*a*) For $L = 20$ in. and $S = 30$ in.

$$h = \sqrt{\frac{(30)^2 - (20)^2}{3}}$$

$$= 12.9099 \text{ in.} \qquad \text{or } h = 12.91 \text{ in.} ◀$$

(*b*) We have $W = 10$ lb

and $$\theta = \sin^{-1}\left(\frac{2h}{s}\right)$$

$$= \sin^{-1}\left[\frac{2(12.9099)}{30}\right]$$

$$\theta = 59.391°$$

PROBLEM 4.87 (Continued)

From the force triangle

$$T = \frac{W}{\sin\theta}$$

$$= \frac{10 \text{ lb}}{\sin 59.391°}$$

$$= 11.6190 \text{ lb}$$

or $T = 11.62$ lb ◀

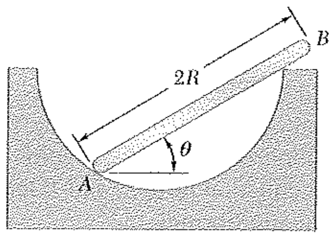
(c)

$$B = \frac{W}{\tan\theta}$$

$$= \frac{10 \text{ lb}}{\tan 59.391°}$$

$$= 5.9161 \text{ lb}$$

or $\mathbf{B} = 5.92$ lb ⟵ ◀



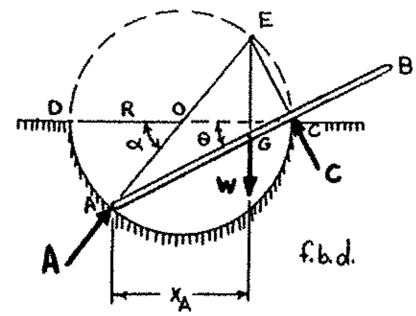
PROBLEM 4.88

A uniform rod AB of length $2R$ rests inside a hemispherical bowl of radius R as shown. Neglecting friction, determine the angle θ corresponding to equilibrium.

SOLUTION

Based on the f.b.d., the uniform rod AB is a three-force body. Point E is the point of intersection of the three forces. Since force A passes through O, the center of the circle, and since force **C** is perpendicular to the rod, triangle ACE is a right triangle inscribed in the circle. Thus, E is a point on the circle.

Note that the angle α of triangle DOA is the central angle corresponding to the inscribed angle θ of triangle DCA.



$$\alpha = 2\theta$$

The horizontal projections of $AE, (x_{AE})$, and $AG, (x_{AG})$, are equal.

$$x_{AE} = x_{AG} = x_A$$

or $$(AE)\cos 2\theta = (AG)\cos\theta$$

and $$(2R)\cos 2\theta = R\cos\theta$$

Now $$\cos 2\theta = 2\cos^2\theta - 1$$

then $$4\cos^2\theta - 2 = \cos\theta$$

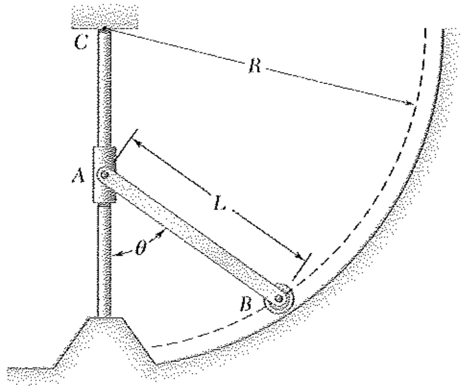
or $$4\cos^2\theta - \cos\theta - 2 = 0$$

Applying the quadratic equation

$$\cos\theta = 0.84307 \quad \text{and} \quad \cos\theta = -0.59307$$

$$\theta = 32.534° \quad \text{and} \quad \theta = 126.375°(\text{Discard})$$

or $\theta = 32.5°$ ◀

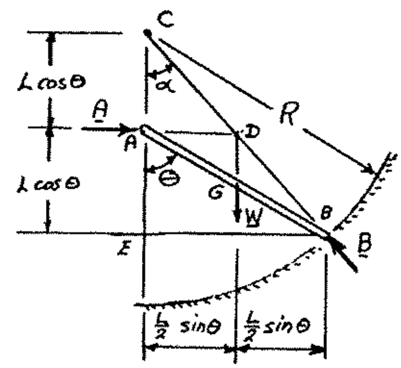


PROBLEM 4.89

A slender rod of length L and weight W is attached to a collar at A and is fitted with a small wheel at B. Knowing that the wheel rolls freely along a cylindrical surface of radius R, and neglecting friction, derive an equation in θ, L, and R that must be satisfied when the rod is in equilibrium.

SOLUTION

Free-Body Diagram (Three-Force body)



Reaction **B** must pass through D where **B** and **W** intersect.

Note that ΔABC and ΔBGD are similar.

$$AC = AE = L\cos\theta$$

In ΔABC:

$$(CE)^2 + (BE)^2 = (BC)^2$$

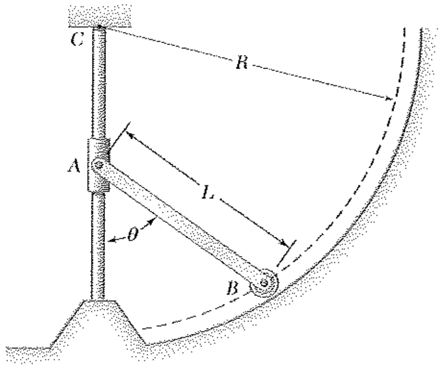
$$(2L\cos\theta)^2 + (L\sin\theta)^2 = R^2$$

$$\left(\frac{R}{L}\right)^2 = 4\cos^2\theta + \sin^2\theta$$

$$\left(\frac{R}{L}\right)^2 = 4\cos^2\theta + 1 - \cos^2\theta$$

$$\left(\frac{R}{L}\right)^2 = 3\cos^2\theta + 1$$

$$\cos^2\theta = \frac{1}{3}\left[\left(\frac{R}{L}\right)^2 - 1\right] \blacktriangleleft$$



PROBLEM 4.90

Knowing that for the rod of Problem 4.89, $L = 15$ in., $R = 20$ in., and $W = 10$ lb, determine (*a*) the angle θ corresponding to equilibrium, (*b*) the reactions at A and B.

SOLUTION

See the solution to Problem 4.89 for free-body diagram and analysis leading to the following equation

$$\cos^2\theta = \frac{1}{3}\left[\left(\frac{R}{L}\right)^2 - 1\right]$$

For $L = 15$ in., $R = 20$ in., and $W = 10$ lb.

(*a*)

$$\cos^2\theta = \frac{1}{3}\left[\left(\frac{20\text{ in.}}{15\text{ in.}}\right)^2 - 1\right];\quad \theta = 59.39°$$

$\theta = 59.4°$ ◀

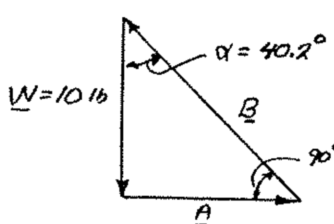
In ΔABC:

$$\tan\alpha = \frac{BE}{CE} = \frac{L\sin\theta}{2L\cos\theta} = \frac{1}{2}\tan\theta$$

$$\tan\alpha = \frac{1}{2}\tan 59.39° = 0.8452$$

$$\alpha = 40.2°$$

Force triangle

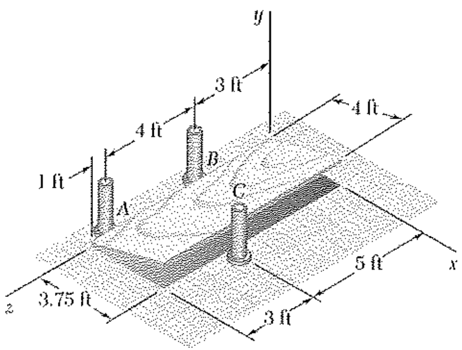


$$A = W\tan\alpha = (10\text{ lb})\tan 40.2° = 8.45\text{ lb}$$

$$B = \frac{W}{\cos\alpha} = \frac{(10\text{ lb})}{\cos 40.2°} = 13.09\text{ lb}$$

(*b*) $\mathbf{A} = 8.45$ lb ⟶ ◀

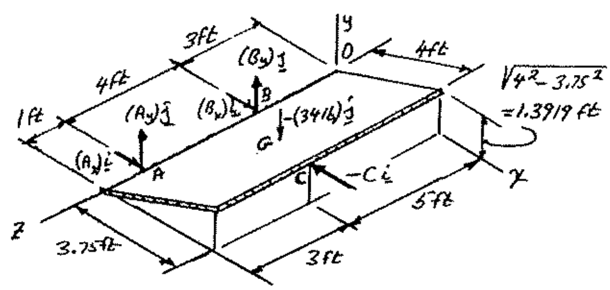
(*c*) $\mathbf{B} = 13.09$ lb ⦪ 49.8° ◀



PROBLEM 4.91

A 4×8-ft sheet of plywood weighing 34 lb has been temporarily placed among three pipe supports. The lower edge of the sheet rests on small collars at A and B and its upper edge leans against pipe C. Neglecting friction at all surfaces, determine the reactions at A, B, and C.

SOLUTION



$$\mathbf{r}_{G/B} = \frac{3.75}{2}\mathbf{i} + \frac{1.3919}{2}\mathbf{j} + \mathbf{k}$$

We have 5 unknowns and 6 Eqs. of equilibrium.

Plywood sheet is free to move in z direction, but equilibrium is maintained $(\Sigma F_z = 0)$.

$$\Sigma M_B = 0: \quad r_{A/B} \times (A_x\mathbf{i} + A_y\mathbf{j}) + r_{C/B} \times (-C\mathbf{i}) + r_{G/B} \times (-w\mathbf{j}) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 4 \\ A_x & A_y & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3.75 & 1.3919 & 2 \\ -C & 0 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.875 & 0.696 & 1 \\ 0 & -34 & 0 \end{vmatrix} = 0$$

$$-4A_y\mathbf{i} + 4A_x\mathbf{j} - 2C\mathbf{j} + 1.3919C\mathbf{k} + 34\mathbf{i} - 63.75\mathbf{k} = 0$$

Equating coefficients of unit vectors to zero:

$\mathbf{i}$: $\quad -4A_y + 34 = 0 \qquad A_y = 8.5 \text{ lb}$

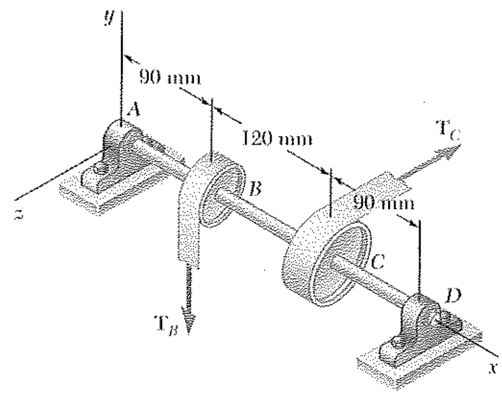
$\mathbf{j}$: $\quad -2C + 4A_x = 0 \qquad A_x = \frac{1}{2}C = \frac{1}{2}(45.80) = 22.9 \text{ lb}$

$\mathbf{k}$: $\quad 1.3919C - 63.75 = 0 \qquad C = 45.80 \text{ lb} \qquad C = 45.8 \text{ lb}$

$\Sigma F_x = 0$: $\quad A_x + B_x - C = 0$: $\qquad B_x = 45.8 - 22.9 = 22.9 \text{ lb}$

$\Sigma F_y = 0$: $\quad A_y + B_y - W = 0$: $\qquad B_y = 34 - 8.5 = 25.5 \text{ lb}$

$$\mathbf{A} = (22.9 \text{ lb})\mathbf{i} + (8.5 \text{ lb})\mathbf{j} \quad \mathbf{B} = (22.9 \text{ lb})\mathbf{i} + (25.5 \text{ lb})\mathbf{j} \quad \mathbf{C} = -(45.8 \text{ lb})\mathbf{i} \blacktriangleleft$$

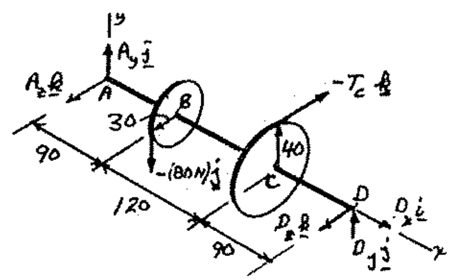


PROBLEM 4.92

Two tape spools are attached to an axle supported by bearings at A and D. The radius of spool B is 30 mm and the radius of spool C is 40 mm. Knowing that $T_B = 80$ N and that the system rotates at a constant rate, determine the reactions at A and D. Assume that the bearing at A does not exert any axial thrust and neglect the weights of the spools and axle.

SOLUTION

Dimensions in mm



We have six unknowns and six Eqs. of equilibrium.

$$\Sigma M_A = 0: \quad (90\mathbf{i} + 30\mathbf{k}) \times (-80\mathbf{j}) + (210\mathbf{i} + 40\mathbf{j}) \times (-T_C\mathbf{k}) + (300\mathbf{i}) \times (D_x\mathbf{i} + D_y\mathbf{j} + D_z\mathbf{k}) = 0$$

$$-7200\mathbf{k} + 2400\mathbf{i} + 210T_C\mathbf{j} - 40T_C\mathbf{i} + 300D_y\mathbf{k} - 300D_z\mathbf{j} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: \quad 2400 - 40T_C = 0 \qquad T_C = 60 \text{ N}$$

$$\mathbf{j}: \quad 210T_C - 300D_z = 0 \quad (210)(60) - 300D_z = 0 \qquad D_z = 42 \text{ N}$$

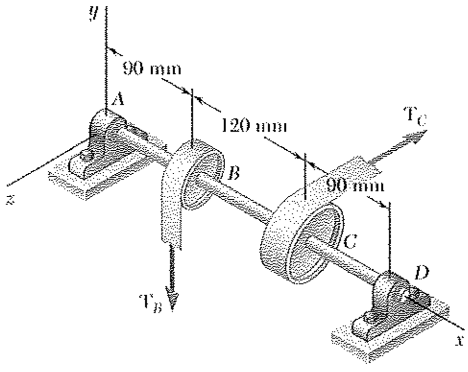
$$\mathbf{k}: \quad -7200 + 300D_y = 0 \qquad D_y = 24 \text{ N}$$

$$\Sigma F_x = 0: \quad D_x = 0$$

$$\Sigma F_y = 0: \quad A_y + D_y - 80 \text{ N} = 0 \qquad A_y = 80 - 24 = 56 \text{ N}$$

$$\Sigma F_z = 0: \quad A_z + D_z - 60 \text{ N} = 0 \qquad A_z = 60 - 42 = 18 \text{ N}$$

$$\mathbf{A} = (56.0 \text{ N})\mathbf{j} + (18.00 \text{ N})\mathbf{k} \quad \mathbf{D} = (24.0 \text{ N})\mathbf{j} + (42.0 \text{ N})\mathbf{k} \blacktriangleleft$$



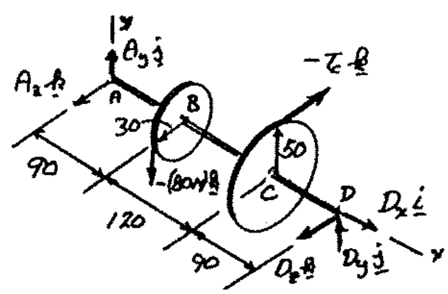
PROBLEM 4.93

Solve Problem 4.92, assuming that the spool C is replaced by a spool of radius 50 mm.

PROBLEM 4.92 Two tape spools are attached to an axle supported by bearings at A and D. The radius of spool B is 30 mm and the radius of spool C is 40 mm. Knowing that $T_B = 80$ N and that the system rotates at a constant rate, determine the reactions at A and D. Assume that the bearing at A does not exert any axial thrust and neglect the weights of the spools and axle.

SOLUTION

Dimensions in mm



We have six unknowns and six Eqs. of equilibrium.

$$\Sigma M_A = 0: \quad (90\mathbf{i} + 30\mathbf{k}) \times (-80\mathbf{j}) + (210\mathbf{i} + 50\mathbf{j}) \times (-T_C\mathbf{k}) + (300\mathbf{i}) \times (D_x\mathbf{i} + D_y\mathbf{j} + D_z\mathbf{k}) = 0$$

$$-7200\mathbf{k} + 2400\mathbf{i} + 210T_C\mathbf{j} - 50T_C\mathbf{i} + 300D_y\mathbf{k} - 300D_z\mathbf{j} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: \quad 2400 - 50T_C = 0 \qquad T_C = 48\text{ N}$$

$$\mathbf{j}: \quad 210T_C - 300D_z = 0 \quad (210)(48) - 300D_z = 0 \qquad D_z = 33.6\text{ N}$$

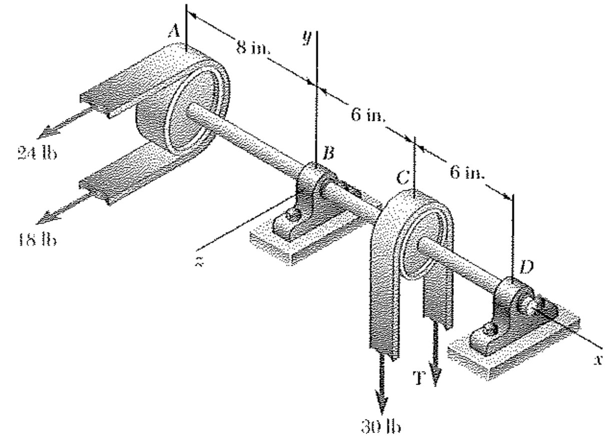
$$\mathbf{k}: \quad -7200 + 300D_y = 0 \qquad D_y = 24\text{ N}$$

$$\Sigma F_x = 0: \quad D_x = 0$$

$$\Sigma F_y = 0: \quad A_y + D_y - 80\text{ N} = 0 \qquad A_y = 80 - 24 = 56\text{ N}$$

$$\Sigma F_z = 0: \quad A_z + D_z - 48 = 0 \qquad A_z = 48 - 33.6 = 14.4\text{ N}$$

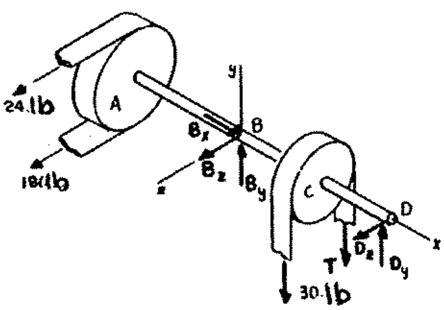
$$\mathbf{A} = (56.0\text{ N})\mathbf{j} + (14.40\text{ N})\mathbf{k} \quad \mathbf{D} = (24.0\text{ N})\mathbf{j} + (33.6\text{ N})\mathbf{k} \blacktriangleleft$$



PROBLEM 4.94

Two transmission belts pass over sheaves welded to an axle supported by bearings at B and D. The sheave at A has a radius of 2.5 in., and the sheave at C has a radius of 2 in. Knowing that the system rotates at a constant rate, determine (*a*) the tension T, (*b*) the reactions at B and D. Assume that the bearing at D does not exert any axial thrust and neglect the weights of the sheaves and axle.

SOLUTION



Assume moment reactions at the bearing supports are zero. From f.b.d. of shaft

(*a*) $\Sigma M_{x\text{-axis}} = 0$: $(24 \text{ lb} - 18 \text{ lb})(5 \text{ in.}) + (30 \text{ lb} - T)(4 \text{ in.}) = 0$ $\qquad T = 37.5 \text{ lb}$ ◀

(*b*) $\Sigma F_x = 0$: $B_x = 0$

$\Sigma M_{D(z\text{-axis})} = 0$: $(30 \text{ lb} + 37.5 \text{ lb})(6 \text{ in.}) - B_y(12 \text{ in.}) = 0$

$$B_y = 33.75 \text{ lb}$$

$\Sigma M_{D(y\text{-axis})} = 0$: $(24 \text{ lb} + 18 \text{ lb})(20 \text{ in.}) + B_z(12 \text{ in.}) = 0$

$$B_z = -70.0 \text{ lb}$$

or $\mathbf{B} = (33.8 \text{ lb})\mathbf{j} - (70.0 \text{ lb})\mathbf{k}$ ◀

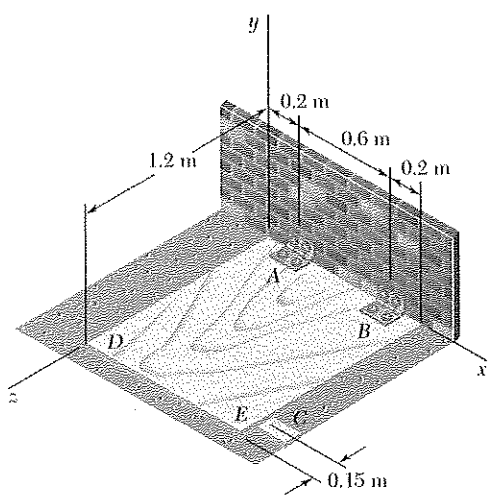
$\Sigma M_{B(z\text{-axis})} = 0$: $-(30 \text{ lb} + 37.5 \text{ lb})(6 \text{ in.}) + D_y(12 \text{ in.}) = 0$

$$D_y = 33.75 \text{ lb}$$

$\Sigma M_{B(y\text{-axis})} = 0$: $(24 \text{ lb} + 18 \text{ lb})(8 \text{ in.}) + D_z(12 \text{ in.}) = 0$

$$D_z = -28.0 \text{ lb}$$

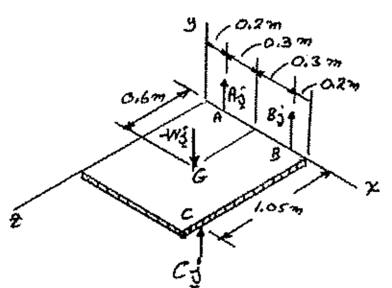
or $\mathbf{D} = (33.8 \text{ lb})\mathbf{j} - (28.0 \text{ lb})\mathbf{k}$ ◀



PROBLEM 4.97

An opening in a floor is covered by a 1×1.2-m sheet of plywood of mass 18 kg. The sheet is hinged at *A* and *B* and is maintained in a position slightly above the floor by a small block *C*. Determine the vertical component of the reaction (*a*) at *A*, (*b*) at *B*, (*c*) at *C*.

SOLUTION



$$\mathbf{r}_{B/A} = 0.6\mathbf{i}$$
$$\mathbf{r}_{C/A} = 0.8\mathbf{i} + 1.05\mathbf{k}$$
$$\mathbf{r}_{G/A} = 0.3\mathbf{i} + 0.6\mathbf{k}$$

$$W = mg = (18 \text{ kg})9.81$$
$$W = 176.58 \text{ N}$$

$$\Sigma M_A = 0: \quad \mathbf{r}_{B/A} \times B\mathbf{j} + \mathbf{r}_{C/A} \times C\mathbf{j} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) = 0$$

$$(0.6\mathbf{i}) \times B\mathbf{j} + (0.8\mathbf{i} + 1.05\mathbf{k}) \times C\mathbf{j} + (0.3\mathbf{i} + 0.6\mathbf{k}) \times (-W\mathbf{j}) = 0$$

$$0.6B\mathbf{k} + 0.8C\mathbf{k} - 1.05C\mathbf{i} - 0.3W\mathbf{k} + 0.6W\mathbf{i} = 0$$

Equate coefficients of unit vectors of zero:

$$\mathbf{i}: \quad 1.05C + 0.6W = 0 \quad C = \left(\frac{0.6}{1.05}\right) 176.58 \text{ N} = 100.90 \text{ N}$$

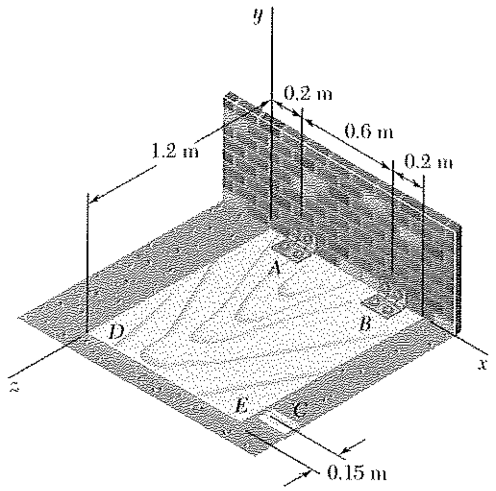
$$\mathbf{k}: \quad 0.6B + 0.8C - 0.3W = 0$$

$$0.6B + 0.8(100.90 \text{ N}) - 0.3(176.58 \text{ N}) = 0 \quad B = -46.24 \text{ N}$$

$$\Sigma F_y = 0: \quad A + B + C - W = 0$$

$$A - 46.24 \text{ N} + 100.90 \text{ N} + 176.58 \text{ N} = 0 \quad A = 121.92 \text{ N}$$

(*a*) $A = 121.9$ N (*b*) $B = -46.2$ N (*c*) $C = 100.9$ N ◀

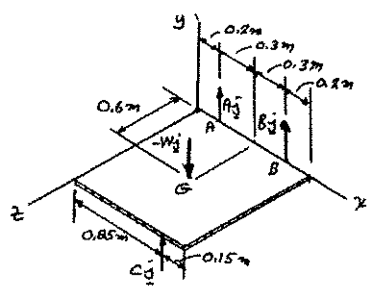


PROBLEM 4.98

Solve Problem 4.97, assuming that the small block C is moved and placed under edge DE at a point 0.15 m from corner E.

PROBLEM 4.97 An opening in a floor is covered by a 1×1.2-m sheet of plywood of mass 18 kg. The sheet is hinged at A and B and is maintained in a position slightly above the floor by a small block C. Determine the vertical component of the reaction (*a*) at A, (*b*) at B, (*c*) at C.

SOLUTION



$$\mathbf{r}_{B/A} = 0.6\mathbf{i}$$
$$\mathbf{r}_{C/A} = 0.65\mathbf{i} + 1.2\mathbf{k}$$
$$\mathbf{r}_{G/A} = 0.3\mathbf{i} + 0.6\mathbf{k}$$

$$W = mg = (18 \text{ kg})\, 9.81 \text{ m/s}^2$$
$$W = 176.58 \text{ N}$$

$$\Sigma M_A = 0: \quad \mathbf{r}_{B/A} \times B\mathbf{j} + \mathbf{r}_{C/A} \times C\mathbf{j} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) = 0$$

$$0.6\mathbf{i} \times B\mathbf{j} + (0.65\mathbf{i} + 1.2\mathbf{k}) \times C\mathbf{j} + (0.3\mathbf{i} + 0.6\mathbf{k}) \times (-W\mathbf{j}) = 0$$

$$0.6B\mathbf{k} + 0.65C\mathbf{k} - 1.2C\mathbf{i} - 0.3W\mathbf{k} + 0.6W\mathbf{i} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: \quad -1.2C + 0.6W = 0 \quad C = \left(\frac{0.6}{1.2}\right) 176.58 \text{ N} = 88.29 \text{ N}$$

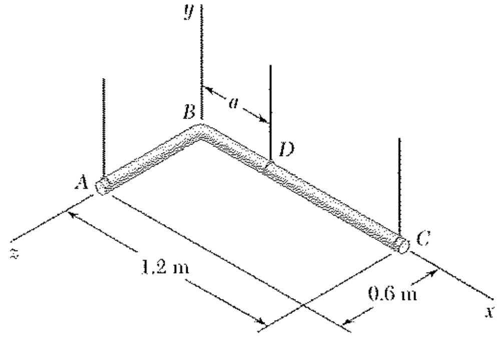
$$\mathbf{k}: \quad 0.6B + 0.65C - 0.3W = 0$$

$$0.6B + 0.65(88.29 \text{ N}) - 0.3(176.58 \text{ N}) = 0 \quad B = -7.36 \text{ N}$$

$$\Sigma F_y = 0: \quad A + B + C - W = 0$$

$$A - 7.36 \text{ N} + 88.29 \text{ N} - 176.58 \text{ N} = 0 \qquad A = 95.648 \text{ N}$$

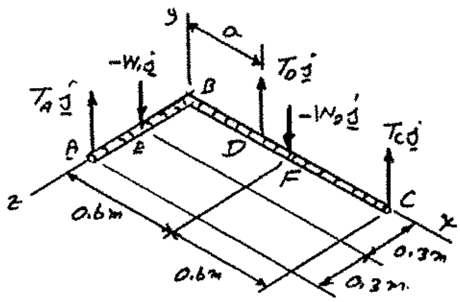
(*a*) $A = 95.6$ N (*b*) -7.36 N (*c*) 88.3 N ◀



PROBLEM 4.102

For the pipe assembly of Problem 4.101, determine (*a*) the largest permissible value of *a* if the assembly is not to tip, (*b*) the corresponding tension in each wire.

SOLUTION



$$W_1 = 0.6m'g$$
$$W_2 = 1.2m'g$$

$$\Sigma M_D = 0: \quad \mathbf{r}_{A/D} \times T_A\mathbf{j} + \mathbf{r}_{E/D} \times (-W_1\mathbf{j}) + \mathbf{r}_{F/D} \times (-W_2\mathbf{j}) + \mathbf{r}_{C/D} \times T_C\mathbf{j} = 0$$

$$(-a\mathbf{i} + 0.6\mathbf{k}) \times T_A\mathbf{j} + (-a\mathbf{i} + 0.3\mathbf{k}) \times (-W_1\mathbf{j}) + (0.6 - a)\mathbf{i} \times (-W_2\mathbf{j}) + (1.2 - a)\mathbf{i} \times T_C\mathbf{j} = 0$$

$$-T_A a\mathbf{k} - 0.6T_A\mathbf{i} + W_1 a\mathbf{k} + 0.3W_1\mathbf{i} - W_2(0.6 - a)\mathbf{k} + T_C(1.2 - a)\mathbf{k} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: \quad -0.6T_A + 0.3W_1 = 0; \quad T_A = \frac{1}{2}W_1 = \frac{1}{2}0.6m'g = 0.3m'g$$

$$\mathbf{k}: \quad -T_A a + W_1 a - W_2(0.6 - a) + T_C(1.2 - a) = 0$$

$$-0.3m'ga + 0.6m'ga - 1.2m'g(0.6 - a) + T_C(1.2 - a) = 0$$

$$T_C = \frac{0.3a - 0.6a + 1.2(0.6 - a)}{1.2 - a} \qquad \text{For Max } a \text{ and no tipping, } T_C = 0$$

(*a*)

$$-0.3a + 1.2(0.6 - a) = 0$$
$$-0.3a + 0.72 - 1.2a = 0$$
$$1.5a = 0.72$$

$a = 0.480$ m ◀

PROBLEM 4.102 (Continued)

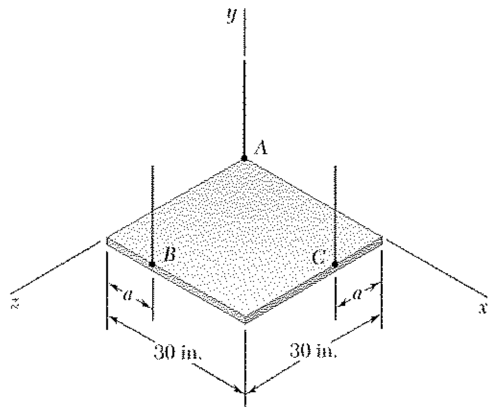
(*b*) Reactions: $m'g = (8 \text{ kg/m})\, 9.81 \text{ m/s}^2 = 78.48 \text{ N/m}$

$$T_A = 0.3m'g = 0.3 \times 78.48 = 23.544 \text{ N} \qquad T_A = 23.5 \text{ N} \blacktriangleleft$$

$$\Sigma F_y = 0: \quad T_A + T_C + T_D - W_1 - W_2 = 0$$

$$T_A + 0 + T_D - 0.6m'g - 1.2m'g = 0$$

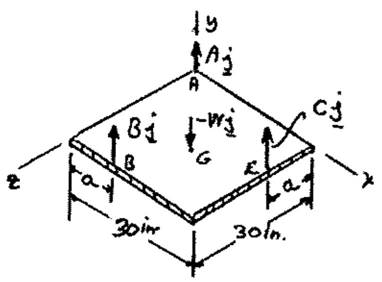
$$T_D = 1.8m'g - T_A = 1.8 \times 78.48 - 23.544 = 117.72 \qquad T_D = 117.7 \text{ N} \blacktriangleleft$$



PROBLEM 4.103

The 24-lb square plate shown is supported by three vertical wires. Determine (*a*) the tension in each wire when $a = 10$ in., (*b*) the value of a for which the tension in each wire is 8 lb.

SOLUTION



$$\mathbf{r}_{B/A} = a\mathbf{i} + 30\mathbf{k}$$
$$\mathbf{r}_{C/A} = 30\mathbf{i} + a\mathbf{k}$$
$$\mathbf{r}_{G/A} = 15\mathbf{i} + 15\mathbf{k}$$

By symmetry: $B = C$

$$\Sigma M_A = 0: \quad \mathbf{r}_{B/A} \times B\mathbf{j} + \mathbf{r}_C \times C\mathbf{j} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) = 0$$

$$(a\mathbf{i} + 30\mathbf{k}) \times B\mathbf{j} + (30\mathbf{i} + a\mathbf{k}) \times B\mathbf{j} + (15\mathbf{i} + 15\mathbf{k}) \times (-W\mathbf{j}) = 0$$

$$Ba\mathbf{k} - 30B\mathbf{i} + 30B\mathbf{k} - Ba\mathbf{i} - 15W\mathbf{k} + 15W\mathbf{i} = 0$$

Equate coefficient of unit vector i to zero:

$$\mathbf{i}: \quad -30B - Ba + 15W = 0$$

$$B = \frac{15W}{30 + a} \qquad C = B = \frac{15W}{30 + a} \qquad (1)$$

$$\Sigma F_y = 0: \quad A + B + C - W = 0$$

$$A + 2\left[\frac{15W}{30 + a}\right] - W = 0; \quad A = \frac{aW}{30 + a} \qquad (2)$$

(*a*) For $a = 10$ in.

Eq. (1) $$C = B = \frac{15(24 \text{ lb})}{30 + 10} = 9.00 \text{ lb}$$

Eq. (2) $$A = \frac{10(24 \text{ lb})}{30 + 10} = 6.00 \text{ lb}$$

$A = 6.00$ lb $\quad B = C = 9.00$ lb ◀

PROBLEM 4.103 (Continued)

(*b*) For tension in each wire = 8 lb

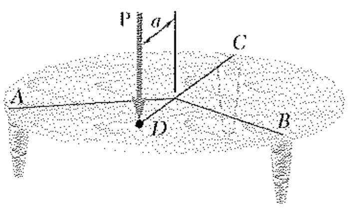
Eq. (1)

$$8\text{ lb} = \frac{15(24\text{ lb})}{30 + a}$$

$$30\text{ in.} + a = 45$$

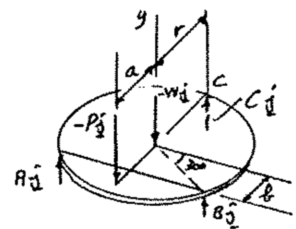
$a = 15.00$ in. ◀

PROBLEM 4.104



The table shown weighs 30 lb and has a diameter of 4 ft. It is supported by three legs equally spaced around the edge. A vertical load **P** of magnitude 100 lb is applied to the top of the table at D. Determine the maximum value of a if the table is not to tip over. Show, on a sketch, the area of the table over which **P** can act without tipping the table.

SOLUTION



$$r = 2\text{ ft} \quad b = r\sin 30° = 1\text{ ft}$$

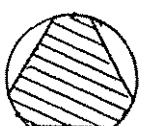
We shall sum moments about AB.

$$(b+r)C + (a-b)P - bW = 0$$

$$(1+2)C + (a-1)100 - (1)30 = 0$$

$$C = \frac{1}{3}[30 - (a-1)100]$$

If table is not to tip, $C \geq 0$



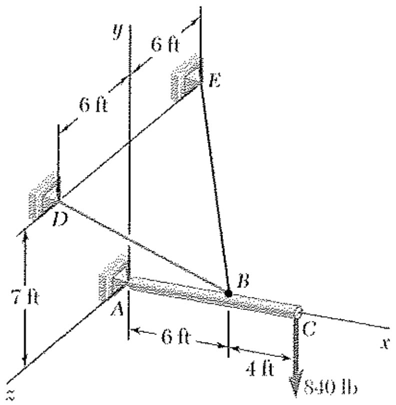
$$[30 - (a-1)100] \geq 0$$

$$30 \geq (a-1)100$$

$$a - 1 \leq 0.3 \quad a \leq 1.3\text{ ft} \quad a = 1.300\text{ ft}$$

Only ⊥ distance from P to AB matters. Same condition must be satisfied for each leg. P must be located in shaded area for no tipping

PROBLEM 4.105

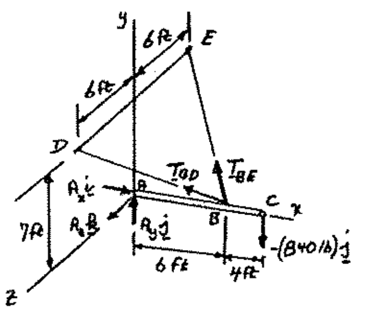


A 10-ft boom is acted upon by the 840-lb force shown. Determine the tension in each cable and the reaction at the ball-and-socket joint at A.

SOLUTION

We have five unknowns and six Eqs. of equilibrium but equilibrium is maintained ($\Sigma M_x = 0$).

Free-Body Diagram:



$$\overrightarrow{BD} = (-6\text{ ft})\mathbf{i} + (7\text{ ft})\mathbf{j} + (6\text{ ft})\mathbf{k} \quad BD = 11\text{ ft}$$

$$BE = (-6\text{ ft})\mathbf{i} + (7\text{ ft})\mathbf{j} - (6\text{ ft})\mathbf{k} \quad BE = 11\text{ ft}$$

$$T_{BD} = T_{BD}\frac{\overrightarrow{BD}}{BD} = \frac{T_{BD}}{11}(-6\mathbf{i} + 7\mathbf{j} + 6\mathbf{k})$$

$$T_{BE} = T_{BE}\frac{\overrightarrow{BE}}{BE} = \frac{T_{BE}}{11}(-6\mathbf{i} + 7\mathbf{j} - 6\mathbf{k})$$

$$\Sigma M_A = 0: \quad \mathbf{r}_B \times T_{BD} + \mathbf{r}_B \times T_{BE} + \mathbf{r}_C \times (-840\mathbf{j}) = 0$$

$$6\mathbf{i} \times \frac{T_{BD}}{11}(-6\mathbf{i} + 7\mathbf{j} + 6\mathbf{k}) + 6\mathbf{i} \times \frac{T_{BE}}{11}(-6\mathbf{i} + 7\mathbf{j} - 6\mathbf{k}) + 10\mathbf{i} \times (-840\mathbf{j}) = 0$$

$$\frac{42}{11}T_{BD}\mathbf{k} - \frac{36}{11}T_{BD}\mathbf{j} + \frac{42}{11}T_{BE}\mathbf{k} + \frac{36}{11}T_{BE}\mathbf{j} - 8400\mathbf{k}$$

Equate coefficients of unit vectors to zero.

$$\mathbf{i}: \quad -\frac{36}{11}T_{BD} + \frac{36}{11}T_{BE} = 0 \quad T_{BE} = T_{BD}$$

$$\mathbf{k}: \quad \frac{42}{11}T_{BD} + \frac{42}{11}T_{BE} - 8400 = 0$$

$$2\left(\frac{42}{11}T_{BD}\right) = 8400$$

$T_{BD} = 1100\text{ lb}$ ◀

$T_{BE} = 1100\text{ lb}$ ◀

PROBLEM 4.105 (Continued)

$$\Sigma F_x = 0: \quad A_x - \frac{6}{11}(1100 \text{ lb}) - \frac{6}{11}(1100 \text{ lb}) = 0$$

$$A_x = 1200 \text{ lb}$$

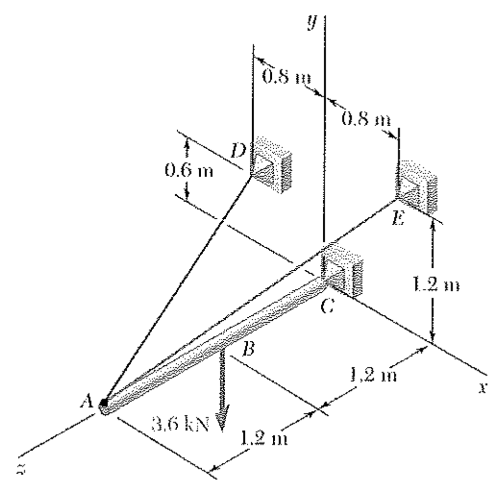
$$\Sigma F_y = 0: \quad A_y + \frac{7}{11}(1100 \text{ lb}) + \frac{7}{11}(1100 \text{ lb}) - 840 \text{ lb} = 0$$

$$A_y = -560 \text{ lb}$$

$$\Sigma F_z = 0: \quad A_z + \frac{6}{11}(1100 \text{ lb}) - \frac{6}{11}(1100 \text{ lb}) = 0$$

$$A_z = 0$$

$$\mathbf{A} = (1200 \text{ lb})\mathbf{i} - (560 \text{ lb})\mathbf{j} \blacktriangleleft$$

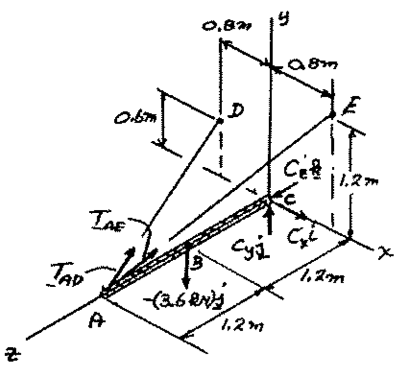


PROBLEM 4.106

A 2.4-m boom is held by a ball-and-socket joint at C and by two cables AD and AE. Determine the tension in each cable and the reaction at C.

SOLUTION

Free-Body Diagram: Five Unknowns and six Eqs. of equilibrium. Equilibrium is maintained ($\Sigma M_{AC} = 0$).



$$\mathbf{r}_B = 1.2\mathbf{k}$$
$$\mathbf{r}_A = 2.4\mathbf{k}$$

$$\overrightarrow{AD} = -0.8\mathbf{i} + 0.6\mathbf{j} - 2.4\mathbf{k} \qquad AD = 2.6 \text{ m}$$
$$\overrightarrow{AE} = 0.8\mathbf{i} + 1.2\mathbf{j} - 2.4\mathbf{k} \qquad AE = 2.8 \text{ m}$$

$$T_{AD} = \frac{\overrightarrow{AD}}{AD} = \frac{T_{AD}}{2.6}(-0.8\mathbf{i} + 0.6\mathbf{j} - 2.4\mathbf{k})$$
$$T_{AE} = \frac{\overrightarrow{AE}}{AE} = \frac{T_{AE}}{2.8}(0.8\mathbf{i} + 1.2\mathbf{j} - 2.4\mathbf{k})$$

$$\Sigma M_C = 0\text{:} \quad \mathbf{r}_A \times \mathbf{T}_{AD} + \mathbf{r}_A \times \mathbf{T}_{AE} + \mathbf{r}_B \times (-3 \text{ kN})\mathbf{j} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2.4 \\ -0.8 & 0.6 & -2.4 \end{vmatrix} \frac{T_{AD}}{2.6} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2.4 \\ 0.8 & 1.2 & -2.4 \end{vmatrix} \frac{T_{AE}}{2.8} + 1.2\mathbf{k} \times (-3.6 \text{ kN})\mathbf{j} = 0$$

Equate coefficients of unit vectors to zero.

$$\mathbf{i}\text{:} \quad -0.55385\, T_{AD} - 1.02857\, T_{AE} + 4.32 = 0 \qquad (1)$$
$$\mathbf{j}\text{:} \quad -0.73846\, T_{AD} + 0.68671\, T_{AE} = 0$$

$$T_{AD} = 0.92857\, T_{AE} \qquad (2)$$

Eq. (1):

$$-0.55385(0.92857)\, T_{AE} - 1.02857\, T_{AE} + 4.32 = 0$$

$$1.54286\, T_{AE} = 4.32$$
$$T_{AE} = 2.800 \text{ kN}$$

$T_{AE} = 2.80 \text{ kN}$ ◀

PROBLEM 4.106 (Continued)

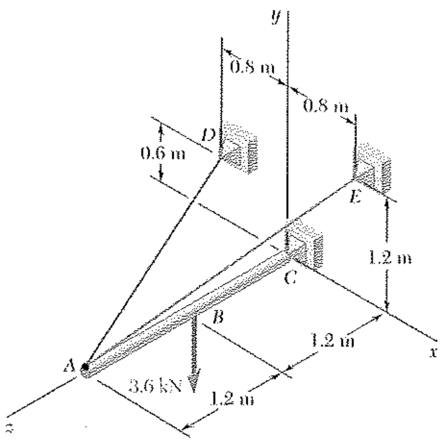
Eq. (2): $T_{AD} = 0.92857(2.80) = 2.600\text{ kN}$ $\quad T_{AD} = 2.60\text{ kN}$ ◀

$\Sigma F_x = 0$: $C_x - \frac{0.8}{2.6}(2.6\text{ kN}) + \frac{0.8}{2.8}(2.8\text{ kN}) = 0$ $\quad C_x = 0$

$\Sigma F_y = 0$: $C_y + \frac{0.6}{2.6}(2.6\text{ kN}) + \frac{1.2}{2.8}(2.8\text{ kN}) - (3.6\text{ kN}) = 0$ $\quad C_y = 1.800\text{ kN}$

$\Sigma F_z = 0$: $C_z - \frac{2.4}{2.6}(2.6\text{ kN}) - \frac{2.4}{2.8}(2.8\text{ kN}) = 0$ $\quad C_z = 4.80\text{ kN}$

$\mathbf{C} = (1.800\text{ kN})\mathbf{j} + (4.80\text{ kN})\mathbf{k}$ ◀



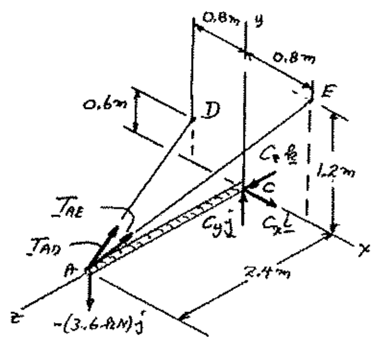
PROBLEM 4.107

Solve Problem 4.106, assuming that the 3.6-kN load is applied at Point A.

PROBLEM 4.106 A 2.4-m boom is held by a ball-and-socket joint at C and by two cables AD and AE. Determine the tension in each cable and the reaction at C.

SOLUTION

Free-Body Diagram: Five unknowns and six Eqs. of equilibrium. Equilibrium is maintained $(\Sigma M_{AC} = 0)$.



$$\overrightarrow{AD} = -0.8\mathbf{i} + 0.6\mathbf{j} - 2.4\mathbf{k} \qquad AD = 2.6 \text{ m}$$

$$\overrightarrow{AE} = 0.8\mathbf{i} + 1.2\mathbf{j} - 2.4\mathbf{k} \qquad AE = 2.8 \text{ m}$$

$$T_{AD} = \frac{\overrightarrow{AD}}{AD} = \frac{T_{AD}}{2.6}(-0.8\mathbf{i} + 0.6\mathbf{j} - 2.4\mathbf{k})$$

$$T_{AE} = \frac{\overrightarrow{AE}}{AE} = \frac{T_{AE}}{2.8}(0.8\mathbf{i} + 1.2\mathbf{j} - 2.4\mathbf{k})$$

$$\Sigma M_C = 0: \quad \mathbf{r}_A \times \mathbf{T}_{AD} + \mathbf{r}_A \times \mathbf{T}_{AE} + \mathbf{r}_A \times (-3.6 \text{ kN})\mathbf{j}$$

Factor r_A: $\quad \mathbf{r}_A \times (\mathbf{T}_{AD} + \mathbf{T}_{AE} - (3.6 \text{ kN})\mathbf{j})$

or: $\quad \mathbf{T}_{AD} + \mathbf{T}_{AE} - (3 \text{ kN})\mathbf{j} = 0 \quad$ (Forces concurrent at A)

Coefficient of **i**:

$$-\frac{T_{AD}}{2.6}(0.8) + \frac{T_{AE}}{2.8}(0.8) = 0$$

$$T_{AD} = \frac{2.6}{2.8} T_{AE} \qquad (1)$$

Coefficient of **j**:

$$\frac{T_{AD}}{2.6}(0.6) + \frac{T_{AE}}{2.8}(1.2) - 3.6 \text{ kN} = 0$$

$$\frac{2.6}{2.8} T_{AE}\left(\frac{0.6}{2.6}\right) + \frac{1.2}{2.8} T_{AE} - 3.6 \text{ kN} = 0$$

$$T_{AE}\left(\frac{0.6 + 1.2}{2.8}\right) = 3.6 \text{ kN}$$

$$T_{AE} = 5.600 \text{ kN} \qquad T_{AE} = 5.60 \text{ kN} \blacktriangleleft$$

PROBLEM 4.107 (Continued)

Eq. (1): $T_{AD} = \frac{2.6}{2.8}(5.6) = 5.200 \text{ kN}$ $\qquad T_{AD} = 5.20 \text{ kN}$ ◀

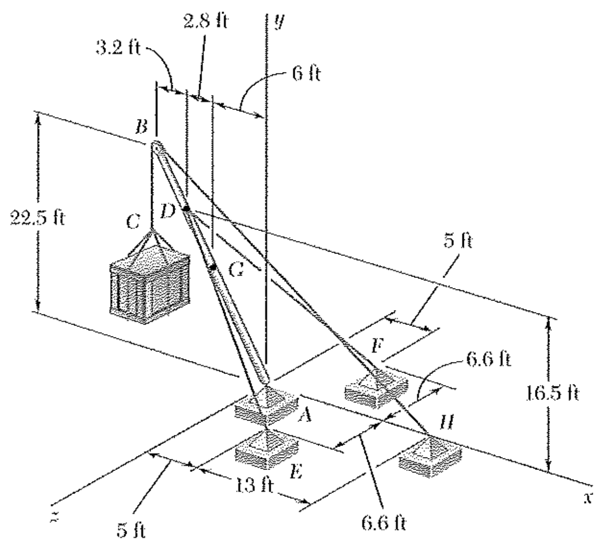
$\Sigma F_x = 0: \quad C_x - \frac{0.8}{2.6}(5.2 \text{ kN}) + \frac{0.8}{2.8}(5.6 \text{ kN}) = 0; \qquad C_x = 0$

$\Sigma F_y = 0: \quad C_y + \frac{0.6}{2.6}(5.2 \text{ kN}) + \frac{1.2}{2.8}(5.6 \text{ kN}) - 3.6 \text{ kN} = 0 \qquad C_y = 0$

$\Sigma F_z = 0: \quad C_z - \frac{2.4}{2.6}(5.2 \text{ kN}) - \frac{2.4}{2.8}(5.6 \text{ kN}) = 0 \qquad C_z = 9.60 \text{ kN}$

$\mathbf{C} = (9.60 \text{ kN})\mathbf{k}$ ◀

Note: Since forces and reaction are concurrent at A, we could have used the methods of Chapter 2.

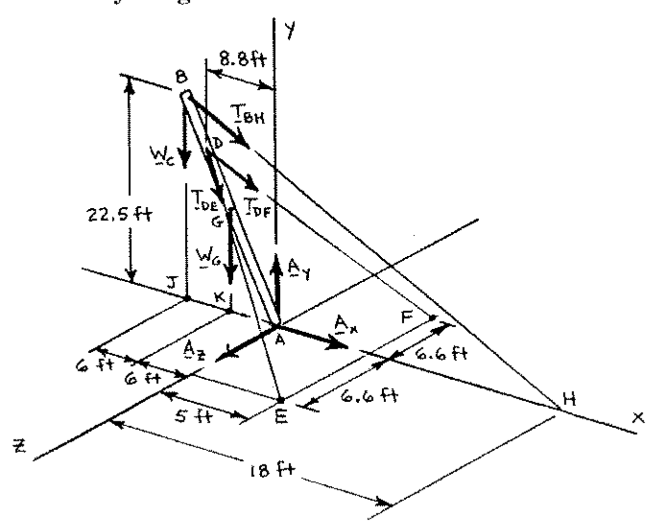


PROBLEM 4.108

A 600-lb crate hangs from a cable that passes over a pulley B and is attached to a support at H. The 200-lb boom AB is supported by a ball-and-socket joint at A and by two cables DE and DF. The center of gravity of the boom is located at G. Determine (*a*) the tension in cables DE and DF, (*b*) the reaction at A.

SOLUTION

Free-Body Diagram:



$W_C = 600$ lb
$W_G = 200$ lb

We have five unknowns $(T_{DE}, T_{DF}, A_x, A_y, A_z)$ and five equilibrium equations. The boom is free to spin about the AB axis, but equilibrium is maintained, since $\Sigma M_{AB} = 0$.

We have $\overrightarrow{BH} = (30 \text{ ft})\mathbf{i} - (22.5 \text{ ft})\mathbf{j}$ $\qquad BH = 37.5$ ft

$$\overrightarrow{DE} = (13.8 \text{ ft})\mathbf{i} - \frac{8.8}{12}(22.5 \text{ ft})\mathbf{j} + (6.6 \text{ ft})\mathbf{k}$$

$$= (13.8 \text{ ft})\mathbf{i} - (16.5 \text{ ft})\mathbf{j} + (6.6 \text{ ft})\mathbf{k} \qquad DE = 22.5 \text{ ft}$$

$$\overrightarrow{DF} = (13.8 \text{ ft})\mathbf{i} - (16.5 \text{ ft})\mathbf{j} - (6.6 \text{ ft})\mathbf{k} \qquad DF = 22.5 \text{ ft}$$

PROBLEM 4.108 (Continued)

Thus:

$$\mathbf{T}_{BH} = T_{BH}\frac{\overrightarrow{BH}}{BH} = (600\text{ lb})\frac{30\mathbf{i} - 22.5\mathbf{j}}{37.5} = (480\text{ lb})\mathbf{i} - (360\text{ lb})\mathbf{j}$$

$$\mathbf{T}_{DE} = T_{DE}\frac{\overrightarrow{DE}}{DE} = \frac{T_{DE}}{22.5}(13.8\mathbf{i} - 16.5\mathbf{j} + 6.6\mathbf{k})$$

$$\mathbf{T}_{DF} = T_{DF}\frac{\overrightarrow{DF}}{DF} = \frac{T_{DE}}{22.5}(13.8\mathbf{i} - 16.5\mathbf{j} - 6.6\mathbf{k})$$

(*a*) $\Sigma \mathbf{M}_A = 0$: $(\mathbf{r}_J \times \mathbf{W}_C) + (\mathbf{r}_K \times \mathbf{W}_G) + (\mathbf{r}_H \times \mathbf{T}_{BH}) + (\mathbf{r}_E \times \mathbf{T}_{DE}) + (\mathbf{r}_F \times \mathbf{T}_{DF}) = 0$

$$-(12\mathbf{i}) \times (-600\mathbf{j}) - (6\mathbf{i}) \times (-200\mathbf{j}) + (18\mathbf{i}) \times (480\mathbf{i} - 360\mathbf{j})$$

$$+\frac{T_{DE}}{22.5}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 0 & 6.6 \\ 13.8 & -16.5 & 6.6 \end{vmatrix} + \frac{T_{DF}}{22.5}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 0 & -6.6 \\ 13.8 & -16.5 & -6.6 \end{vmatrix} = 0$$

or,

$$7200\mathbf{k} + 1200\mathbf{k} - 6480\mathbf{k} + 4.84(T_{DE} - T_{DF})\mathbf{i}$$

$$+\frac{58.08}{22.5}(T_{DE} - T_{DF})\mathbf{j} - \frac{82.5}{22.5}(T_{DE} + T_{DF})\mathbf{k} = 0$$

Equating to zero the coefficients of the unit vectors:

i or **j**: $T_{DE} - T_{DF} = 0 \qquad T_{DE} = T_{DF}$*

k: $7200 + 1200 - 6480 - \frac{82.5}{22.5}(2T_{DE}) = 0 \qquad T_{DE} = 261.82\text{ lb}$

$T_{DE} = T_{DF} = 262\text{ lb}$ ◀

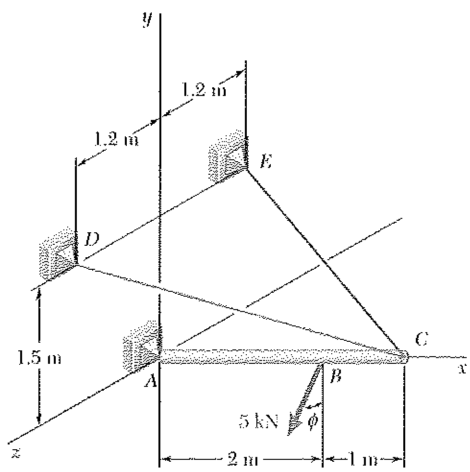
(*b*) $\Sigma F_x = 0$: $A_x + 480 + 2\left(\frac{13.8}{22.5}\right)(261.82) = 0 \qquad A_x = -801.17\text{ lb}$

$\Sigma F_y = 0$: $A_y - 600 - 200 - 360 - 2\left(\frac{16.5}{22.5}\right)(261.82) = 0 \qquad A_y = 1544.00\text{ lb}$

$\Sigma F_z = 0$: $A_z = 0$

$\mathbf{A} = -(801\text{ lb})\mathbf{i} + (1544\text{ lb})\mathbf{j}$ ◀

*Remark: The fact that $T_{DE} = T_{DF}$ could have been noted at the outset from the symmetry of structure with respect to *xy* plane.



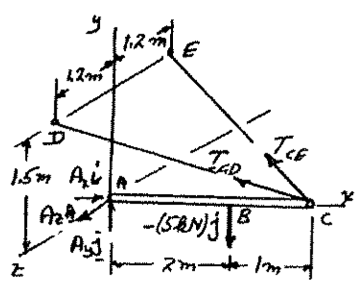
PROBLEM 4.109

A 3-m pole is supported by a ball-and-socket joint at A and by the cables CD and CE. Knowing that the 5-kN force acts vertically downward ($\phi = 0$), determine (*a*) the tension in cables CD and CE, (*b*) the reaction at A.

SOLUTION

Free-Body Diagram:

By symmetry with xy plane



$$T_{CD} = T_{CE} = T$$

$$\overrightarrow{CD} = -3\mathbf{i} + 1.5\mathbf{j} + 1.2\mathbf{k}$$

$$CD = 3.562 \text{ m}$$

$$T_{CD} = T\frac{\overrightarrow{CD}}{CD} = T\frac{-3\mathbf{i} + 1.5\mathbf{j} + 1.2\mathbf{k}}{3.562}$$

$$T_{CE} = T\frac{-3\mathbf{i} + 1.5\mathbf{j} - 1.2\mathbf{k}}{3.562}$$

$$\mathbf{r}_{B/A} = 2\mathbf{i} \qquad \mathbf{r}_{C/A} = 3\mathbf{i}$$

$$\Sigma M_A = 0: \quad \mathbf{r}_{C/A} \times \mathbf{T}_{CD} + \mathbf{r}_{C/A} \times \mathbf{T}_{CE} + \mathbf{r}_{B/A} \times (-5 \text{ kN})\mathbf{j} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 0 \\ -3 & 1.5 & 1.2 \end{vmatrix} \frac{T}{3.562} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 0 \\ -3 & 1.5 & -1.2 \end{vmatrix} \frac{T}{3.562} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & 0 \\ 0 & -5 & 0 \end{vmatrix} = 0$$

Coefficient of **k**:

$$2\left[3 \times 1.5 \times \frac{T}{3.562}\right] - 10 = 0 \quad T = 3.958 \text{ kN}$$

$$\Sigma F = 0: \quad A + T_{CD} + T_{CE} - 5\mathbf{j} = 0$$

PROBLEM 4.109 (Continued)

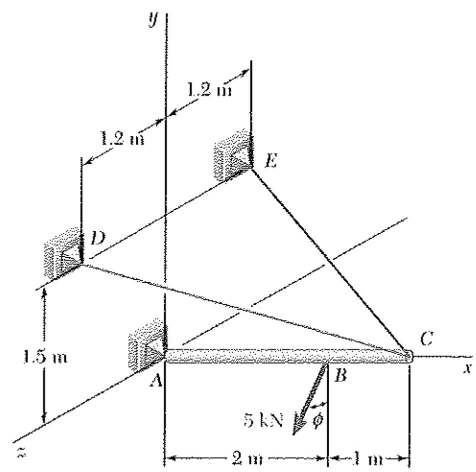
Coefficient of **k**: $A_z = 0$

Coefficient of **i**: $A_x - 2[3.958\times 3/3.562] = 0 \qquad A_x = 6.67\text{ kN}$

Coefficient of **j**: $A_y + 2[3.958\times 1.5/3.562] - 5 = 0 \qquad A_y = 1.667\text{ kN}$

(*a*) $T_{CD} = T_{CE} = 3.96\text{ kN}$

(*b*) $\mathbf{A} = (6.67\text{ kN})\mathbf{i} + (1.667\text{ kN})\mathbf{j}$ ◀



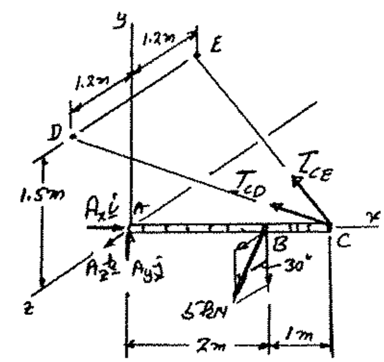
PROBLEM 4.110

A 3-m pole is supported by a ball-and-socket joint at A and by the cables CD and CE. Knowing that the line of action of the 5-kN force forms an angle $\phi = 30°$ with the vertical xy plane, determine (*a*) the tension in cables CD and CE, (*b*) the reaction at A.

SOLUTION

Free-Body Diagram:

Five unknowns and six Eqs. of equilibrium but equilibrium is maintained $(\Sigma M_{AC} = 0)$



$$\mathbf{r}_{B/A} = 2\mathbf{i}$$

$$\mathbf{r}_{C/A} = 3\mathbf{i}$$

Load at B.

$$= -(5\cos 30)\mathbf{j} + (5\sin 30)\mathbf{k}$$
$$= -4.33\mathbf{j} + 2.5\mathbf{k}$$

$$\overrightarrow{CD} = -3\mathbf{i} + 1.5\mathbf{j} + 1.2\mathbf{k} \qquad CD = 3.562 \text{ m}$$

$$\mathbf{T}_{CD} = T_{CD}\frac{\overrightarrow{CD}}{CD} = \frac{T}{3.562}(-3\mathbf{i} + 1.5\mathbf{j} + 1.2\mathbf{k})$$

Similarly,

$$\mathbf{T}_{CE} = \frac{T}{3.562}(-3\mathbf{i} + 1.5\mathbf{j} - 1.2\mathbf{k})$$

$$\Sigma M_A = 0: \quad \mathbf{r}_{C/A} \times \mathbf{T}_{CD} + \mathbf{r}_{C/A} \times \mathbf{T}_{CE} + \mathbf{r}_{B/A} \times (-4.33\mathbf{j} + 2.5\mathbf{k}) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 0 \\ -3 & 1.5 & 1.2 \end{vmatrix} \frac{T_{CD}}{3.562} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 0 \\ -3 & 1.5 & -1.2 \end{vmatrix} \frac{T_{CE}}{3.562} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & 0 \\ 0 & -4.33 & 2.5 \end{vmatrix} = 0$$

Equate coefficients of unit vectors to zero.

$$\mathbf{j}: \quad -3.6\frac{T_{CD}}{3.562} + 3.6\frac{T_{CE}}{3.562} - 5 = 0$$

$$-3.6T_{CD} + 3.6T_{CE} - 17.810 = 0 \qquad (1)$$

PROBLEM 4.110 (Continued)

$$\mathbf{k}:\quad 4.5\frac{T_{CD}}{3.562} + 4.5\frac{T_{CE}}{3.562} - 8.66 = 0$$

$$4.5T_{CD} + 4.5T_{CE} = 30.846 \qquad (2)$$

(2) + 1.25(1): $9T_{CE} - 53.11 = 0 \quad T_{CE} = 5.901 \text{ kN}$

Eq. (1): $-3.6T_{CD} + 3.6(5.901) - 17.810 = 0$

$$T_{CD} = 0.954 \text{ kN}$$

$$\Sigma F = 0:\quad \mathbf{A} + \mathbf{T}_{CD} + \mathbf{T}_{CE} - 4.33\mathbf{j} + 2.5\mathbf{k} = 0$$

$$\mathbf{i}:\quad A_x + \frac{0.954}{3.562}(-3) + \frac{5.901}{3.562}(-3) = 0$$

$$A_x = 5.77 \text{ kN}$$

$$\mathbf{j}:\quad A_y + \frac{0.954}{3.562}(1.5) + \frac{5.901}{3.562}(1.5) - 4.33 = 0$$

$$A_y = 1.443 \text{ kN}$$

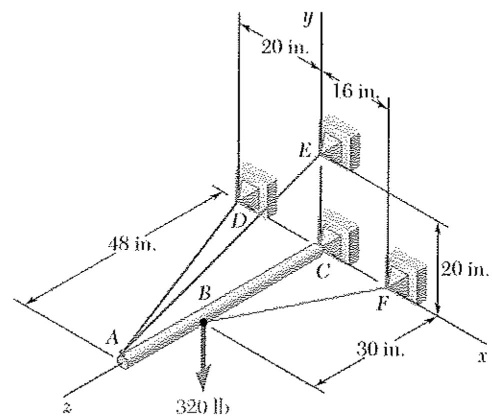
$$\mathbf{k}:\quad A_z + \frac{0.954}{3.562}(1.2) + \frac{5.901}{3.562}(-1.2) + 2.5 = 0$$

$$A_z = -0.833 \text{ kN}$$

Answers:

(*a*) $T_{CD} = 0.954 \text{ kN}$ $\qquad T_{CE} = 5.90 \text{ kN}$ ◀

(*b*) $\mathbf{A} = (5.77 \text{ kN})\mathbf{i} + (1.443 \text{ kN})\mathbf{j} - (0.833 \text{ kN})\mathbf{k}$ ◀

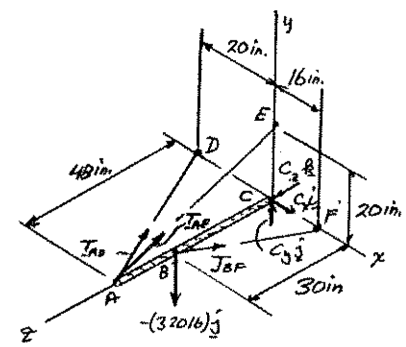


PROBLEM 4.111

A 48-in. boom is held by a ball-and-socket joint at *C* and by two cables *BF* and *DAE*; cable *DAE* passes around a frictionless pulley at *A*. For the loading shown, determine the tension in each cable and the reaction at *C*.

SOLUTION

Free-Body Diagram:



Five unknowns and six Eqs. of equilibrium but equilibrium is maintained ($\Sigma M_{AC} = 0$).

T = Tension in both parts of cable *DAE*.

$$\mathbf{r}_B = 30\mathbf{k}$$
$$\mathbf{r}_A = 48\mathbf{k}$$

$$\overrightarrow{AD} = -20\mathbf{i} - 48\mathbf{k} \qquad AD = 52 \text{ in.}$$
$$\overrightarrow{AE} = 20\mathbf{j} - 48\mathbf{k} \qquad AE = 52 \text{ in.}$$
$$\overrightarrow{BF} = 16\mathbf{i} - 30\mathbf{k} \qquad BF = 34 \text{ in.}$$

$$\mathbf{T}_{AD} = T\frac{\overrightarrow{AD}}{AD} = \frac{T}{52}(-20\mathbf{i} - 48\mathbf{k}) = \frac{T}{13}(-5\mathbf{i} - 12\mathbf{k})$$
$$\mathbf{T}_{AE} = T\frac{\overrightarrow{AE}}{AE} = \frac{T}{52}(20\mathbf{j} - 48\mathbf{k}) = \frac{T}{13}(5\mathbf{j} - 12\mathbf{k})$$
$$\mathbf{T}_{BF} = T_{BF}\frac{\overrightarrow{BF}}{BF} = \frac{T_{BF}}{34}(16\mathbf{i} - 30\mathbf{k}) = \frac{T_{BF}}{17}(8\mathbf{i} - 15\mathbf{k})$$

$$\Sigma\mathbf{M}_C = 0{:}\quad \mathbf{r}_A \times \mathbf{T}_{AD} + \mathbf{r}_A \times \mathbf{T}_{AE} + \mathbf{r}_B \times \mathbf{T}_{BF} + \mathbf{r}_B \times (-320 \text{ lb})\mathbf{j} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 48 \\ -5 & 0 & -12 \end{vmatrix}\frac{T}{13} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 48 \\ 0 & 5 & -12 \end{vmatrix}\frac{T}{13} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 30 \\ 8 & 0 & -15 \end{vmatrix}\frac{T_{BF}}{17} + (30\mathbf{k}) \times (-320\mathbf{j}) = 0$$

Coefficient of **i**:

$$-\frac{240}{13}T + 9600 = 0 \qquad T = 520 \text{ lb}$$

PROBLEM 4.111 (Continued)

Coefficient of **j**: $-\frac{240}{13}T + \frac{240}{17}T_{BD} = 0$

$$T_{BD} = \frac{17}{13}T = \frac{17}{13}(520) \quad T_{BD} = 680 \text{ lb}$$

$\Sigma \mathbf{F} = 0$: $\mathbf{T}_{AD} + \mathbf{T}_{AE} + \mathbf{T}_{BF} - 320\mathbf{j} + C = 0$

Coefficient of **i**: $-\frac{20}{52}(520) + \frac{8}{17}(680) + C_x = 0$

$$-200 + 320 + C_x = 0 \qquad C_x = -120 \text{ lb}$$

Coefficient of **j**: $\frac{20}{52}(520) - 320 + C_y = 0$

$$200 - 320 + C_y = 0 \qquad C_y = 120 \text{ lb}$$

Coefficient of **k**: $-\frac{48}{52}(520) - \frac{48}{52}(520) - \frac{30}{34}(680) + C_z = 0$

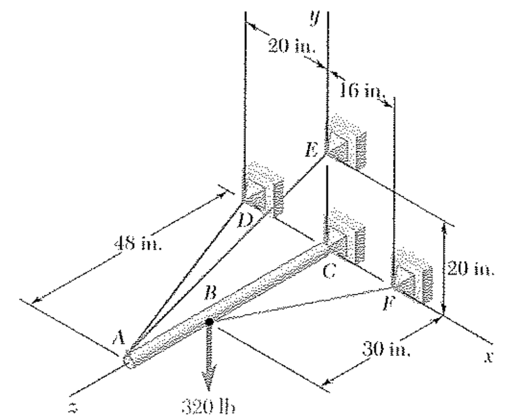
$$-480 - 480 - 600 + C_z = 0$$

$$C_z = 1560 \text{ lb}$$

Answers: $T_{DAE} = T$ $\quad T_{DAE} = 520 \text{ lb}$ ◀

$T_{BD} = 680 \text{ lb}$ ◀

$\mathbf{C} = -(120.0 \text{ lb})\mathbf{i} + (120.0 \text{ lb})\mathbf{j} + (1560 \text{ lb})\mathbf{k}$ ◀



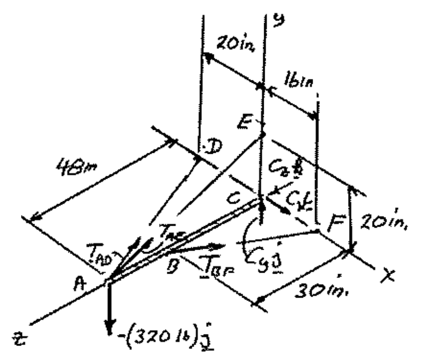
PROBLEM 4.112

Solve Problem 4.111, assuming that the 320-lb load is applied at A.

PROBLEM 4.111 A 48-in. boom is held by a ball-and-socket joint at C and by two cables BF and DAE; cable DAE passes around a frictionless pulley at A. For the loading shown, determine the tension in each cable and the reaction at C.

SOLUTION

Free-Body Diagram:



Five unknowns and six Eqs. of equilibrium but equilibrium is maintained $(\Sigma M_{AC} = 0)$.

T = tension in both parts of cable DAE.

$$\mathbf{r}_B = 30\mathbf{k}$$

$$\mathbf{r}_A = 48\mathbf{k}$$

$$\overrightarrow{AD} = -20\mathbf{i} - 48\mathbf{k} \qquad AD = 52 \text{ in.}$$

$$\overrightarrow{AE} = 20\mathbf{j} - 48\mathbf{k} \qquad AE = 52 \text{ in.}$$

$$\overrightarrow{BF} = 16\mathbf{i} - 30\mathbf{k} \qquad BF = 34 \text{ in.}$$

$$\mathbf{T}_{AD} = T\frac{\overrightarrow{AD}}{AD} = \frac{T}{52}(-20\mathbf{i} - 48\mathbf{k}) = \frac{T}{13}(-5\mathbf{i} - 12\mathbf{k})$$

$$\mathbf{T}_{AE} = T\frac{\overrightarrow{AE}}{AE} = \frac{T}{52}(20\mathbf{j} - 48\mathbf{k}) = \frac{T}{13}(5\mathbf{j} - 12\mathbf{k})$$

$$\mathbf{T}_{BF} = T_{BF}\frac{\overrightarrow{BF}}{BF} = \frac{T_{BF}}{34}(16\mathbf{i} - 30\mathbf{k}) = \frac{T_{BF}}{17}(8\mathbf{i} - 15\mathbf{k})$$

$$\Sigma M_C = 0: \quad \mathbf{r}_A \times \mathbf{T}_{AD} + \mathbf{r}_A \times \mathbf{T}_{AE} + \mathbf{r}_B \times \mathbf{T}_{BF} + \mathbf{r}_A \times (-320 \text{ lb})\mathbf{j} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 48 \\ -5 & 0 & -12 \end{vmatrix} \frac{T}{13} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 48 \\ 0 & 5 & -12 \end{vmatrix} \frac{T}{13} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 30 \\ 8 & 0 & -15 \end{vmatrix} \frac{T_{BF}}{17} + 48\mathbf{k} \times (-320\mathbf{j}) = 0$$

Coefficient of $\mathbf{i}$:

$$-\frac{240}{13}T + 15360 = 0 \qquad T = 832 \text{ lb}$$

Coefficient of **j**: $-\frac{240}{13}T + \frac{240}{17}T_{BD} = 0$

$$T_{BD} = \frac{17}{13}T = \frac{17}{13}(832) \qquad T_{BD} = 1088 \text{ lb}$$

$$\Sigma \mathbf{F} = 0: \quad \mathbf{T}_{AD} + \mathbf{T}_{AE} + \mathbf{T}_{BF} - 320\mathbf{j} + \mathbf{C} = 0$$

Coefficient of **i**: $-\frac{20}{52}(832) + \frac{8}{17}(1088) + C_x = 0$

$$-320 + 512 + C_x = 0 \qquad C_x = -192 \text{ lb}$$

Coefficient of **j**: $\frac{20}{52}(832) - 320 + C_y = 0$

$$320 - 320 + C_y = 0 \qquad C_y = 0$$

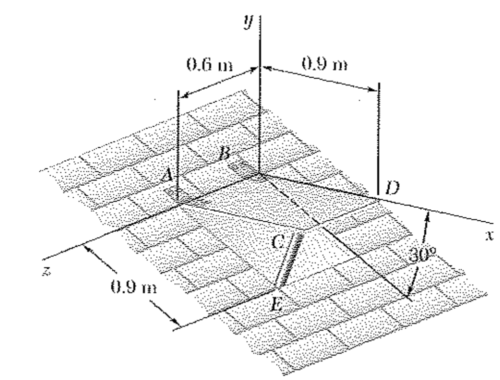
Coefficient of **k**: $-\frac{48}{52}(832) - \frac{48}{52}(852) - \frac{30}{34}(1088) + C_z = 0$

$$-768 - 768 - 960 + C_z = 0 \qquad C_z = 2496 \text{ lb}$$

Answers: $T_{DAE} = T$ $\qquad T_{DAE} = 832 \text{ lb}$ ◀

$T_{BD} = 1088 \text{ lb}$ ◀

$\mathbf{C} = -(192.0 \text{ lb})\mathbf{i} + (2496 \text{ lb})\mathbf{k}$ ◀

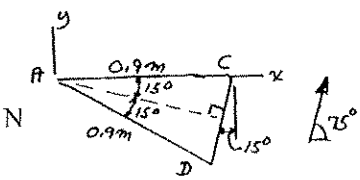


PROBLEM 4.113

A 20-kg cover for a roof opening is hinged at corners A and B. The roof forms an angle of 30° with the horizontal, and the cover is maintained in a horizontal position by the brace CE. Determine (a) the magnitude of the force exerted by the brace, (b) the reactions at the hinges. Assume that the hinge at A does not exert any axial thrust.

SOLUTION

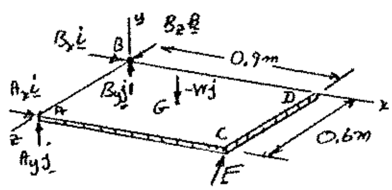
Force exerted by CD



$$\mathbf{F} = F(\sin 75°)\mathbf{i} + F(\cos 75°)\mathbf{j}$$

$$\mathbf{F} = F(0.2588\mathbf{i} + 0.9659\mathbf{j})$$

$$W = mg = 20 \text{ kg}(9.81 \text{ m/s}^2) = 196.2 \text{ N}$$



$$\mathbf{r}_{A/B} = 0.6\mathbf{k}$$

$$\mathbf{r}_{C/B} = 0.9\mathbf{i} + 0.6\mathbf{k}$$

$$\mathbf{r}_{G/B} = 0.45\mathbf{i} + 0.3\mathbf{k}$$

$$\mathbf{F} = F(0.2588\mathbf{i} + 0.9659\mathbf{j})$$

$$\Sigma \mathbf{M}_B = 0: \quad \mathbf{r}_{G/B} \times (-196.2\mathbf{j}) + \mathbf{r}_{C/B} \times \mathbf{F} + \mathbf{r}_{A/B} \times \mathbf{A} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.45 & 0 & 0.3 \\ 0 & -196.2 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.9 & 0 & 0.6 \\ 0.2588 & +0.9659 & 0 \end{vmatrix} F + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.6 \\ A_x & A_y & 0 \end{vmatrix} = 0$$

Coefficient of $\mathbf{i}$: $\quad +58.86 - 0.5796F - 0.6A_y = 0 \qquad (1)$

Coefficient of $\mathbf{j}$: $\quad +0.1553F + 0.6A_x = 0 \qquad (2)$

Coefficient of $\mathbf{k}$: $\quad -88.29 + 0.8693F = 0: \quad F = 101.56 \text{ N}$

Eq. (2): $\quad +58.86 - 0.5796(101.56) - 0.6A_y = 0 \quad A_y = 0$

Eq. (3): $\quad +0.1553(101.56) + 0.6A_x = 0 \quad A_x = -26.29 \text{ N}$

$$F = 101.6 \text{ N}$$

$\mathbf{A} = -(26.3 \text{ N})\mathbf{i}$ ◀

$$\Sigma \mathbf{F}: \quad \mathbf{A} + \mathbf{B} + \mathbf{F} - W_\mathbf{j} = 0$$

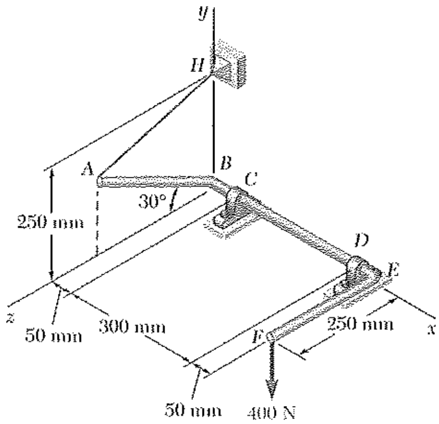
Coefficient of $\mathbf{i}$: $\quad 26.29 + B_x + 0.2588(101.56) = 0 \quad B_x = 0$

Coefficient of $\mathbf{j}$: $\quad B_y + 0.9659(101.56) - 196.2 = 0 \quad B_y = 98.1 \text{ N}$

Coefficient of $\mathbf{k}$: $\quad B_z = 0$

$\mathbf{B} = (98.1 \text{ N})\mathbf{j}$ ◀

PROBLEM 4.114



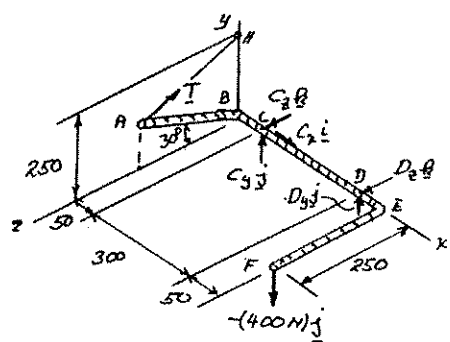
The bent rod *ABEF* is supported by bearings at *C* and *D* and by wire *AH*. Knowing that portion *AB* of the rod is 250 mm long, determine (*a*) the tension in wire *AH*, (*b*) the reactions at *C* and *D*. Assume that the bearing at *D* does not exert any axial thrust.

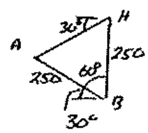
SOLUTION

Free-Body Diagram:

Dimensions in mm

ΔABH is equilateral





$$\mathbf{r}_{H/C} = -50\mathbf{i} + 250\mathbf{j}$$
$$\mathbf{r}_{D/C} = 300\mathbf{i}$$
$$\mathbf{r}_{F/C} = 350\mathbf{i} + 250\mathbf{k}$$
$$\mathbf{T} = T(\sin 30°)\mathbf{j} - T(\cos 30°)\mathbf{k} = T(0.5\mathbf{j} - 0.866\mathbf{k})$$

$$\Sigma \mathbf{M}_C = 0: \quad \mathbf{r}_{H/C} \times \mathbf{T} + \mathbf{r}_D \times \mathbf{D} + \mathbf{r}_{F/C} \times (-400\mathbf{j}) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -50 & 250 & 0 \\ 0 & 0.5 & -0.866 \end{vmatrix} T + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 300 & 0 & 0 \\ 0 & D_y & D_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 350 & 0 & 250 \\ 0 & -400 & 0 \end{vmatrix} = 0$$

Coefficient **i**: $\quad -216.5T + 100 \times 10^3 = 0$

$$T = 461.9 \text{ N}$$

$T = 462$ N ◀

Coefficient of **j**: $\quad -43.3T - 300D_z = 0$

$$-43.3(461.9) - 300D_z = 0 \qquad D_z = -66.67 \text{ N}$$

PROBLEM 4.114 (Continued)

Coefficient of **k**: $-25T + 300D_y - 140\times10^3 = 0$

$-25(461.9) + 300D_y - 140\times10^3 = 0 \qquad D_y = 505.1\text{ N}$

$\mathbf{D} = (505\text{ N})\mathbf{j} - (66.7\text{ N})\mathbf{k}$ ◀

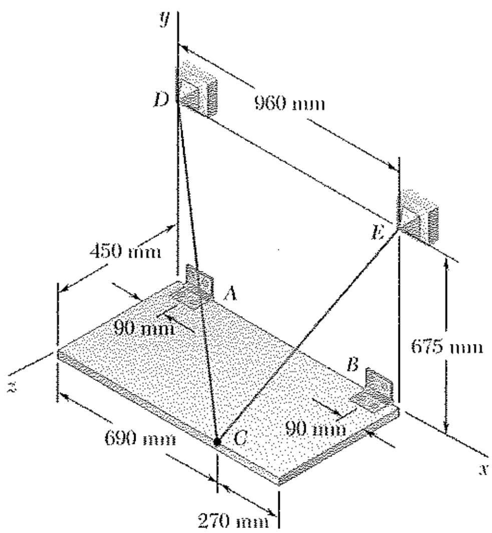
$\Sigma\mathbf{F} = 0\text{:} \quad \mathbf{C} + \mathbf{D} + \mathbf{T} - 400\mathbf{j} = 0$

Coefficient **i**: $C_x = 0 \qquad C_x = 0$

Coefficient **j**: $C_y + (461.9)0.5 + 505.1 - 400 = 0 \qquad C_y = -336\text{ N}$

Coefficient **k**: $C_z - (461.9)0.866 - 66.67 = 0 \qquad C_z = 467\text{ N}$

$\mathbf{C} = -(336\text{ N})\mathbf{j} + (467\text{ N})\mathbf{k}$ ◀

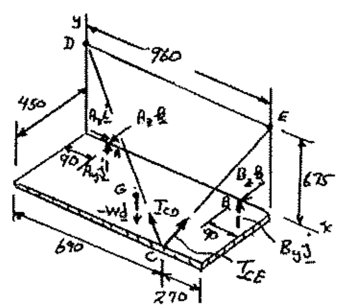


PROBLEM 4.115

A 100-kg uniform rectangular plate is supported in the position shown by hinges *A* and *B* and by cable *DCE* that passes over a frictionless hook at *C*. Assuming that the tension is the same in both parts of the cable, determine (*a*) the tension in the cable, (*b*) the reactions at *A* and *B*. Assume that the hinge at *B* does not exert any axial thrust.

SOLUTION

Dimensions in mm



$$\mathbf{r}_{B/A}(960-180)\mathbf{i} = 780\mathbf{i}$$

$$\mathbf{r}_{G/A} = \left(\frac{960}{2} - 90\right)\mathbf{i} + \frac{450}{2}\mathbf{k}$$

$$= 390\mathbf{i} + 225\mathbf{k}$$

$$\mathbf{r}_{C/A} = 600\mathbf{i} + 450\mathbf{k}$$

T = Tension in cable DCE

$$\overrightarrow{CD} = -690\mathbf{i} + 675\mathbf{j} - 450\mathbf{k} \qquad CD = 1065 \text{ mm}$$

$$\overrightarrow{CE} = 270\mathbf{i} + 675\mathbf{j} - 450\mathbf{k} \qquad CE = 855 \text{ mm}$$

$$\mathbf{T}_{CD} = \frac{T}{1065}(-690\mathbf{i} + 675\mathbf{j} - 450\mathbf{k})$$

$$\mathbf{T}_{CE} = \frac{T}{855}(270\mathbf{i} + 675\mathbf{j} - 450\mathbf{k})$$

$$\mathbf{W} = -mg\mathbf{i} = -(100 \text{ kg})(9.81 \text{ m/s}^2)\mathbf{j} = -(981 \text{ N})\mathbf{j}$$

$$\Sigma\mathbf{M}_A = 0: \quad \mathbf{r}_{C/A} \times \mathbf{T}_{CD} + \mathbf{r}_{C/A} \times \mathbf{T}_{CE} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) + \mathbf{r}_{B/A} \times \mathbf{B} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 600 & 0 & 450 \\ -690 & 675 & -450 \end{vmatrix} \frac{T}{1065} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 600 & 0 & 450 \\ 270 & 675 & -450 \end{vmatrix} \frac{T}{855} +$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 390 & 0 & 225 \\ 0 & -981 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 780 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} = 0$$

PROBLEM 4.115 (Continued)

Coefficient of **i**: $-(450)(675)\frac{T}{1065}-(450)(675)\frac{T}{855}+220.725\times10^3=0$

$T = 344.6\text{ N}$ $\quad$ $T = 345\text{ N}$ ◀

Coefficient of **j**: $(-690\times450+600\times450)\frac{344.6}{1065}+(270\times450+600\times450)\frac{344.6}{855}-780B_z=0$

$B_z = 185.49\text{ N}$

Coefficient of **k**: $(600)(675)\frac{344.6}{1065}+(600)(675)\frac{344.6}{855}-382.59\times10^3+780B_y$ $\quad$ $B_y = 113.2\text{N}$

$\mathbf{B} = (113.2\text{ N})\mathbf{j} + (185.5\text{ N})\mathbf{k}$ ◀

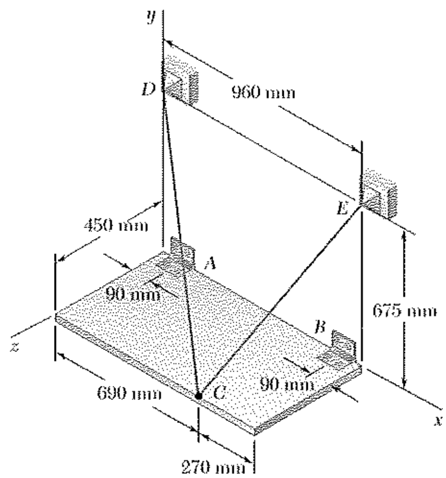
$\Sigma\mathbf{F}=0$: $\quad \mathbf{A}+\mathbf{B}+\mathbf{T}_{CD}+\mathbf{T}_{CE}+\mathbf{W}=0$

Coefficient of **i**: $A_x-\frac{690}{1065}(344.6)+\frac{270}{855}(344.6)=0$ $\quad$ $A_x = 114.4\text{ N}$

Coefficient of **j**: $A_y+113.2+\frac{675}{1065}(344.6)+\frac{675}{855}(344.6)-981=0$ $\quad$ $A_y = 377\text{ N}$

Coefficient of **k**: $A_z+185.5-\frac{450}{1065}(344.6)-\frac{450}{855}(344.6)=0$ $\quad$ $A_z = 141.5\text{ N}$

$\mathbf{A} = (114.4\text{ N})\mathbf{i} + (377\text{ N})\mathbf{j} + (144.5\text{ N})\mathbf{k}$ ◀



PROBLEM 4.116

Solve Problem 4.115, assuming that cable *DCE* is replaced by a cable attached to Point *E* and hook *C*.

PROBLEM 4.115 A 100-kg uniform rectangular plate is supported in the position shown by hinges *A* and *B* and by cable *DCE* that passes over a frictionless hook at *C*. Assuming that the tension is the same in both parts of the cable, determine (*a*) the tension in the cable, (*b*) the reactions at *A* and *B*. Assume that the hinge at *B* does not exert any axial thrust.

SOLUTION

See solution to Problem 4.115 for free-body diagram and analysis leading to the following:

$$CD = 1065 \text{ mm}$$

$$CE = 855 \text{ mm}$$

Now:

$$\mathbf{T}_{CD} = \frac{T}{1065}(-690\mathbf{i} + 675\mathbf{j} - 450\mathbf{k})$$

$$\mathbf{T}_{CE} = \frac{T}{855}(270\mathbf{i} + 675\mathbf{j} - 450\mathbf{k})$$

$$\mathbf{W} = -mg\mathbf{i} = -(100 \text{ kg})(9.81 \text{ m/s}^2)\mathbf{j} = -(981 \text{ N})\mathbf{j}$$

$$\Sigma\mathbf{M}_A = 0: \quad \mathbf{r}_{C/A} \times T_{CE} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) + \mathbf{r}_{B/A} \times B = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 600 & 0 & 450 \\ 270 & 675 & -450 \end{vmatrix} \frac{T}{855} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 390 & 0 & 225 \\ 0 & -981 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 780 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} = 0$$

Coefficient of **i**:

$$-(450)(675)\frac{T}{855} + 220.725 \times 10^3 = 0$$

$$T = 621.3 \text{ N} \qquad T = 621 \text{ N} ◀$$

Coefficient of **j**:

$$(270 \times 450 + 600 \times 450)\frac{621.3}{855} - 980B_z = 0 \quad B_z = 364.7 \text{ N}$$

Coefficient of **k**:

$$(600)(675)\frac{621.3}{855} - 382.59 \times 10^3 + 780B_y = 0 \quad B_y = 113.2 \text{ N}$$

$$\mathbf{B} = (113.2 \text{ N})\mathbf{j} + (365 \text{ N})\mathbf{k} ◀$$

PROBLEM 4.116 (Continued)

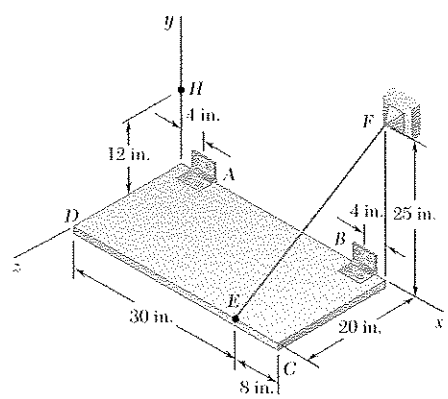
$$\Sigma \mathbf{F} = 0: \quad \mathbf{A} + \mathbf{B} + \mathbf{T}_{CE} + \mathbf{W} = 0$$

Coefficient of **i**: $A_x + \frac{270}{855}(621.3) = 0 \quad A_x = -196.2 \text{ N}$

Coefficient of **j**: $A_y + 113.2 + \frac{675}{855}(621.3) - 981 = 0 \quad A_y = 377.3 \text{ N}$

Coefficient of **k**: $A_z + 364.7 - \frac{450}{855}(621.3) = 0 \qquad A_z = -37.7 \text{ N}$

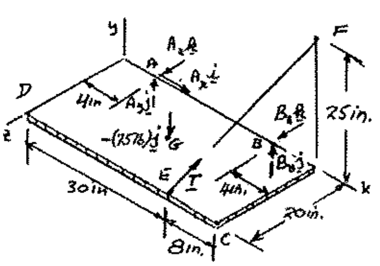
$$\mathbf{A} = -(196.2 \text{ N})\mathbf{i} + (377 \text{ N})\mathbf{j} - (37.7 \text{ N})\mathbf{k} \blacktriangleleft$$



PROBLEM 4.117

The rectangular plate shown weighs 75 lb and is held in the position shown by hinges at A and B and by cable EF. Assuming that the hinge at B does not exert any axial thrust, determine (a) the tension in the cable, (b) the reactions at A and B.

SOLUTION



$$\mathbf{r}_{B/A} = (38-8)\mathbf{i} = 30\mathbf{i}$$

$$\mathbf{r}_{E/A} = (30-4)\mathbf{i} + 20\mathbf{k}$$

$$= 26\mathbf{i} + 20\mathbf{k}$$

$$\mathbf{r}_{G/A} = \frac{38}{2}\mathbf{i} + 10\mathbf{k}$$

$$= 19\mathbf{i} + 10\mathbf{k}$$

$$\overrightarrow{EF} = 8\mathbf{i} + 25\mathbf{j} - 20\mathbf{k}$$

$$EF = 33 \text{ in.}$$

$$\mathbf{T} = T\frac{\overrightarrow{AE}}{AE} = \frac{T}{33}(8\mathbf{i} + 25\mathbf{j} - 20\mathbf{k})$$

$$\Sigma\mathbf{M}_A = 0: \quad \mathbf{r}_{E/A} \times T + \mathbf{r}_{G/A} \times (-75\mathbf{j}) + \mathbf{r}_{B/A} \times B = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 26 & 0 & 20 \\ 8 & 25 & -20 \end{vmatrix} \frac{T}{33} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 19 & 0 & 10 \\ 0 & -75 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 30 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} = 0$$

Coefficient of $\mathbf{i}$: $\quad -(25)(20)\frac{T}{33} + 750 = 0:$ $\quad T = 49.5 \text{ lb}$ ◀

Coefficient of $\mathbf{j}$: $\quad (160+520)\frac{49.5}{33} - 30B_z = 0: \quad B_z = 34 \text{ lb}$

Coefficient of $\mathbf{k}$: $\quad (26)(25)\frac{49.5}{33} - 1425 + 30B_y = 0: \quad B_y = 15 \text{ lb}$ $\quad \mathbf{B} = (15 \text{ lb})\mathbf{j} + (34 \text{ lb})\mathbf{k}$ ◀

PROBLEM 4.117 (Continued)

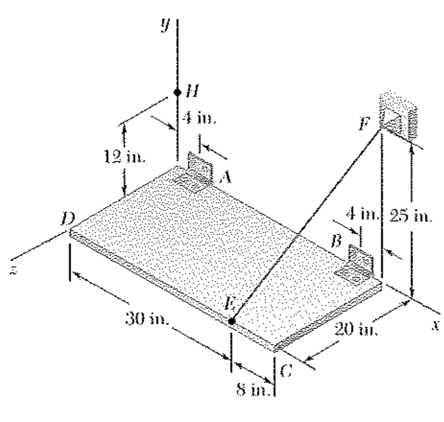
$$\Sigma \mathbf{F} = 0: \quad \mathbf{A} + \mathbf{B} + \mathbf{T} - (75 \text{ lb})\mathbf{j} = 0$$

Coefficient of **i**: $A_x + \frac{8}{33}(49.5) = 0 \quad A_x = -12.00 \text{ lb}$

Coefficient of **j**: $A_y + 15 + \frac{25}{33}(49.5) - 75 = 0 \quad A_y = 22.5 \text{ lb}$

Coefficient of **k**: $A_z + 34 - \frac{20}{33}(49.5) = 0 \quad A_z = -4.00 \text{ lb}$

$\mathbf{A} = -(12.00 \text{ lb})\mathbf{i} + (22.5 \text{ lb})\mathbf{j} - (4.00 \text{ lb})\mathbf{k}$ ◀

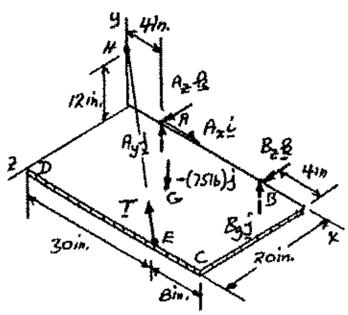


PROBLEM 4.118

Solve Problem 4.117, assuming that cable EF is replaced by a cable attached at points E and H.

PROBLEM 4.117 The rectangular plate shown weighs 75 lb and is held in the position shown by hinges at A and B and by cable EF. Assuming that the hinge at B does not exert any axial thrust, determine (*a*) the tension in the cable, (*b*) the reactions at A and B.

SOLUTION



$$\mathbf{r}_{B/A} = (38-8)\mathbf{i} = 30\mathbf{i}$$

$$\mathbf{r}_{E/A} = (30-4)\mathbf{i} + 20\mathbf{k} = 26\mathbf{i} + 20\mathbf{k}$$

$$\mathbf{r}_{G/A} = \frac{38}{2}\mathbf{i} + 10\mathbf{k} = 19\mathbf{i} + 10\mathbf{k}$$

$$\overrightarrow{EH} = -30\mathbf{i} + 12\mathbf{j} - 20\mathbf{k}$$

$$EH = 38 \text{ in.}$$

$$\mathbf{T} = T\frac{\overrightarrow{EH}}{EH} = \frac{T}{38}(-30\mathbf{i} + 12\mathbf{j} - 20\mathbf{k})$$

$$\Sigma \mathbf{M}_A = 0: \quad \mathbf{r}_{E/A} \times \mathbf{T} + \mathbf{r}_{G/A} \times (-75\mathbf{j}) + \mathbf{r}_{B/A} \times \mathbf{B} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 26 & 0 & 20 \\ -30 & 12 & -20 \end{vmatrix} \frac{T}{38} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 19 & 0 & 10 \\ 0 & -75 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 30 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} = 0$$

Coefficient of **i**: $\quad -(12)(20)\frac{T}{38} + 750 = 0 \quad T = 118.75$ $\qquad T = 118.8\,\text{lb}$ ◀

Coefficient of **j**: $\quad (-600 + 520)\frac{118.75}{38} - 30B_z = 0 \quad B_z = -8.33\,\text{lb}$

Coefficient of **k**: $\quad (26)(12)\frac{118.75}{38} - 1425 + 30B_y = 0 \quad B_y = 15.00\,\text{lb}$ $\qquad \mathbf{B} = (15.00\,\text{lb})\mathbf{j} - (8.33\,\text{lb})\mathbf{k}$ ◀

PROBLEM 4.118 (Continued)

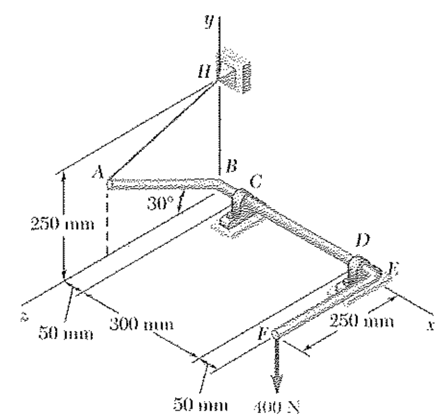
$$\Sigma \mathbf{F} = 0: \quad \mathbf{A} + \mathbf{B} + \mathbf{T} - (75 \text{ lb})\mathbf{j} = 0$$

Coefficient of **i**: $A_x - \frac{30}{38}(118.75) = 0 \quad A_x = 93.75 \text{ lb}$

Coefficient of **j**: $A_y + 15 + \frac{12}{38}(118.75) - 75 = 0 \quad A_y = 22.5 \text{ lb}$

Coefficient of **k**: $A_z - 8.33 - \frac{20}{38}(118.75) = 0 \quad A_z = 70.83 \text{ lb}$

$\mathbf{A} = (93.8 \text{ lb})\mathbf{i} + (22.5 \text{ lb})\mathbf{j} + (70.8 \text{ lb})\mathbf{k}$ ◄



PROBLEM 4.119

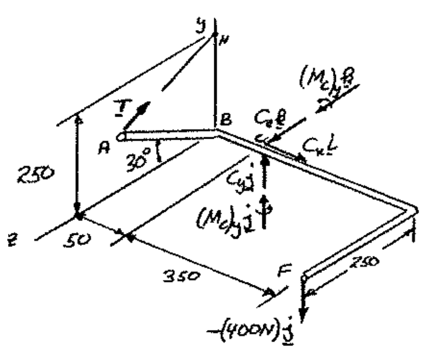
Solve Problem 4.114, assuming that the bearing at D is removed and that the bearing at C can exert couples about axes parallel to the y and z axes.

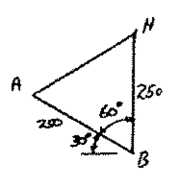
PROBLEM 4.114 The bent rod $ABEF$ is supported by bearings at C and D and by wire AH. Knowing that portion AB of the rod is 250 mm long, determine (a) the tension in wire AH, (b) the reactions at C and D. Assume that the bearing at D does not exert any axial thrust.

SOLUTION

Free-Body Diagram: ΔABH is Equilateral

Dimensions in mm





$$\mathbf{r}_{H/C} = -50\mathbf{i} + 250\mathbf{j}$$

$$\mathbf{r}_{F/C} = 350\mathbf{i} + 250\mathbf{k}$$

$$\mathbf{T} = T(\sin 30°)\mathbf{j} - T(\cos 30°)\mathbf{k} = T(0.5\mathbf{j} - 0.866\mathbf{k})$$

$$\Sigma \mathbf{M}_C = 0: \quad \mathbf{r}_{F/C} \times (-400\mathbf{j}) + \mathbf{r}_{H/C} \times T + (M_C)_y \mathbf{j} + (M_C)_z \mathbf{k} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 350 & 0 & 250 \\ 0 & -400 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -50 & 250 & 0 \\ 0 & 0.5 & -0.866 \end{vmatrix} T + (M_C)_y \mathbf{j} + (M_C)_z \mathbf{k} = 0$$

Coefficient of **i**: $+100 \times 10^3 - 216.5T = 0 \quad T = 461.9 \text{ N}$ $\qquad T = 462 \text{ N}$ ◀

Coefficient of **j**: $-43.3(461.9) + (M_C)_y = 0$

$$(M_C)_y = 20 \times 10^3 \text{ N} \cdot \text{mm}$$

$$(M_C)_y = 20.0 \text{ N} \cdot \text{m}$$

PROBLEM 4.119 (Continued)

Coefficient of **k**: $-140\times10^3 - 25(461.9) + (M_C)_z = 0$

$$(M_C)_z = 151.54\times10^3 \text{ N}\cdot\text{mm}$$
$$(M_C)_z = 151.5 \text{ N}\cdot\text{m}$$

$$\Sigma F = 0: \quad C + T - 400\mathbf{j} = 0$$

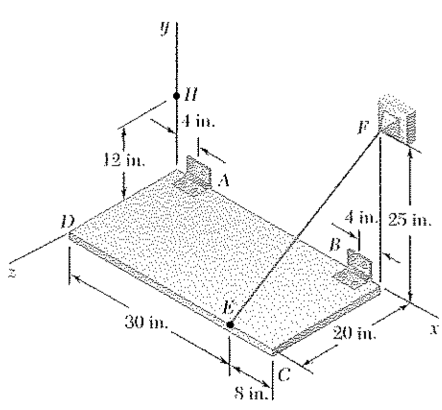
$\mathbf{M}_C = (20.0 \text{ N}\cdot\text{m})\mathbf{j} + (151.5 \text{ N}\cdot\text{m})\mathbf{k}$ ◀

Coefficient of **i**: $C_x = 0$

Coefficient of **j**: $C_y + 0.5(461.9) - 400 = 0 \quad C_y = 169.1 \text{ N}$

Coefficient of **k**: $C_z - 0.866(461.9) = 0 \quad C_z = 400 \text{ N}$

$\mathbf{C} = (169.1 \text{ N})\mathbf{j} + (400 \text{ N})\mathbf{k}$ ◀

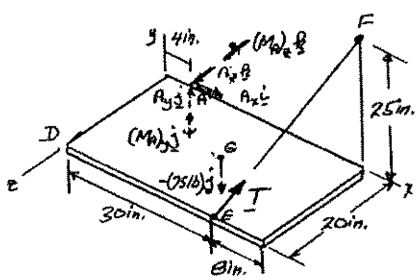


PROBLEM 4.120

Solve Problem 4.117, assuming that the hinge at B is removed and that the hinge at A can exert couples about axes parallel to the y and z axes.

PROBLEM 4.117 The rectangular plate shown weighs 75 lb and is held in the position shown by hinges at A and B and by cable EF. Assuming that the hinge at B does not exert any axial thrust, determine (a) the tension in the cable, (b) the reactions at A and B.

SOLUTION



$$\mathbf{r}_{E/A} = (30-4)\mathbf{i} + 20\mathbf{k} = 26\mathbf{i} + 20\mathbf{k}$$

$$\mathbf{r}_{G/A} = (0.5\times 38)\mathbf{i} + 10\mathbf{k} = 19\mathbf{i} + 10\mathbf{k}$$

$$\overrightarrow{AE} = 8\mathbf{i} + 25\mathbf{j} - 20\mathbf{k}$$

$$AE = 33 \text{ in.}$$

$$T = T\frac{\overrightarrow{AE}}{AE} = \frac{T}{33}(8\mathbf{i} + 25\mathbf{j} - 20\mathbf{k})$$

$$\Sigma \mathbf{M}_A = 0: \quad \mathbf{r}_{E/A} \times \mathbf{T} + \mathbf{r}_{G/A} \times (-75\mathbf{j}) + (M_A)_y \mathbf{j} + (M_A)_z \mathbf{k} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 26 & 0 & 20 \\ 8 & 25 & -20 \end{vmatrix} \frac{T}{33} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 19 & 0 & 10 \\ 0 & -75 & 0 \end{vmatrix} + (M_A)_y \mathbf{j} + (M_A)_z \mathbf{k} = 0$$

Coefficient of **i**: $\quad -(20)(25)\frac{T}{33} + 750 = 0 \qquad T = 49.5 \text{ lb}$ ◀

Coefficient of **j**: $\quad (160 + 520)\frac{49.5}{33} + (M_A)_y = 0 \quad (M_A)_y = -1020 \text{ lb}\cdot\text{in.}$

Coefficient of **k**: $\quad (26)(25)\frac{49.5}{33} - 1425 + (M_A)_z = 0 \quad (M_A)_z = 450 \text{ lb}\cdot\text{in.}$

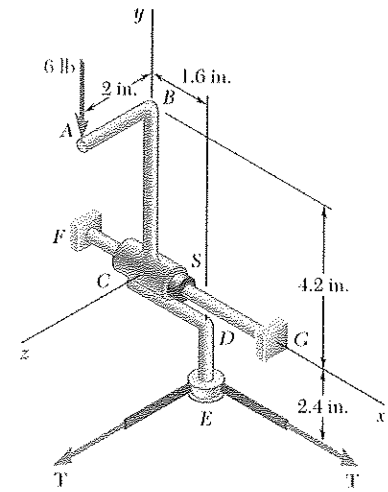
$$\Sigma F = 0: \quad A + T - 75\mathbf{j} = 0 \qquad \mathbf{M}_A = -(1020 \text{ lb}\cdot\text{in})\mathbf{j} + (450 \text{ lb}\cdot\text{in.})\mathbf{k} \text{ ◀}$$

Coefficient of **i**: $\quad A_x + \frac{8}{33}(49.5) = 0 \quad A_x = 12.00 \text{ lb}$

Coefficient of **j**: $\quad A_y + \frac{25}{33}(49.5) - 75 = 0 \quad A_y = 37.5 \text{ lb}$

Coefficient of **k**: $\quad A_z - \frac{20}{33}(49.5) \qquad A_z = 30.0 \text{ lb}$

$$\mathbf{A} = -(12.00 \text{ lb})\mathbf{i} + (37.5 \text{ lb})\mathbf{j} + (30.0 \text{ lb})\mathbf{k} \text{ ◀}$$

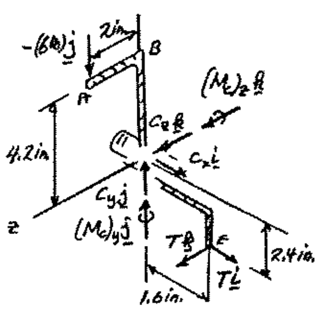


PROBLEM 4.121

The assembly shown is used to control the tension T in a tape that passes around a frictionless spool at E. Collar C is welded to rods ABC and CDE. It can rotate about shaft FG but its motion along the shaft is prevented by a washer S. For the loading shown, determine (*a*) the tension T in the tape, (*b*) the reaction at C.

SOLUTION

Free-Body Diagram:



$$\mathbf{r}_{A/C} = 4.2\mathbf{j} + 2\mathbf{k}$$
$$\mathbf{r}_{E/C} = 1.6\mathbf{i} - 2.4\mathbf{j}$$

$$\Sigma M_C = 0: \quad \mathbf{r}_{A/C} \times (-6\mathbf{j}) + \mathbf{r}_{E/C} \times T(\mathbf{i} + \mathbf{k}) + (M_C)_y \mathbf{j} + (M_C)_z \mathbf{k} = 0$$

$$(4.2\,\mathbf{j} + 2\mathbf{k}) \times (-6\mathbf{j}) + (1.6\mathbf{i} - 2.4\,\mathbf{j}) \times T(\mathbf{i} + \mathbf{k}) + (M_C)_y \mathbf{j} + (M_C)_z \mathbf{k} = 0$$

Coefficient of **i**: $\quad 12 - 2.4T = 0 \qquad T = 5 \text{ lb}$ ◀

Coefficient of **j**: $\quad -1.6(5 \text{ lb}) + (M_C)_y = 0 \quad (M_C)_y = 8 \text{ lb} \cdot \text{in.}$

Coefficient of **k**: $\quad 2.4(5 \text{ lb}) + (M_C)_z = 0 \quad (M_C)_z = -12 \text{ lb} \cdot \text{in.}$

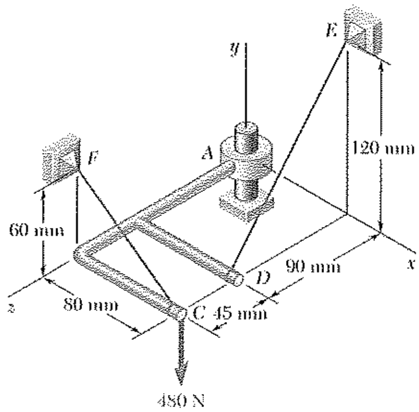
$$\mathbf{M}_C = (8 \text{ lb} \cdot \text{in.})\mathbf{j} - (12 \text{ lb} \cdot \text{in.})\mathbf{k}$$ ◀

$$\Sigma F = 0: \quad C_x\mathbf{i} + C_y\mathbf{j} + C_z\mathbf{k} - (6 \text{ lb})\mathbf{j} + (5 \text{ lb})\mathbf{i} + (5 \text{ lb})\mathbf{k} = 0$$

Equate coefficients of unit vectors to zero.

$C_x = -5 \text{ lb} \quad C_y = 6 \text{ lb} \quad C_z = -5 \text{ lb}$ ◀

$\mathbf{C} = -(5.00 \text{ lb})\mathbf{i} + (6.00 \text{ lb})\mathbf{j} - (5.00 \text{ lb})\mathbf{k}$ ◀

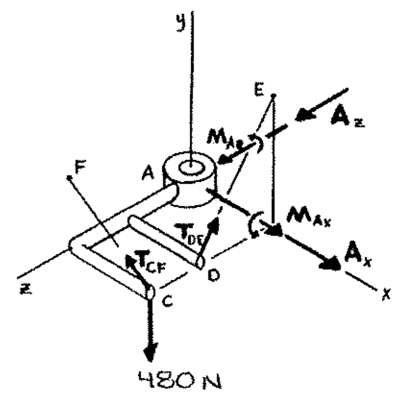


PROBLEM 4.122

The assembly shown is welded to collar A that fits on the vertical pin shown. The pin can exert couples about the x and z axes but does not prevent motion about or along the y axis. For the loading shown, determine the tension in each cable and the reaction at A.

SOLUTION

Free-Body Diagram:



First note

$$\mathbf{T}_{CF} = \boldsymbol{\lambda}_{CF} T_{CF} = \frac{-(0.08\text{ m})\mathbf{i} + (0.06\text{ m})\mathbf{j}}{\sqrt{(0.08)^2 + (0.06)^2}\text{ m}} T_{CF}$$

$$= T_{CF}(-0.8\mathbf{i} + 0.6\mathbf{j})$$

$$\mathbf{T}_{DE} = \boldsymbol{\lambda}_{DE} T_{DE} = \frac{(0.12\text{ m})\mathbf{j} - (0.09\text{ m})\mathbf{k}}{\sqrt{(0.12)^2 + (0.09)^2}\text{ m}} T_{DE}$$

$$= T_{DE}(0.8\mathbf{j} - 0.6\mathbf{k})$$

(*a*) From f.b.d. of assembly

$$\Sigma F_y = 0: \quad 0.6T_{CF} + 0.8T_{DE} - 480\text{ N} = 0$$

or $$0.6T_{CF} + 0.8T_{DE} = 480\text{ N} \qquad (1)$$

$$\Sigma M_y = 0: \quad -(0.8T_{CF})(0.135\text{ m}) + (0.6T_{DE})(0.08\text{ m}) = 0$$

or $$T_{DE} = 2.25T_{CF} \qquad (2)$$

Substituting Equation (2) into Equation (1)

$$0.6T_{CF} + 0.8[(2.25)T_{CF}] = 480\text{ N}$$

$$T_{CF} = 200.00\text{ N}$$

or $$T_{CF} = 200\text{ N} \blacktriangleleft$$

and from Equation (2) $$T_{DE} = 2.25(200.00\text{ N}) = 450.00$$

or $$T_{DE} = 450\text{ N} \blacktriangleleft$$

PROBLEM 4.122 (Continued)

(*b*) From f.b.d. of assembly

$$\Sigma F_z = 0: \quad A_z - (0.6)(450.00 \text{ N}) = 0 \quad A_z = 270.00 \text{ N}$$

$$\Sigma F_x = 0: \quad A_x - (0.8)(200.00 \text{ N}) = 0 \quad A_x = 160.000 \text{ N}$$

or $\mathbf{A} = (160.0 \text{ N})\mathbf{i} + (270 \text{ N})\mathbf{k}$ ◀

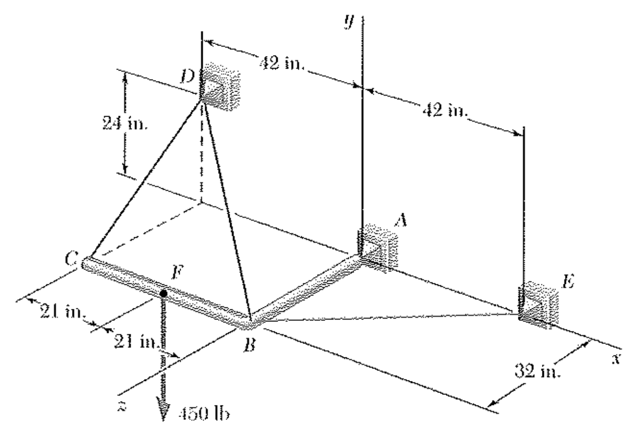
$$\Sigma M_x = 0: \quad M_{A_x} + (480 \text{ N})(0.135 \text{ m}) - [(200.00 \text{ N})(0.6)](0.135 \text{ m}) - [(450 \text{ N})(0.8)](0.09 \text{ m}) = 0$$

$$M_{A_x} = -16.2000 \text{ N}\cdot\text{m}$$

$$\Sigma M_z = 0: \quad M_{A_z} - (480 \text{ N})(0.08 \text{ m}) + [(200.00 \text{ N})(0.6)](0.08 \text{ m}) + [(450 \text{ N})(0.8)](0.08 \text{ m}) = 0$$

$$M_{A_z} = 0$$

or $\mathbf{M}_A = -(16.20 \text{ N}\cdot\text{m})\mathbf{i}$ ◀

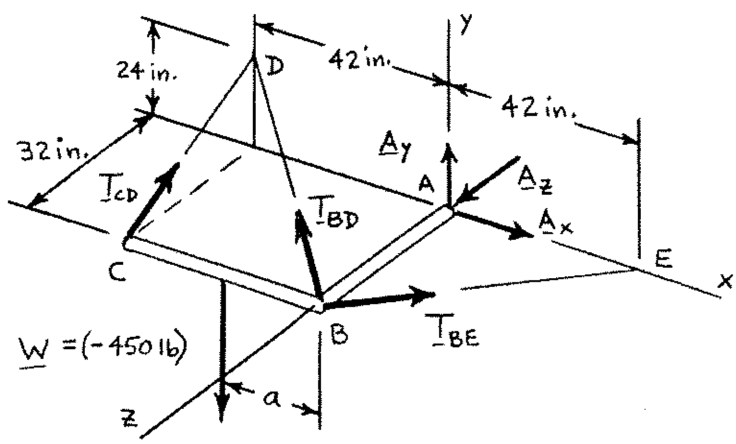


PROBLEM 4.123

The rigid L-shaped member ABC is supported by a ball-and-socket joint at A and by three cables. If a 450-lb load is applied at F, determine the tension in each cable.

SOLUTION

Free-Body Diagram:



In this problem: $a = 21$ in.

We have

$$\overrightarrow{CD} = (24 \text{ in.})\mathbf{j} - (32 \text{ in.})\mathbf{k} \quad CD = 40 \text{ in.}$$

$$\overrightarrow{BD} = -(42 \text{ in.})\mathbf{i} + (24 \text{ in.})\mathbf{j} - (32 \text{ in.})\mathbf{k} \quad BD = 58 \text{ in.}$$

$$\overrightarrow{BE} = (42 \text{ in.})\mathbf{i} - (32 \text{ in.})\mathbf{k} \quad BE = 52.802 \text{ in.}$$

Thus

$$T_{CD} = T_{CD}\frac{\overrightarrow{CD}}{CD} = T_{CD}(0.6\mathbf{j} - 0.8\mathbf{k})$$

$$T_{BD} = T_{BD}\frac{\overrightarrow{BD}}{BD} = T_{BD}(-0.72414\mathbf{i} + 0.41379\mathbf{j} - 0.55172\mathbf{k})$$

$$T_{BE} = T_{BE}\frac{\overrightarrow{BE}}{BE} = T_{BE}(0.79542\mathbf{i} - 0.60604\mathbf{k})$$

$$\Sigma\mathbf{M}_A = 0: \quad (\mathbf{r}_C \times \mathbf{T}_{CD}) + (\mathbf{r}_B \times \mathbf{T}_{BD}) + (\mathbf{r}_B \times \mathbf{T}_{BE}) + (\mathbf{r}_W \times \mathbf{W}) = 0$$

Noting that

$$\mathbf{r}_C = -(42\text{ in.})\mathbf{i} + (32\text{ in.})\mathbf{k}$$
$$\mathbf{r}_B = (32\text{ in.})\mathbf{k}$$
$$\mathbf{r}_W = -a\mathbf{i} + (32\text{ in.})\mathbf{k}$$

and using determinants, we write

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -42 & 0 & 32 \\ 0 & 0.6 & -0.8 \end{vmatrix} T_{CD} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 32 \\ -0.72414 & 0.41379 & -0.55172 \end{vmatrix} T_{BD}$$
$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 32 \\ 0.79542 & 0 & -0.60604 \end{vmatrix} T_{BE} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -a & 0 & 32 \\ 0 & -450 & 0 \end{vmatrix} = 0$$

Equating to zero the coefficients of the unit vectors:

$$\mathbf{i}: \quad -19.2T_{CD} - 13.241T_{BD} + 14400 = 0 \qquad (1)$$

$$\mathbf{j}: \quad -33.6T_{CD} - 23.172T_{BD} + 25.453T_{BE} = 0 \qquad (2)$$

$$\mathbf{k}: \quad -25.2T_{CD} + 450a = 0 \qquad (3)$$

Recalling that $a = 21$ in., Eq. (3) yields

$$T_{CD} = \frac{450(21)}{25.2} = 375\text{ lb}$$

$T_{CD} = 375$ lb ◀

From (1):

$$-19.2(375) - 13.241T_{BD} + 14400 = 0$$

$$T_{BD} = 543.77\text{ lb}$$

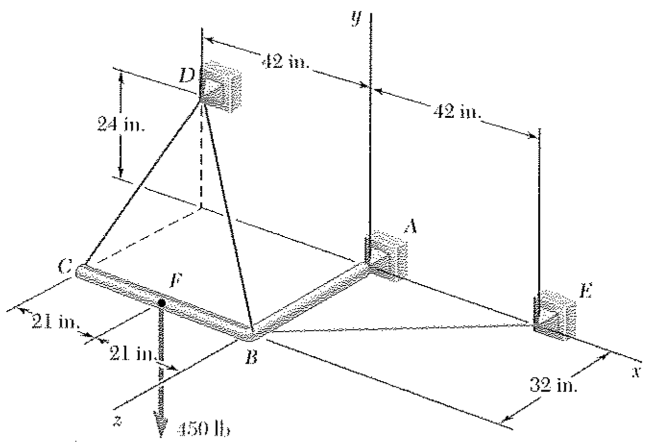
$T_{BD} = 544$ lb ◀

From (2):

$$-33.6(375) - 23.172(543.77) + 25.453T_{BE} = 0$$

$$T_{BE} = 990.07\text{ lb}$$

$T_{BE} = 990$ lb ◀



PROBLEM 4.124

Solve Problem 4.123, assuming that the 450-lb load is applied at C.

PROBLEM 4.123 The rigid L-shaped member ABC is supported by a ball-and-socket joint at A and by three cables. If a 450-lb load is applied at F, determine the tension in each cable.

SOLUTION

See solution of Problem 4.123 for free-body diagram and derivation of Eqs. (1), (2), and (3):

$$-19.2T_{CD} - 13.241T_{BD} + 14400 = 0 \quad (1)$$

$$-33.6T_{CD} - 23.172T_{BD} + 25.453T_{BE} = 0 \quad (2)$$

$$-25.2T_{CD} + 450a = 0 \quad (3)$$

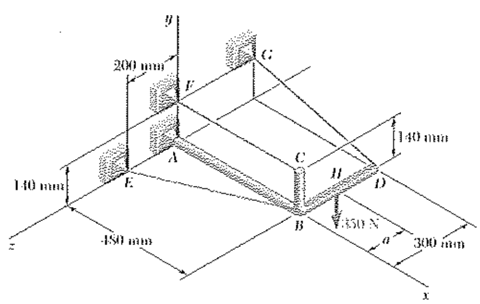
In this problem, the 450-lb load is applied at C and we have $a = 42$ in. Carrying into (3) and solving for T_{CD},

$$T_{CD} = 750 \text{ lb}$$

$T_{CD} = 750$ lb ◀

From (1): $-19.2(750) - 13.241T_{BD} + 14400 = 0$ $T_{BD} = 0$ ◀

From (2): $-33.6(750) - 0 + 25.453T_{BE} = 0$ $T_{BE} = 990$ lb ◀



PROBLEM 4.125

Frame $ABCD$ is supported by a ball-and-socket joint at A and by three cables. For $a = 150$ mm, determine the tension in each cable and the reaction at A.

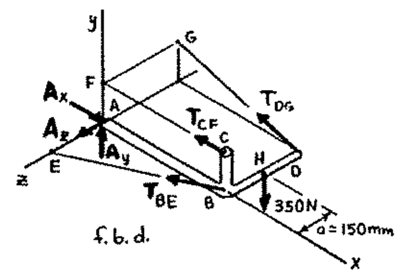
SOLUTION

First note

$$\mathbf{T}_{DG} = \boldsymbol{\lambda}_{DG} T_{DG} = \frac{-(0.48 \text{ m})\mathbf{i} + (0.14 \text{ m})\mathbf{j}}{\sqrt{(0.48)^2 + (0.14)^2} \text{ m}} T_{DG}$$

$$= \frac{-0.48\mathbf{i} + 0.14\mathbf{j}}{0.50} T_{DG}$$

$$= \frac{T_{DG}}{25}(24\mathbf{i} + 7\mathbf{j})$$



$$\mathbf{T}_{BE} = \boldsymbol{\lambda}_{BE} T_{BE} = \frac{-(0.48 \text{ m})\mathbf{i} + (0.2 \text{ m})\mathbf{k}}{\sqrt{(0.48)^2 + (0.2)^2} \text{ m}} T_{BE}$$

$$= \frac{-0.48\mathbf{i} + 0.2\mathbf{k}}{0.52} T_{BE}$$

$$= \frac{T_{BE}}{13}(-12\mathbf{j} + 5\mathbf{k})$$

From f.b.d. of frame $ABCD$

$$\Sigma M_x = 0: \quad \left(\frac{7}{25} T_{DG}\right)(0.3 \text{ m}) - (350 \text{ N})(0.15 \text{ m}) = 0$$

or $T_{DG} = 625 \text{ N}$ ◀

$$\Sigma M_y = 0: \quad \left(\frac{24}{25} \times 625 \text{ N}\right)(0.3 \text{ m}) - \left(\frac{5}{13} T_{BE}\right)(0.48 \text{ m}) = 0$$

or $T_{BE} = 975 \text{ N}$ ◀

$$\Sigma M_z = 0: \quad T_{CF}(0.14 \text{ m}) + \left(\frac{7}{25} \times 625 \text{ N}\right)(0.48 \text{ m}) - (350 \text{ N})(0.48 \text{ m}) = 0$$

or $T_{CF} = 600 \text{ N}$ ◀

PROBLEM 4.125 (Continued)

$\Sigma F_x = 0\colon\quad A_x + T_{CF} + (T_{BE})_x + (T_{DG})_x = 0$

$$A_x - 600\text{ N} - \left(\frac{12}{13}\times 975\text{ N}\right) - \left(\frac{24}{25}\times 625\text{ N}\right) = 0$$

$$A_x = 2100\text{ N}$$

$\Sigma F_y = 0\colon\quad A_y + (T_{DG})_y - 350\text{ N} = 0$

$$A_y + \left(\frac{7}{25}\times 625\text{ N}\right) - 350\text{ N} = 0$$

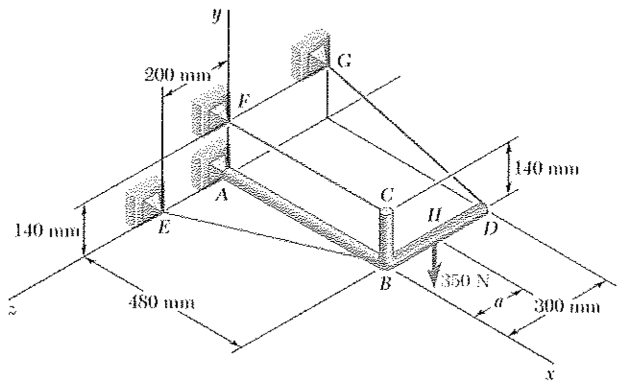
$$A_y = 175.0\text{ N}$$

$\Sigma F_z = 0\colon\quad A_z + (T_{BE})_z = 0$

$$A_z + \left(\frac{5}{13}\times 975\text{ N}\right) = 0$$

$$A_z = -375\text{ N}$$

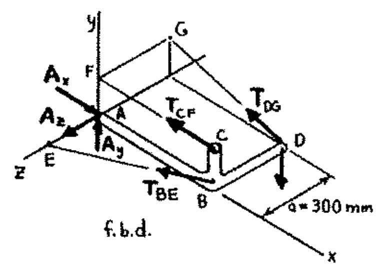
Therefore $\mathbf{A} = (2100\text{ N})\mathbf{i} + (175.0\text{ N})\mathbf{j} - (375\text{ N})\mathbf{k}$ ◀



PROBLEM 4.126

Frame *ABCD* is supported by a ball-and-socket joint at *A* and by three cables. Knowing that the 350-N load is applied at *D* ($a = 300$ mm), determine the tension in each cable and the reaction at *A*.

SOLUTION



f.b.d.

First note

$$\mathbf{T}_{DG} = \boldsymbol{\lambda}_{DG} T_{DG} = \frac{-(0.48\text{ m})\mathbf{i} + (0.14\text{ m})\mathbf{j}}{\sqrt{(0.48)^2 + (0.14)^2}\text{ m}} T_{DG}$$

$$= \frac{-0.48\mathbf{i} + 0.14\mathbf{j}}{0.50} T_{DG}$$

$$= \frac{T_{DG}}{25}(24\mathbf{i} + 7\mathbf{j})$$

$$\mathbf{T}_{BE} = \boldsymbol{\lambda}_{BE} T_{BE} = \frac{-(0.48\text{ m})\mathbf{i} + (0.2\text{ m})\mathbf{k}}{\sqrt{(0.48)^2 + (0.2)^2}\text{ m}} T_{BE}$$

$$= \frac{-0.48\mathbf{i} + 0.2\mathbf{k}}{0.52} T_{BE}$$

$$= \frac{T_{BE}}{13}(-12\mathbf{i} + 5\mathbf{k})$$

From f.b.d. of frame *ABCD*

$$\Sigma M_x = 0: \quad \left(\frac{7}{25} T_{DG}\right)(0.3\text{ m}) - (350\text{ N})(0.3\text{ m}) = 0$$

or $\qquad T_{DG} = 1250\text{ N}$ ◀

$$\Sigma M_y = 0: \quad \left(\frac{24}{25} \times 1250\text{ N}\right)(0.3\text{ m}) - \left(\frac{5}{13} T_{BE}\right)(0.48\text{ m}) = 0$$

or $\qquad T_{BE} = 1950\text{ N}$ ◀

$$\Sigma M_z = 0: \quad T_{CF}(0.14\text{ m}) + \left(\frac{7}{25} \times 1250\text{ N}\right)(0.48\text{ m}) - (350\text{ N})(0.48\text{ m}) = 0$$

or $\qquad T_{CF} = 0$ ◀

PROBLEM 4.126 (Continued)

$$\Sigma F_x = 0: \quad A_x + T_{CF} + (T_{BE})_x + (T_{DG})_x = 0$$

$$A_x + 0 - \left(\frac{12}{13} \times 1950 \text{ N}\right) - \left(\frac{24}{25} \times 1250 \text{ N}\right) = 0$$

$$A_x = 3000 \text{ N}$$

$$\Sigma F_y = 0: \quad A_y + (T_{DG})_y - 350 \text{ N} = 0$$

$$A_y + \left(\frac{7}{25} \times 1250 \text{ N}\right) - 350 \text{ N} = 0$$

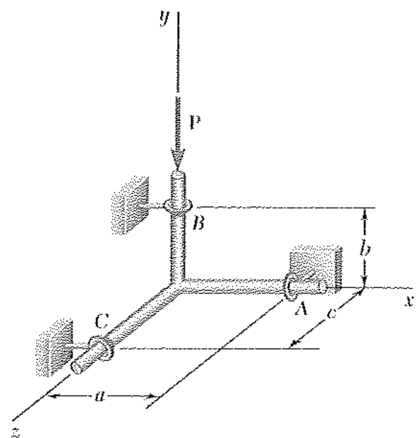
$$A_y = 0$$

$$\Sigma F_z = 0: \quad A_z + (T_{BE})_z = 0$$

$$A_z + \left(\frac{5}{13} \times 1950 \text{ N}\right) = 0$$

$$A_z = -750 \text{ N}$$

Therefore $\mathbf{A} = (3000 \text{ N})\mathbf{i} - (750 \text{ N})\mathbf{k}$ ◀

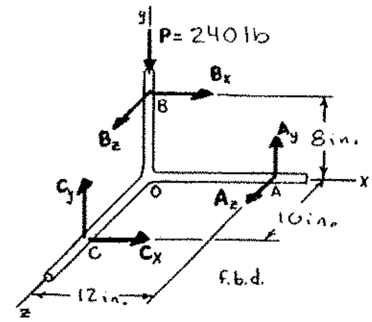


PROBLEM 4.127

Three rods are welded together to form a "corner" that is supported by three eyebolts. Neglecting friction, determine the reactions at A, B, and C when $P = 240$ lb, $a = 12$ in., $b = 8$ in., and $c = 10$ in.

SOLUTION

From f.b.d. of weldment



$$\Sigma \mathbf{M}_O = 0: \quad \mathbf{r}_{A/O} \times \mathbf{A} + \mathbf{r}_{B/O} \times \mathbf{B} + \mathbf{r}_{C/O} \times \mathbf{C} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & 0 \\ 0 & A_y & A_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 8 & 0 \\ B_x & 0 & B_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 10 \\ C_x & C_y & 0 \end{vmatrix} = 0$$

$$(-12A_z\mathbf{j} + 12A_y\mathbf{k}) + (8B_z\mathbf{i} - 8B_x\mathbf{k}) + (-10C_y\mathbf{i} + 10C_x\mathbf{j}) = 0$$

From **i**-coefficient $\quad 8B_z - 10C_y = 0$

or $\quad B_z = 1.25C_y \quad (1)$

j-coefficient $\quad -12A_z + 10C_x = 0$

or $\quad C_x = 1.2A_z \quad (2)$

k-coefficient $\quad 12A_y - 8B_x = 0$

or $\quad B_x = 1.5A_y \quad (3)$

$$\Sigma \mathbf{F} = 0: \quad \mathbf{A} + \mathbf{B} + \mathbf{C} - \mathbf{P} = 0$$

or $\quad (B_x + C_x)\mathbf{i} + (A_y + C_y - 240 \text{ lb})\mathbf{j} + (A_z + B_z)\mathbf{k} = 0$

From **i**-coefficient $\quad B_x + C_x = 0$

or $\quad C_x = -B_x \quad (4)$

j-coefficient $\quad A_y + C_y - 240 \text{ lb} = 0$

or $\quad A_y + C_y = 240 \text{ lb} \quad (5)$

PROBLEM 4.127 (Continued)

k-coefficient $A_z + B_z = 0$

or $$A_z = -B_z \quad (6)$$

Substituting C_x from Equation (4) into Equation (2)

$$-B_z = 1.2A_z \quad (7)$$

Using Equations (1), (6), and (7)

$$C_y = \frac{B_z}{1.25} = \frac{-A_z}{1.25} = \frac{1}{1.25}\left(\frac{B_x}{1.2}\right) = \frac{B_x}{1.5} \quad (8)$$

From Equations (3) and (8)

$$C_y = \frac{1.5A_y}{1.5} \quad \text{or} \quad C_y = A_y$$

and substituting into Equation (5)

$$2A_y = 240 \text{ lb}$$

$$A_y = C_y = 120 \text{ lb} \quad (9)$$

Using Equation (1) and Equation (9)

$$B_z = 1.25(120 \text{ lb}) = 150.0 \text{ lb}$$

Using Equation (3) and Equation (9)

$$B_x = 1.5(120 \text{ lb}) = 180.0 \text{ lb}$$

From Equation (4) $C_x = -180.0 \text{ lb}$

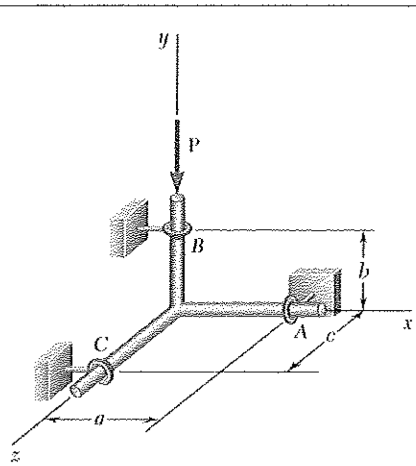
From Equation (6) $A_z = -150.0 \text{ lb}$

Therefore $\mathbf{A} = (120.0 \text{ lb})\mathbf{j} - (150.0 \text{ lb})\mathbf{k}$ ◀

$\mathbf{B} = (180.0 \text{ lb})\mathbf{i} + (150.0 \text{ lb})\mathbf{k}$ ◀

$\mathbf{C} = -(180.0 \text{ lb})\mathbf{i} + (120.0 \text{ lb})\mathbf{j}$ ◀

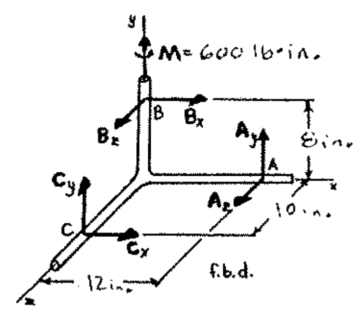
PROBLEM 4.128



Solve Problem 4.127, assuming that the force **P** is removed and is replaced by a couple $\mathbf{M} = +(600 \text{ lb} \cdot \text{in.})\mathbf{j}$ acting at B.

PROBLEM 4.127 Three rods are welded together to form a "corner" that is supported by three eyebolts. Neglecting friction, determine the reactions at A, B, and C when $P = 240$ lb, $a = 12$ in., $b = 8$ in., and $c = 10$ in.

SOLUTION



From f.b.d. of weldment

$$\Sigma \mathbf{M}_O = 0: \quad \mathbf{r}_{A/O} \times \mathbf{A} + \mathbf{r}_{B/O} \times \mathbf{B} + \mathbf{r}_{C/O} \times \mathbf{C} + \mathbf{M} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & 0 \\ 0 & A_y & A_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 8 & 0 \\ B_x & 0 & B_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 10 \\ C_x & C_y & 0 \end{vmatrix} + (600 \text{ lb} \cdot \text{in.})\mathbf{j} = 0$$

$$(-12A_z\mathbf{j} + 12A_y\mathbf{k}) + (8B_z\mathbf{j} - 8B_x\mathbf{k}) + (-10C_y\mathbf{i} + 10C_x\mathbf{j}) + (600 \text{ lb} \cdot \text{in.})\mathbf{j} = 0$$

From **i**-coefficient $\quad 8B_z - 10C_y = 0$

or $\quad C_y = 0.8B_z \quad (1)$

j-coefficient $\quad -12A_z + 10C_x + 600 = 0$

or $\quad C_x = 1.2A_z - 60 \quad (2)$

k-coefficient $\quad 12A_y - 8B_x = 0$

or $\quad B_x = 1.5A_y \quad (3)$

$$\Sigma \mathbf{F} = 0: \quad \mathbf{A} + \mathbf{B} + \mathbf{C} = 0$$

$$(B_x + C_x)\mathbf{i} + (A_y + C_y)\mathbf{j} + (A_z + B_z)\mathbf{k} = 0$$

From **i**-coefficient $\quad C_x = -B_x \quad (4)$

j-coefficient $\quad C_y = -A_y \quad (5)$

k-coefficient $\quad A_z = -B_z \quad (6)$

PROBLEM 4.128 (Continued)

Substituting C_x from Equation (4) into Equation (2)

$$A_z = 50 - \left(\frac{B_x}{1.2}\right) \quad (7)$$

Using Equations (1), (6), and (7)

$$C_y = 0.8B_z = -0.8A_z = \left(\frac{2}{3}\right)B_x - 40 \quad (8)$$

From Equations (3) and (8)

$$C_y = A_y - 40$$

Substituting into Equation (5) $\quad 2A_y = 40$

$$A_y = 20.0 \text{ lb}$$

From Equation (5) $\quad C_y = -20.0 \text{ lb}$

Equation (1) $\quad B_z = -25.0 \text{ lb}$

Equation (3) $\quad B_x = 30.0 \text{ lb}$

Equation (4) $\quad C_x = -30.0 \text{ lb}$

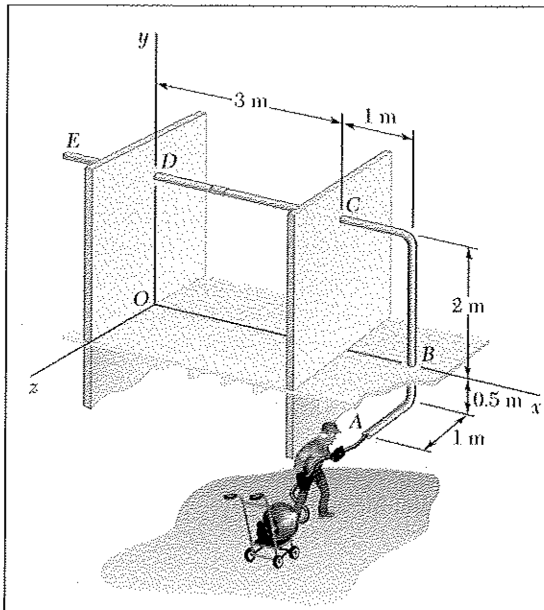
Equation (6) $\quad A_z = 25.0 \text{ lb}$

Therefore

$\mathbf{A} = (20.0 \text{ lb})\mathbf{j} + (25.0 \text{ lb})\mathbf{k}$ ◀

$\mathbf{B} = (30.0 \text{ lb})\mathbf{i} - (25.0 \text{ lb})\mathbf{k}$ ◀

$\mathbf{C} = -(30.0 \text{ lb})\mathbf{i} - (20.0 \text{ lb})\mathbf{j}$ ◀

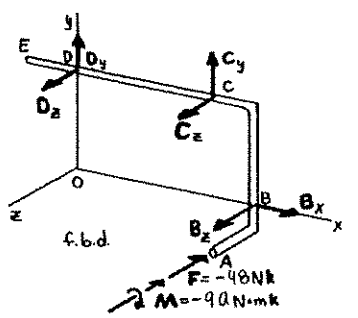


PROBLEM 4.129

In order to clean the clogged drainpipe AE, a plumber has disconnected both ends of the pipe and inserted a power snake through the opening at A. The cutting head of the snake is connected by a heavy cable to an electric motor that rotates at a constant speed as the plumber forces the cable into the pipe. The forces exerted by the plumber and the motor on the end of the cable can be represented by the wrench $\mathbf{F} = -(48\text{ N})\mathbf{k}, \mathbf{M} = -(90\text{ N}\cdot\text{m})\mathbf{k}$. Determine the additional reactions at B, C, and D caused by the cleaning operation. Assume that the reaction at each support consists of two force components perpendicular to the pipe.

SOLUTION

From f.b.d. of pipe assembly $ABCD$



$$\Sigma F_x = 0: \quad B_x = 0$$

$$\Sigma M_{D(x\text{-axis})} = 0: \quad (48\text{ N})(2.5\text{ m}) - B_z(2\text{ m}) = 0$$

$$B_z = 60.0\text{ N}$$

and $\mathbf{B} = (60.0\text{ N})\mathbf{k}$ ◀

$$\Sigma M_{D(z\text{-axis})} = 0: \quad C_y(3\text{ m}) - 90\text{ N}\cdot\text{m} = 0$$

$$C_y = 30.0\text{ N}$$

$$\Sigma M_{D(y\text{-axis})} = 0: \quad -C_z(3\text{ m}) - (60.0\text{ N})(4\text{ m}) + (48\text{ N})(4\text{ m}) = 0$$

$$C_z = -16.00\text{ N}$$

and $\mathbf{C} = (30.0\text{ N})\mathbf{j} - (16.00\text{ N})\mathbf{k}$ ◀

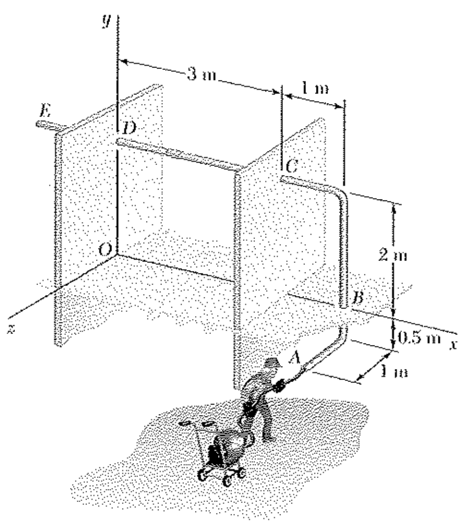
$$\Sigma F_y = 0: \quad D_y + 30.0 = 0$$

$$D_y = -30.0\text{ N}$$

$$\Sigma F_z = 0: \quad D_z - 16.00\text{ N} + 60.0\text{ N} - 48\text{ N} = 0$$

$$D_z = 4.00\text{ N}$$

and $\mathbf{D} = -(30.0\text{ N})\mathbf{j} + (4.00\text{ N})\mathbf{k}$ ◀



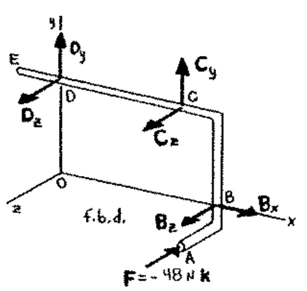
PROBLEM 4.130

Solve Problem 4.129, assuming that the plumber exerts a force $\mathbf{F} = -(48\text{ N})\mathbf{k}$ and that the motor is turned off ($\mathbf{M} = 0$).

PROBLEM 4.129 In order to clean the clogged drainpipe *AE*, a plumber has disconnected both ends of the pipe and inserted a power snake through the opening at *A*. The cutting head of the snake is connected by a heavy cable to an electric motor that rotates at a constant speed as the plumber forces the cable into the pipe. The forces exerted by the plumber and the motor on the end of the cable can be represented by the wrench $\mathbf{F} = -(48\text{ N})\mathbf{k}, \mathbf{M} = -(90\text{ N}\cdot\text{m})\mathbf{k}$. Determine the additional reactions at *B*, *C*, and *D* caused by the cleaning operation. Assume that the reaction at each support consists of two force components perpendicular to the pipe.

SOLUTION

From f.b.d. of pipe assembly *ABCD*



$$\Sigma F_x = 0: \quad B_x = 0$$

$$\Sigma M_{D(x\text{-axis})} = 0: \quad (48\text{ N})(2.5\text{ m}) - B_z(2\text{ m}) = 0$$

$$B_z = 60.0\text{ N} \qquad \text{and } \mathbf{B} = (60.0\text{ N})\mathbf{k} \blacktriangleleft$$

$$\Sigma M_{D(z\text{-axis})} = 0: \quad C_y(3\text{ m}) - B_x(2\text{ m}) = 0$$

$$C_y = 0$$

$$\Sigma M_{D(y\text{-axis})} = 0: \quad C_z(3\text{ m}) - (60.0\text{ N})(4\text{ m}) + (48\text{ N})(4\text{ m}) = 0$$

$$C_z = -16.00\text{ N} \qquad \text{and } \mathbf{C} = -(16.00\text{ N})\mathbf{k} \blacktriangleleft$$

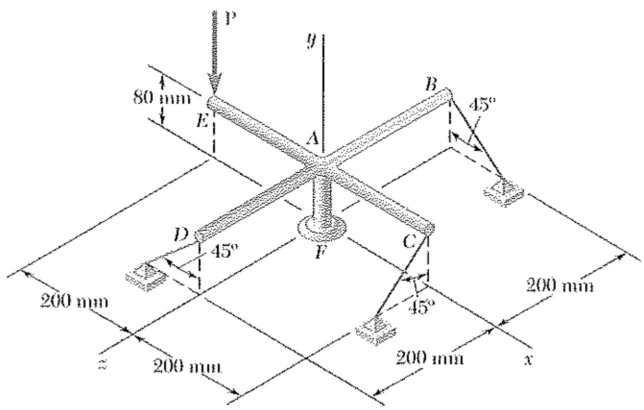
$$\Sigma F_y = 0: \quad D_y + C_y = 0$$

$$D_y = 0$$

$$\Sigma F_z = 0: \quad D_z + B_z + C_z - F = 0$$

$$D_z + 60.0\text{ N} - 16.00\text{ N} - 48\text{ N} = 0$$

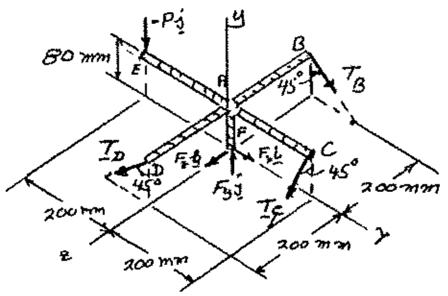
$$D_z = 4.00\text{ N} \qquad \text{and } \mathbf{D} = (4.00\text{ N})\mathbf{k} \blacktriangleleft$$



PROBLEM 4.131

The assembly shown consists of an 80-mm rod AF that is welded to a cross consisting of four 200-mm arms. The assembly is supported by a ball-and-socket joint at F and by three short links, each of which forms an angle of 45° with the vertical. For the loading shown, determine (a) the tension in each link, (b) the reaction at F.

SOLUTION



$$\mathbf{r}_{E/F} = -200\,\mathbf{i} + 80\mathbf{j}$$

$$\mathbf{T}_B = T_B(\mathbf{i} - \mathbf{j})/\sqrt{2} \qquad \mathbf{r}_{B/F} = 80\mathbf{j} - 200\mathbf{k}$$

$$\mathbf{T}_C = T_C(-\mathbf{j} + \mathbf{k})/\sqrt{2} \qquad \mathbf{r}_{C/F} = 200\mathbf{i} + 80\mathbf{j}$$

$$\mathbf{T}_D = T_D(-\mathbf{i} + \mathbf{j})/\sqrt{2} \qquad \mathbf{r}_{D/E} = 80\mathbf{j} + 200\mathbf{k}$$

$$\Sigma M_F = 0: \quad \mathbf{r}_{B/F} \times \mathbf{T}_B + \mathbf{r}_{C/F} \times \mathbf{T}_C + \mathbf{r}_{D/F} \times T_D + \mathbf{r}_{E/F} \times (-P\mathbf{j}) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 80 & -200 \\ 1 & -1 & 0 \end{vmatrix} \frac{T_B}{\sqrt{2}} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 200 & 80 & 0 \\ 0 & -1 & 1 \end{vmatrix} \frac{T_c}{\sqrt{2}} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 80 & 200 \\ -1 & -1 & 0 \end{vmatrix} \frac{T_D}{\sqrt{2}} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -200 & 80 & 0 \\ 0 & -P & 0 \end{vmatrix} = 0$$

Equate coefficients of unit vectors to zero and multiply each equation by $\sqrt{2}$.

$$\mathbf{i}: \quad -200T_B + 80T_C + 200\,T_D = 0 \tag{1}$$

$$\mathbf{j}: \quad -200T_B - 200T_C - 200T_D = 0 \tag{2}$$

$$\mathbf{k}: \quad -80T_B - 200T_C + 80T_D + 200\sqrt{2}P = 0 \tag{3}$$

$$\frac{80}{200}(2): \quad -80\,T_B - 80\,T_C - 80\,T_D = 0 \tag{4}$$

Eqs. (3) + (4): $\qquad -160T_B - 280T_C + 200\sqrt{2}P = 0 \qquad (5)$

Eqs. (1) + (2): $\qquad -400T_B - 120T_C = 0$

$$T_B = -\frac{120}{400}T_C - 0.3T_C \tag{6}$$

PROBLEM 4.131 (Continued)

Eqs. (6) ⟶ (5): $-160(-0.3T_C) - 280T_C + 200\sqrt{2}P = 0$

$$-232T_C + 200\sqrt{2}P = 0$$

$$T_C = 1.2191P \qquad T_C = 1.219P \blacktriangleleft$$

From Eq. (6): $T_B = -0.3(1.2191P) = -0.36574 = P \qquad T_B = -0.366P \blacktriangleleft$

From Eq. (2): $-200(-0.3657P) - 200(1.2191P) - 200T_{0D} = 0$

$$T_D = -0.8534P \qquad T_D = -0.853P \blacktriangleleft$$

$\Sigma\mathbf{F} = 0$: $\mathbf{F} + \mathbf{T}_B + \mathbf{T}_C + \mathbf{T}_D - P\mathbf{j} = 0$

$\mathbf{i}$: $F_x + \dfrac{(-0.36574P)}{\sqrt{2}} - \dfrac{(-0.8534P)}{\sqrt{2}} = 0$

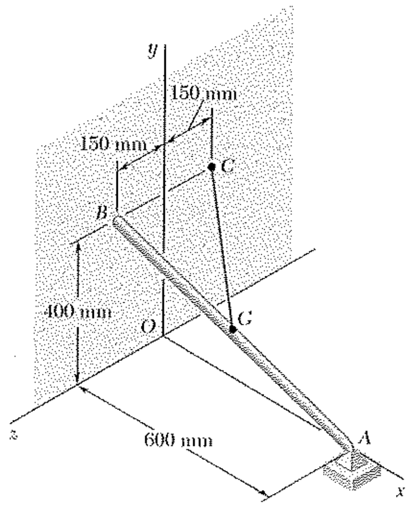
$F_x = -0.3448P \qquad F_x = -0.345P$

$\mathbf{j}$: $F_y - \dfrac{(-0.36574P)}{\sqrt{2}} - \dfrac{(1.2191P)}{\sqrt{2}} - \dfrac{(-0.8534P)}{\sqrt{2}} - 200 = 0$

$F_y = P \qquad F_y = P$

$\mathbf{k}$: $F_z + \dfrac{(1.2191P)}{\sqrt{2}} = 0$

$F_z = -0.8620P \quad F_z = -0.862P \qquad \mathbf{F} = -0.345P\mathbf{i} + P\mathbf{j} - 0.862P\mathbf{k} \blacktriangleleft$

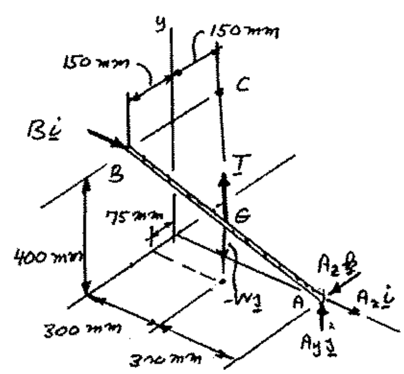


PROBLEM 4.132

The uniform 10-kg rod AB is supported by a ball-and-socket joint at A and by the cord CG that is attached to the midpoint G of the rod. Knowing that the rod leans against a frictionless vertical wall at B, determine (*a*) the tension in the cord, (*b*) the reactions at A and B.

SOLUTION

Free-Body Diagram:



Five unknowns and six Eqs. of equilibrium. But equilibrium is maintained $(\Sigma M_{AB} = 0)$

$$W = mg$$
$$= (10\text{ kg})9.81\text{ m/s}^2$$
$$W = 98.1\text{ N}$$

$$\overrightarrow{GC} = -300\mathbf{i} + 200\mathbf{j} - 225\mathbf{k} \quad GC = 425\text{ mm}$$

$$\mathbf{T} = T\frac{\overrightarrow{GC}}{GC} = \frac{T}{425}(-300\mathbf{i} + 200\mathbf{j} - 225\mathbf{k})$$

$$\mathbf{r}_{B/A} = -600\mathbf{i} + 400\mathbf{j} + 150\text{ mm}$$

$$\mathbf{r}_{G/A} = -300\mathbf{i} + 200\mathbf{j} + 75\text{ mm}$$

$$\Sigma M_A = 0: \quad \mathbf{r}_{B/A} \times \mathbf{B} + \mathbf{r}_{G/A} \times \mathbf{T} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -600 & 400 & 150 \\ B & 0 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -300 & 200 & 75 \\ -300 & 200 & -225 \end{vmatrix} \frac{T}{425} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -300 & 200 & 75 \\ 0 & -98.1 & 0 \end{vmatrix}$$

Coefficient of $\mathbf{i}$: $(-105.88 - 35.29)T + 7357.5 = 0$

$$T = 52.12\text{ N}$$

$T = 52.1\text{ N}$ ◀

PROBLEM 4.132 (Continued)

Coefficient of $\mathbf{j}$: $150B-(300\times 75+300\times 225)\dfrac{52.12}{425}=0$

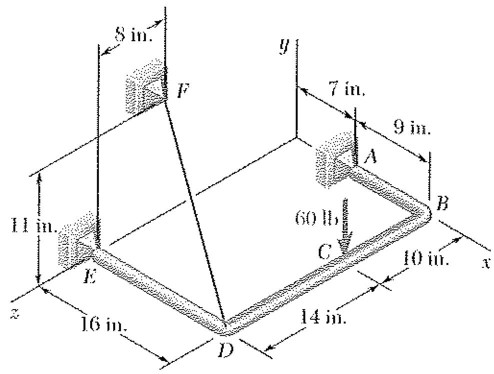
$B=73.58$ N $\qquad \mathbf{B}=(73.6\text{ N})\mathbf{i}$ ◀

$\Sigma\mathbf{F}=0$: $\mathbf{A}+\mathbf{B}+\mathbf{T}-W\mathbf{j}=0$

Coefficient of $\mathbf{i}$: $A_x+73.58-52.15\dfrac{300}{425}=0$ $\qquad A_x=36.8$ N ◀

Coefficient of $\mathbf{j}$: $A_y+52.15\dfrac{200}{425}-98.1=0$ $\qquad A_y=73.6$ N ◀

Coefficient of $\mathbf{k}$: $A_z-52.15\dfrac{225}{425}=0$ $\qquad A_z=27.6$ N ◀

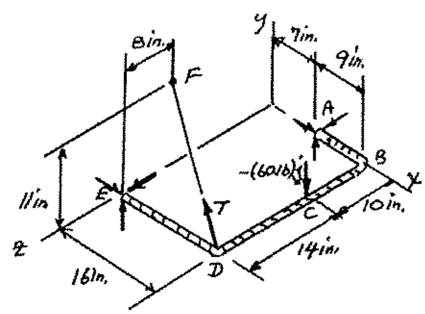


PROBLEM 4.133

The bent rod *ABDE* is supported by ball-and-socket joints at *A* and *E* and by the cable *DF*. If a 60-lb load is applied at *C* as shown, determine the tension in the cable.

SOLUTION

Free-Body Diagram:



$$\overrightarrow{DF} = -16\mathbf{i} + 11\mathbf{j} - 8\mathbf{k} \qquad DF = 21 \text{ in.}$$

$$\mathbf{T} = T\frac{\overrightarrow{DE}}{DF} = \frac{T}{21}(-16\mathbf{i} + 11\mathbf{j} - 8\mathbf{k})$$

$$\mathbf{r}_{D/E} = 16\,\mathbf{i}$$

$$\mathbf{r}_{C/E} = 16\mathbf{i} - 14\mathbf{k}$$

$$\boldsymbol{\lambda}_{EA} = \frac{\overrightarrow{EA}}{EA} = \frac{7\mathbf{i} - 24\mathbf{k}}{25}$$

$$\Sigma M_{EA} = 0: \quad \boldsymbol{\lambda}_{EA} \cdot (\mathbf{r}_{B/E} \times \mathbf{T}) + \boldsymbol{\lambda}_{EA} \cdot (\mathbf{r}_{C/E} \cdot (-60\mathbf{j})) = 0$$

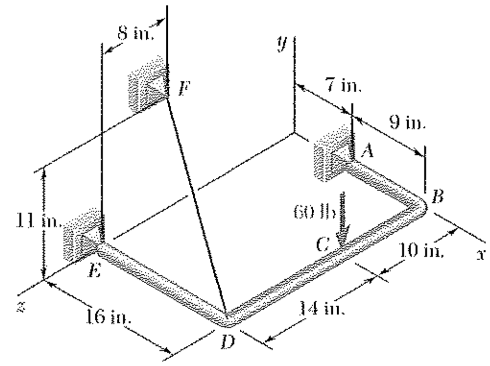
$$\begin{vmatrix} 7 & 0 & -24 \\ 16 & 0 & 0 \\ -16 & 11 & -8 \end{vmatrix} \frac{T}{21 \times 25} + \begin{vmatrix} 7 & 0 & -24 \\ 16 & 0 & -14 \\ 0 & -60 & 0 \end{vmatrix} \frac{1}{25} = 0$$

$$-\frac{24 \times 16 \times 11}{21 \times 25}T + \frac{-7 \times 14 \times 60 + 24 \times 16 \times 60}{25} = 0$$

$$201.14T + 17{,}160 = 0$$

$$T = 85.314 \text{ lb}$$

$T = 85.3$ lb ◀

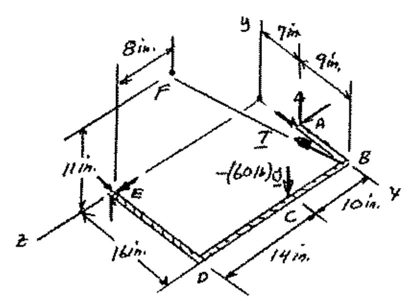


PROBLEM 4.134

Solve Problem 4.133, assuming that cable DF is replaced by a cable connecting B and F.

SOLUTION

Free-Body Diagram:



$$\mathbf{r}_{B/A} = 9\mathbf{i}$$

$$\mathbf{r}_{C/A} = 9\mathbf{i} + 10\mathbf{k}$$

$$\overrightarrow{BF} = -16\mathbf{i} + 11\mathbf{j} + 16\mathbf{k} \qquad BF = 25.16 \text{ in.}$$

$$\mathbf{T} = T\frac{\overrightarrow{BF}}{BF} = \frac{T}{25.16}(-16\mathbf{i} + 11\mathbf{j} + 16\mathbf{k})$$

$$\boldsymbol{\lambda}_{AE} = \frac{\overrightarrow{AE}}{AE} = \frac{7\mathbf{i} - 24\mathbf{k}}{\mathbf{25}}$$

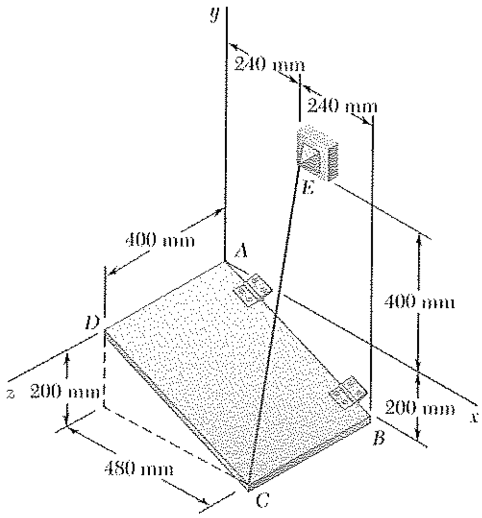
$$\Sigma M_{AE} = 0: \quad \boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) + \boldsymbol{\lambda}_{AE} \cdot (\mathbf{r}_{C/A} \cdot (-60\mathbf{j})) = 0$$

$$\begin{vmatrix} 7 & 0 & -24 \\ 9 & 0 & 0 \\ -16 & 11 & 16 \end{vmatrix} \frac{T}{25 \times 25.16} + \begin{vmatrix} 7 & 0 & -24 \\ 9 & 0 & 10 \\ 0 & -60 & 0 \end{vmatrix} \frac{1}{25} = 0$$

$$-\frac{24 \times 9 \times 11}{25 \times 25.16}T + \frac{24 \times 9 \times 60 + 7 \times 10 \times 60}{25}$$

$$94.436T - 17{,}160 = 0$$

$T = 181.7$ lb ◀

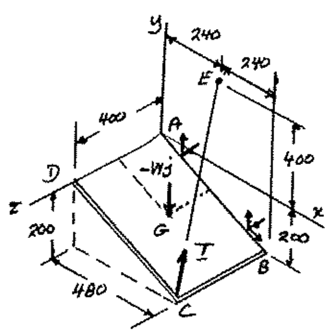


PROBLEM 4.135

The 50-kg plate *ABCD* is supported by hinges along edge *AB* and by wire *CE*. Knowing that the plate is uniform, determine the tension in the wire.

SOLUTION

Free-Body Diagram:



$$W = mg = (50\text{ kg})(9.81\text{ m/s}^2)$$

$$W = 490.50\text{ N}$$

$$\overrightarrow{CE} = -240\mathbf{i} + 600\mathbf{j} - 400\mathbf{k}$$

$$CE = 760\text{ mm}$$

$$\mathbf{T} = T\frac{\overrightarrow{CE}}{CE} = \frac{T}{760}(-240\mathbf{i} + 600\mathbf{j} - 400\mathbf{k})$$

$$\boldsymbol{\lambda}_{AB} = \frac{\overrightarrow{AB}}{AB} = \frac{480\mathbf{i} - 200\mathbf{j}}{520} = \frac{1}{13}(12\mathbf{i} - 5\mathbf{j})$$

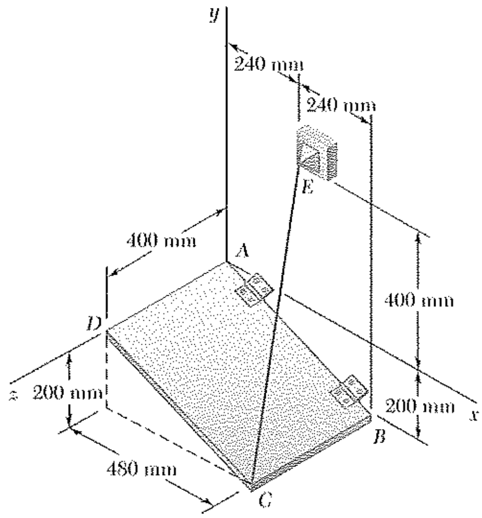
$$\Sigma\mathbf{M}_{AB} = 0: \quad \boldsymbol{\lambda}_{AB}\cdot(\mathbf{r}_{E/A}\times T) + \boldsymbol{\lambda}_{AB}\cdot(\mathbf{r}_{G/A}\times -W\mathbf{j}) = 0$$

$$\mathbf{r}_{E/A} = 240\mathbf{i} + 400\mathbf{j}; \quad \mathbf{r}_{G/A} = 240\mathbf{i} - 100\mathbf{j} + 200\mathbf{k}$$

$$\begin{vmatrix} 12 & -5 & 0 \\ 240 & 400 & 0 \\ -240 & 600 & -400 \end{vmatrix}\frac{T}{13\times 20} + \begin{vmatrix} 12 & -5 & 0 \\ 240 & -100 & 200 \\ 0 & -W & 0 \end{vmatrix}\frac{1}{13} = 0$$

$$(-12\times 400\times 400 - 5\times 240\times 400)\frac{T}{760} + 12\times 200W = 0$$

$$T = 0.76W = 0.76(490.50\text{ N}) \qquad T = 373\text{ N} \blacktriangleleft$$



PROBLEM 4.136

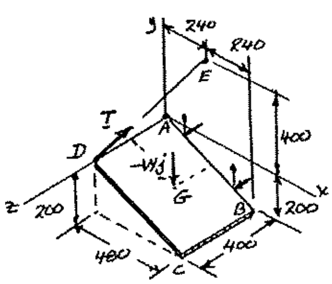
Solve Problem 4.135, assuming that wire CE is replaced by a wire connecting E and D.

PROBLEM 4.135 The 50-kg plate $ABCD$ is supported by hinges along edge AB and by wire CE. Knowing that the plate is uniform, determine the tension in the wire.

SOLUTION

Free-Body Diagram:

Dimensions in mm



$$W = mg = (50\text{ kg})(9.81\text{ m/s}^2)$$

$$W = 490.50\text{ N}$$

$$\overrightarrow{DE} = -240\mathbf{i} + 400\mathbf{j} - 400\mathbf{k}$$

$$DE = 614.5\text{ mm}$$

$$\mathbf{T} = T\frac{\overrightarrow{DE}}{DE} = \frac{T}{614.5}(240\mathbf{i} + 400\mathbf{j} - 400\mathbf{k})$$

$$\boldsymbol{\lambda}_{AB} = \frac{\overrightarrow{AB}}{AB} = \frac{480\mathbf{i} - 200\mathbf{j}}{520} = \frac{1}{13}(12\mathbf{i} - 5\mathbf{j})$$

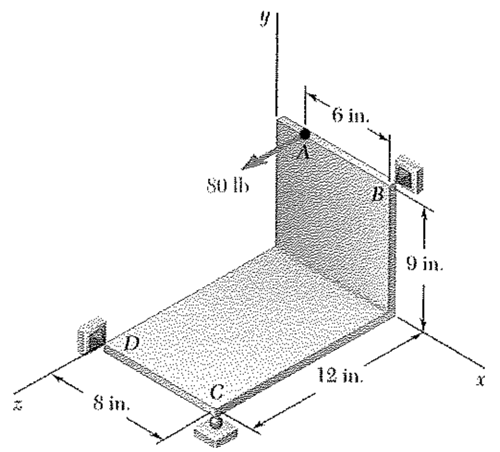
$$\mathbf{r}_{E/A} = 240\mathbf{i} + 400\mathbf{j}; \quad \mathbf{r}_{G/A} = 240\mathbf{i} - 100\mathbf{j} + 200\mathbf{k}$$

$$\begin{vmatrix} 12 & -5 & 0 \\ 240 & 400 & 0 \\ 240 & 400 & -400 \end{vmatrix} \frac{T}{13 \times 614.5} + \begin{vmatrix} 12 & 5 & 0 \\ 240 & -100 & 200 \\ 0 & -W & 0 \end{vmatrix} \frac{1}{13} = 0$$

$$(-12 \times 400 \times 400 - 5 \times 240 \times 400)\frac{T}{614.5} + 12 \times 200 \times W = 0$$

$$T = 0.6145W = 0.6145(490.50\text{ N})$$

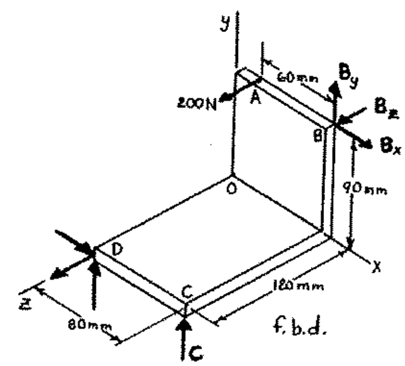
$T = 301$ N ◄



PROBLEM 4.137

Two rectangular plates are welded together to form the assembly shown. The assembly is supported by ball-and-socket joints at B and D and by a ball on a horizontal surface at C. For the loading shown, determine the reaction at C.

SOLUTION



First note

$$\lambda_{BD} = \frac{-(6\text{ in.})\mathbf{i} - (9\text{ in.})\mathbf{j} + (12\text{ in.})\mathbf{k}}{\sqrt{(6)^2 + (9)^2 + (12)^2}\text{ in.}}$$

$$= \frac{1}{16.1555}(-6\mathbf{i} - 9\mathbf{j} + 12\mathbf{k})$$

$$\mathbf{r}_{A/B} = -(6\text{ in.})\mathbf{i}$$

$$\mathbf{P} = (80\text{ lb})\mathbf{k}$$

$$\mathbf{r}_{C/D} = (8\text{ in.})\mathbf{i}$$

$$\mathbf{C} = (C)\mathbf{j}$$

From the f.b.d. of the plates

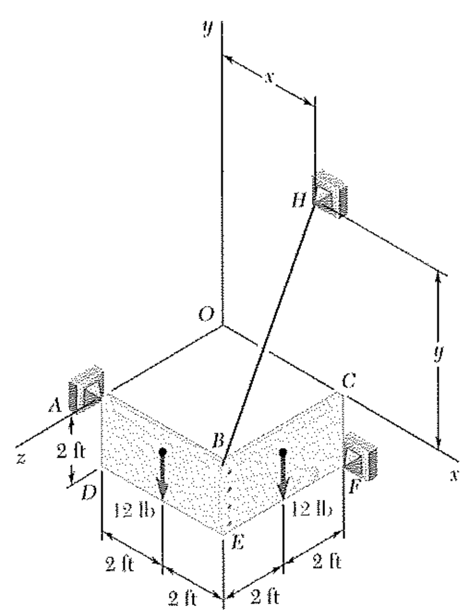
$$\Sigma M_{BD} = 0: \quad \lambda_{BD} \cdot (\mathbf{r}_{A/B} \times \text{P}) + \lambda_{BD} \cdot (\mathbf{r}_{C/D} \times \text{C}) = 0$$

$$\begin{vmatrix} -6 & -9 & 12 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} \left[\frac{6(80)}{16.1555}\right] + \begin{vmatrix} -6 & -9 & 12 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} \left[\frac{C(8)}{16.1555}\right] = 0$$

$$(-9)(6)(80) + (12)(8)C = 0$$

$$C = 45.0\text{ lb}$$

or $\mathbf{C} = (45.0\text{ lb})\mathbf{j}$ ◀

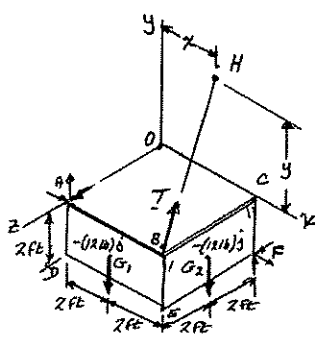


PROBLEM 4.138

Two 2×4-ft plywood panels, each of weight 12 lb, are nailed together as shown. The panels are supported by ball-and-socket joints at A and F and by the wire BH. Determine (*a*) the location of H in the xy plane if the tension in the wire is to be minimum, (*b*) the corresponding minimum tension.

SOLUTION

Free-Body Diagram:



$$\overrightarrow{AF} = 4\mathbf{i} - 2\mathbf{j} - 4\mathbf{k} \quad AF = 6\text{ ft}$$

$$\boldsymbol{\lambda}_{AF} = \frac{1}{3}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$$

$$\mathbf{r}_{G_1/A} = 2\mathbf{i} - \mathbf{j}$$

$$\mathbf{r}_{G_2/A} = 4\mathbf{i} - \mathbf{j} - 2\mathbf{k}$$

$$\mathbf{r}_{B/A} = 4\mathbf{i}$$

$$\Sigma M_{AF} = 0: \quad \boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{G/A} \times (-12\mathbf{j}) + \boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{G2/A} \times (-12\mathbf{j})) + \boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{B/A} \times T) = 0$$

$$\begin{vmatrix} 2 & -1 & -2 \\ 2 & -1 & 0 \\ 0 & -12 & 0 \end{vmatrix} \frac{1}{3} + \begin{vmatrix} 2 & -1 & -2 \\ 4 & -1 & -2 \\ 0 & -12 & 0 \end{vmatrix} \frac{1}{3} + \boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) = 0$$

$$(2 \times 2 \times 12)\frac{1}{3} + (-2 \times 2 \times 12 + 2 \times 4 \times 12)\frac{1}{3} + \boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) = 0$$

$$\boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) = -32 \quad \text{or} \quad \mathbf{T} \cdot (\boldsymbol{\lambda}_{A/F} \times \mathbf{r}_{B/A}) = -32 \qquad (1)$$

Projection of **T** on $(\boldsymbol{\lambda}_{AF} \times \mathbf{r}_{B/A})$ is constant. Thus, $T_{\min}$ is parallel to

$$\boldsymbol{\lambda}_{AF} \times \mathbf{r}_{B/A} = \frac{1}{3}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k}) \times 4\mathbf{i} = \frac{1}{3}(-8\mathbf{j} + 4\mathbf{k})$$

Corresponding unit vector is $\frac{1}{\sqrt{5}}(-2\mathbf{j} + \mathbf{k})$

$$T_{\min} = T(-2\mathbf{j} + \mathbf{k})\frac{1}{\sqrt{5}} \qquad (2)$$

Eq. (1): $$\frac{T}{\sqrt{5}}(-2\mathbf{j} + \mathbf{k}) \cdot \left[\frac{1}{3}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k}) \times 4\mathbf{i}\right] = -32$$

$$\frac{T}{\sqrt{5}}(-2\mathbf{j} + \mathbf{k}) \cdot \frac{1}{3}(-8\mathbf{j} + 4\mathbf{k}) = -32$$

$$\frac{T}{3\sqrt{5}}(16 + 4) = -32 \qquad T = -\frac{3\sqrt{5}(32)}{20} = 4.8\sqrt{5}$$

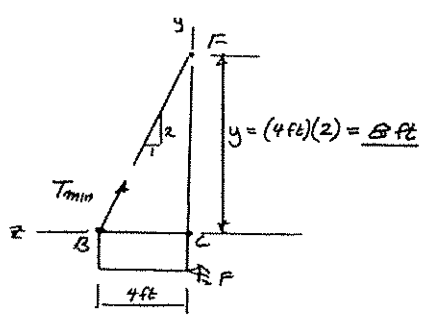
$$T = 10.7331 \text{ lb}$$

Eq. (2) $$T_{\min} = T(-2\mathbf{j} + \mathbf{k})\frac{1}{\sqrt{5}}$$

$$= 4.8\sqrt{5}(-2\mathbf{j} + \mathbf{k})\frac{1}{\sqrt{5}}$$

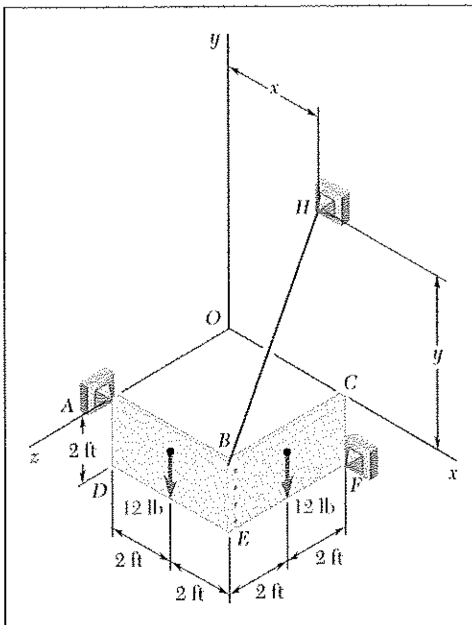
$$\mathbf{T}_{\min} = -(9.6 \text{ lb})\mathbf{j} + (4.8 \text{ lb } \mathbf{k})$$

Since $T_{\min}$ has no **i** component, wire BH is parallel to the yz plane, and $x = 4$ ft.



(*a*) $x = 4.00$ ft; $y = 8.00$ ft ◀

(*b*) $T_{\min} = 10.73$ lb ◀



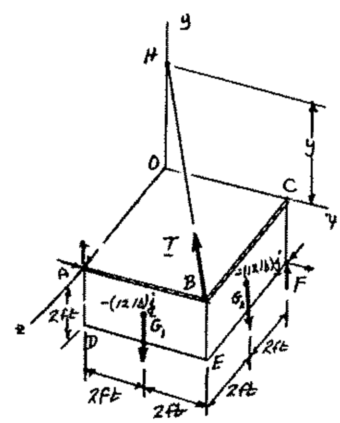
PROBLEM 4.139

Solve Problem 4.138, subject to the restriction that H must lie on the y axis.

PROBLEM 4.138 Two 2×4-ft plywood panels, each of weight 12 lb, are nailed together as shown. The panels are supported by ball-and-socket joints at A and F and by the wire BH. Determine (a) the location of H in the xy plane if the tension in the wire is to be minimum, (b) the corresponding minimum tension.

SOLUTION

Free-Body Diagram:



$$\overrightarrow{AF} = 4\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}$$

$$\boldsymbol{\lambda}_{AF} = \frac{1}{3}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$$

$$\mathbf{r}_{G_1/A} = 2\mathbf{i} - \mathbf{j}$$

$$\mathbf{r}_{G_2/A} = 4\mathbf{i} - \mathbf{j} - 2\mathbf{k}$$

$$\mathbf{r}_{B/A} = 4\mathbf{i}$$

$$\Sigma M_{AF} = 0: \quad \boldsymbol{\lambda}_{AF}\cdot(\mathbf{r}_{G/A}\times(-12\mathbf{j}) + \boldsymbol{\lambda}_{AF}\cdot(\mathbf{r}_{G_2/A}\times(-12\mathbf{j})) + \boldsymbol{\lambda}_{AF}\cdot(\mathbf{r}_{B/A}\times T) = 0$$

$$\begin{vmatrix} 2 & -1 & 2 \\ 2 & -1 & 0 \\ 0 & -12 & 0 \end{vmatrix}\frac{1}{3} + \begin{vmatrix} 2 & -1 & -2 \\ 4 & -1 & -2 \\ 0 & -12 & 0 \end{vmatrix}\frac{1}{3} + \boldsymbol{\lambda}_{AF}\cdot(\mathbf{r}_{B/A}\times\mathbf{T}) = 0$$

$$(2\times 2\times 12)\frac{1}{3} + (-2\times 2\times 12 + 2\times 4\times 12)\frac{1}{3} + \boldsymbol{\lambda}_{AF}\cdot(\mathbf{r}_{B/A}\times\mathbf{T}) = 0$$

$$\boldsymbol{\lambda}_{AF}\cdot(\mathbf{r}_{B/A}\times\mathbf{T}) = -32 \qquad (1)$$

$$\overrightarrow{BH} = -4\mathbf{i} + y\mathbf{j} - 4\mathbf{k} \qquad BH = (32 + y^2)^{1/2}$$

$$\mathbf{T} = T\frac{\overrightarrow{BH}}{BH} = T\frac{-4\mathbf{i} + y\mathbf{j} - 4\mathbf{k}}{(32 + y^2)^{1/2}}$$

PROBLEM 4.139 (Continued)

Eq. (1):

$$\boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{B/A} \times T) = \begin{vmatrix} 2 & -1 & -2 \\ 4 & 0 & 0 \\ -4 & y & -4 \end{vmatrix} \frac{T}{3(32+y^2)^{1/2}} = -32$$

$$(-16 - 8y)T = -3 \times 32(32 + y^2)^{1/2} \qquad T = 96\frac{(32+y^2)^{1/2}}{8y+16} \qquad (2)$$

$$\frac{dT}{dy} = 0: \quad 96\frac{(8y+16)\frac{1}{2}(32+y^2)^{-1/2}(2y) + (32+y^2)^{1/2}(8)}{(8y+16)^2}$$

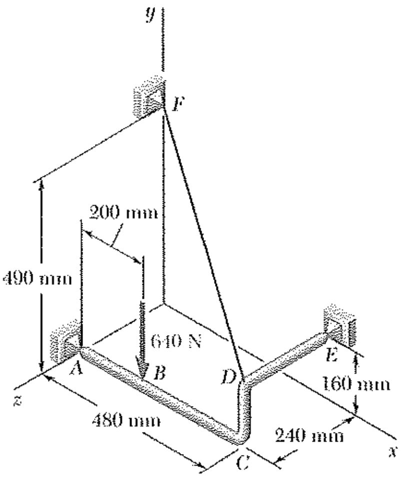
Numerator = 0: $(8y+16)y = (32+y^2)8$

$$8y^2 + 16y = 32 \times 8 + 8y^2$$

$y = 16.00$ ft ◀

Eq. (2): $T = 96\dfrac{(32+16^2)^{1/2}}{8 \times 16 + 16} = 11.3137$ lb

$T_{\min} = 11.31$ lb ◀



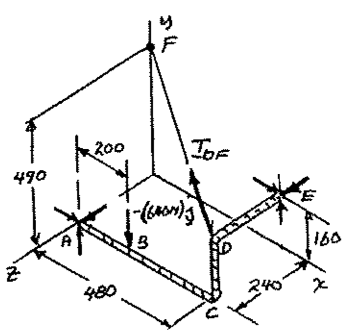
PROBLEM 4.140

The pipe $ACDE$ is supported by ball-and-socket joints at A and E and by the wire DF. Determine the tension in the wire when a 640-N load is applied at B as shown.

SOLUTION

Free-Body Diagram:

Dimensions in mm



$$\overrightarrow{AE} = 480\mathbf{i} + 160\mathbf{j} - 240\mathbf{k}$$

$$AE = 560 \text{ mm}$$

$$\boldsymbol{\lambda}_{AE} = \frac{\overrightarrow{AE}}{AE} = \frac{480\mathbf{i} + 160\mathbf{j} - 240\mathbf{k}}{560}$$

$$\boldsymbol{\lambda}_{AE} = \frac{6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}}{7}$$

$$\mathbf{r}_{B/A} = 200\mathbf{i}$$

$$\mathbf{r}_{D/A} = 480\mathbf{i} + 160\mathbf{j}$$

$$\overrightarrow{DF} = -480\mathbf{i} + 330\mathbf{j} - 240\mathbf{k}; \qquad DF = 630 \text{ mm}$$

$$\mathbf{T}_{DF} = T_{DF}\frac{\overrightarrow{DF}}{DF} = T_{DF}\frac{-480\mathbf{i} + 330\mathbf{j} - 240\mathbf{k}}{630} = T_{DF}\frac{-16\mathbf{i} + 11\mathbf{j} - 8\mathbf{k}}{21}$$

$$\Sigma M_{AE} = \boldsymbol{\lambda}_{AE} \cdot (\mathbf{r}_{D/A} \times \mathbf{T}_{DF}) + \boldsymbol{\lambda}_{AE} \cdot (\mathbf{r}_{B/A} \times (-600\mathbf{j})) = 0$$

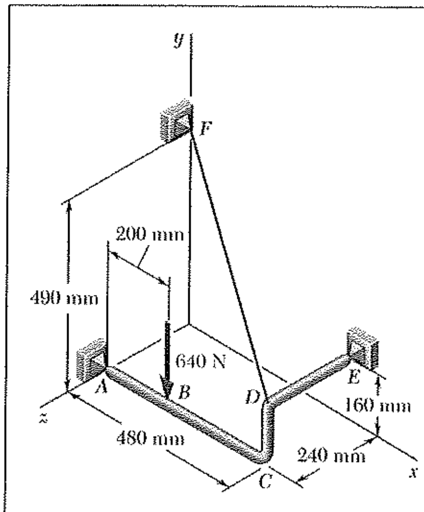
$$\begin{vmatrix} 6 & 2 & -3 \\ 480 & 160 & 0 \\ -16 & 11 & -8 \end{vmatrix} \frac{T_{DE}}{21 \times 7} + \begin{vmatrix} 6 & 2 & -3 \\ 200 & 0 & 0 \\ 0 & -640 & 0 \end{vmatrix} \frac{1}{7} = 0$$

$$\frac{-6 \times 160 \times 8 + 2 \times 480 \times 8 - 3 \times 480 \times 11 - 3 \times 160 \times 16}{21 \times 7} T_{DF} + \frac{3 \times 200 \times 640}{7} = 0$$

$$-1120 T_{DF} + 384 \times 10^3 = 0$$

$$T_{DF} = 342.86 \text{ N}$$

$T_{DF} = 343$ N ◀



PROBLEM 4.141

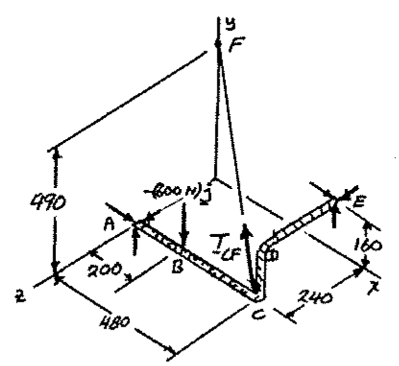
Solve Problem 4.140, assuming that wire *DF* is replaced by a wire connecting *C* and *F*.

PROBLEM 4.140 The pipe *ACDE* is supported by ball-and-socket joints at *A* and *E* and by the wire *DF*. Determine the tension in the wire when a 640-N load is applied at *B* as shown.

SOLUTION

Free-Body Diagram:

Dimensions in mm



$$\overrightarrow{AE} = 480\mathbf{i} + 160\mathbf{j} - 240\mathbf{k}$$

$$AE = 560 \text{ mm}$$

$$\boldsymbol{\lambda}_{AE} = \frac{\overrightarrow{AE}}{AE} = \frac{480\mathbf{i} + 160\mathbf{j} - 240\mathbf{k}}{560}$$

$$\boldsymbol{\lambda}_{AE} = \frac{6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}}{7}$$

$$\mathbf{r}_{B/A} = 200\mathbf{i}$$

$$\mathbf{r}_{C/A} = 480\mathbf{i}$$

$$\overrightarrow{CF} = -480\mathbf{i} + 490\mathbf{j} - 240\mathbf{k}; \quad CF = 726.70 \text{ mm}$$

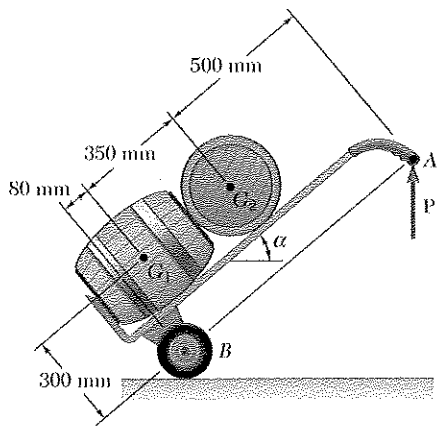
$$\mathbf{T}_{CF} = T_{CF}\frac{\overrightarrow{CE}}{CF} = \frac{-480\mathbf{i} + 490\mathbf{j} - 240\mathbf{k}}{726.70}$$

$$\Sigma M_{AE} = 0: \quad \boldsymbol{\lambda}_{AE} \cdot (\mathbf{r}_{C/A} \times \mathbf{T}_{CF}) + \boldsymbol{\lambda}_{AE} \cdot (\mathbf{r}_{B/A} \times (-600\mathbf{j})) = 0$$

$$\begin{vmatrix} 6 & 2 & -3 \\ 480 & 0 & 0 \\ -480 & +490 & -240 \end{vmatrix} \frac{T_{CF}}{726.7 \times 7} + \begin{vmatrix} 6 & 2 & -3 \\ 200 & 0 & 0 \\ 0 & -640 & 0 \end{vmatrix} \frac{1}{7} = 0$$

$$\frac{2 \times 480 \times 240 - 3 \times 480 \times 490}{726.7 \times 7} T_{CF} + \frac{3 \times 200 \times 640}{7} = 0$$

$$-653.91 T_{CF} + 384 \times 10^3 = 0 \qquad T_{CF} = 587 \text{ N} \blacktriangleleft$$

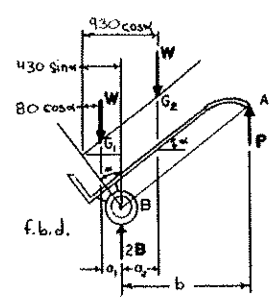


PROBLEM 4.142

A hand truck is used to move two kegs, each of mass 40 kg. Neglecting the mass of the hand truck, determine (*a*) the vertical force **P** that should be applied to the handle to maintain equilibrium when $\alpha = 35°$, (*b*) the corresponding reaction at each of the two wheels.

SOLUTION

Free-Body Diagram:



Dimensions in mm

$$W = mg = (40 \text{ kg})(9.81 \text{ m/s}^2) = 392.40 \text{ N}$$
$$a_1 = (300 \text{ mm})\sin\alpha - (80 \text{ mm})\cos\alpha$$
$$a_2 = (430 \text{ mm})\cos\alpha - (300 \text{ mm})\sin\alpha$$
$$b = (930 \text{ mm})\cos\alpha$$

From free-body diagram of hand truck

$$+\circlearrowleft \Sigma M_B = 0: \quad P(b) - W(a_2) + W(a_1) = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad P - 2W + 2B = 0 \qquad (2)$$

For $\alpha = 35°$

$$a_1 = 300\sin 35° - 80\cos 35° = 106.541 \text{ mm}$$
$$a_2 = 430\cos 35° - 300\sin 35° = 180.162 \text{ mm}$$
$$b = 930\cos 35° = 761.81 \text{ mm}$$

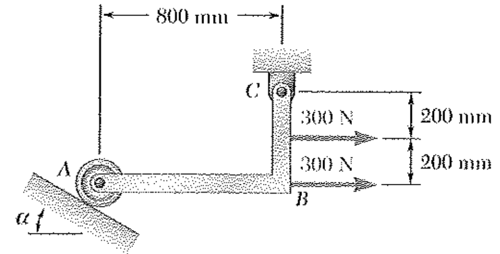
(*a*) From Equation (1)

$$P(761.81 \text{ mm}) - 392.40 \text{ N}(180.162 \text{ mm}) + 392.40 \text{ N}(106.54 \text{ mm}) = 0$$

$P = 37.921$ N — or $\mathbf{P} = 37.9 \text{ N} \uparrow$ ◀

(*b*) From Equation (2)

$37.921 \text{ N} - 2(392.40 \text{ N}) + 2B = 0$ — or $\mathbf{B} = 373 \text{ N} \uparrow$ ◀



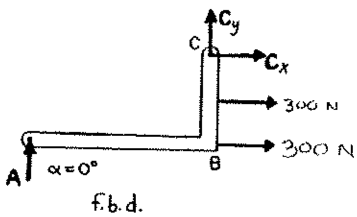
PROBLEM 4.143

Determine the reactions at A and C when (*a*) $\alpha = 0$, (*b*) $\alpha = 30°$.

SOLUTION

(*a*) $\alpha = 0°$

From f.b.d. of member ABC



$$+\circlearrowleft \Sigma M_C = 0: \quad (300 \text{ N})(0.2 \text{ m}) + (300 \text{ N})(0.4 \text{ m}) - A(0.8 \text{ m}) = 0$$

$$A = 225 \text{ N} \qquad \text{or} \qquad \mathbf{A} = 225 \text{ N}\uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad C_y + 225 \text{ N} = 0$$

$$C_y = -225 \text{ N} \quad \text{or} \quad \mathbf{C}_y = 225 \text{ N}\downarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad 300 \text{ N} + 300 \text{ N} + C_x = 0$$

$$C_x = -600 \text{ N} \quad \text{or} \quad \mathbf{C}_x = 600 \text{ N} \longleftarrow$$

Then

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(600)^2 + (225)^2} = 640.80 \text{ N}$$

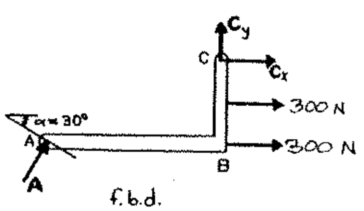
and

$$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{-225}{-600}\right) = 20.556°$$

$$\text{or} \qquad \mathbf{C} = 641 \text{ N} \measuredangle 20.6° \blacktriangleleft$$

(*b*) $\alpha = 30°$

From f.b.d. of member ABC



$$+\circlearrowleft \Sigma M_C = 0: \quad (300 \text{ N})(0.2 \text{ m}) + (300 \text{ N})(0.4 \text{ m}) - (A\cos 30°)(0.8 \text{ m})$$
$$+ (A\sin 30°)(20 \text{ in.}) = 0$$

$$A = 365.24 \text{ N} \qquad \text{or} \qquad \mathbf{A} = 365 \text{ N} \measuredangle 60.0° \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad 300 \text{ N} + 300 \text{ N} + (365.24 \text{ N})\sin 30° + C_x = 0$$

$$C_x = -782.62$$

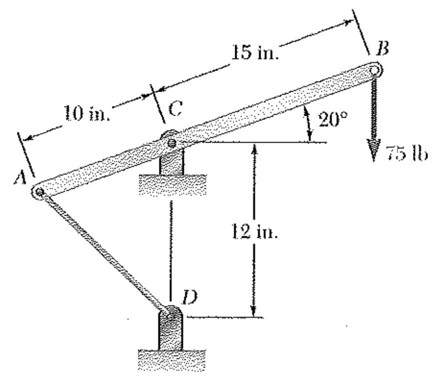
$$+\uparrow \Sigma F_y = 0: \quad C_y + (365.24 \text{ N})\cos 30° = 0$$

$$C_y = -316.31 \text{ N} \quad \text{or} \quad \mathbf{C}_y = 316 \text{ N} \downarrow$$

Then $$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(782.62)^2 + (316.31)^2} = 884.12 \text{ N}$$

and $$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{-316.31}{-782.62}\right) = 22.007°$$

or $$\mathbf{C} = 884 \text{ N} \measuredangle\, 22.0° \blacktriangleleft$$

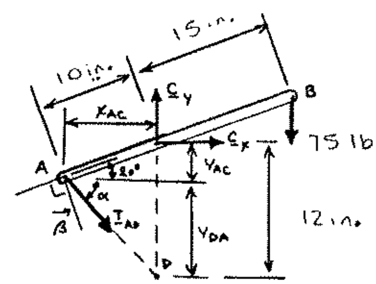


PROBLEM 4.144

A lever AB is hinged at C and attached to a control cable at A. If the lever is subjected to a 75-lb vertical force at B, determine (*a*) the tension in the cable, (*b*) the reaction at C.

SOLUTION

Free-Body Diagram:



Geometry:

$$x_{AC} = (10 \text{ in.})\cos 20° = 9.3969 \text{ in.}$$
$$y_{AC} = (10 \text{ in.})\sin 20° = 3.4202 \text{ in.}$$
$$\Rightarrow y_{DA} = 12 \text{ in.} - 3.4202 \text{ in.} = 8.5798 \text{ in.}$$
$$\alpha = \tan^{-1}\left(\frac{y_{DA}}{x_{AC}}\right) = \tan^{-1}\left(\frac{8.5798}{9.3969}\right) = 42.397°$$
$$\beta = 90° - 20° - 42.397° = 27.603°$$

Equilibrium for lever:

(*a*) $+\circlearrowleft \Sigma M_C = 0$: $T_{AD}\cos 27.603°(10 \text{ in.}) - (75 \text{ lb})[(15 \text{ in.})\cos 20°] = 0$

$$T_{AD} = 119.293 \text{ lb}$$

$T_{AD} = 119.3$ lb ◀

(*b*) $\xrightarrow{+} \Sigma F_x = 0$: $C_x + (119.293 \text{ lb})\cos 42.397° = 0$

$$C_x = -88.097 \text{ lb}$$

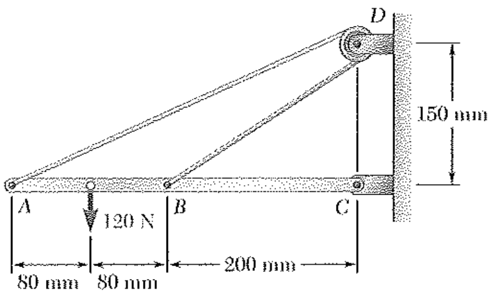
$+\uparrow \Sigma F_y = 0$: $C_y - 75 \text{ lb} - (119.293 \text{ lb})\sin 42.397° = 0$

$$C_y = 155.435$$

Thus: $C = \sqrt{C_x^2 + C_y^2} = \sqrt{(-88.097)^2 + (155.435)^2} = 178.665 \text{ lb}$

and $\theta = \tan^{-1}\dfrac{C_y}{C_x} = \tan^{-1}\dfrac{155.435}{88.097} = 60.456°$

C = 178.7 lb ⦪ 60.5° ◀



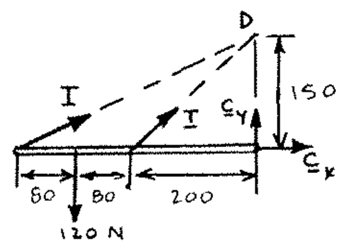
PROBLEM 4.145

Neglecting friction and the radius of the pulley, determine (*a*) the tension in cable *ADB*, (*b*) the reaction at *C*.

SOLUTION

Free-Body Diagram:

Dimensions in mm



Geometry:

Distance $$AD = \sqrt{(0.36)^2 + (0.150)^2} = 0.39 \text{ m}$$

Distance $$BD = \sqrt{(0.2)^2 + (0.15)^2} = 0.25 \text{ m}$$

Equilibrium for beam:

(*a*) $$+\circlearrowleft \Sigma M_C = 0: \quad (120 \text{ N})(0.28 \text{ m}) - \left(\frac{0.15}{0.39}T\right)(0.36 \text{ m}) - \left(\frac{0.15}{0.25}T\right)(0.2 \text{ m}) = 0$$

$$T = 130.000 \text{ N}$$

or $T = 130.0$ N ◀

(*b*) $$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + \left(\frac{0.36}{0.39}\right)(130.000 \text{ N}) + \left(\frac{0.2}{0.25}\right)(130.000 \text{ N}) = 0$$

$$C_x = -224.00 \text{ N}$$

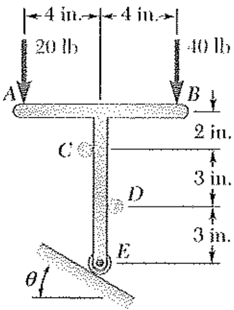
$$+\uparrow \Sigma F_y = 0: \quad C_y + \left(\frac{0.15}{0.39}\right)(130.00 \text{ N}) + \left(\frac{0.15}{0.25}\right)(130.00 \text{ N}) - 120 \text{ N} = 0$$

$$C_y = -8.0000 \text{ N}$$

Thus: $$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(-224)^2 + (-8)^2} = 224.14 \text{ N}$$

and $$\theta = \tan^{-1}\frac{C_y}{C_x} = \tan^{-1}\frac{8}{224} = 2.0454°$$

C = 224 N ⦪ 2.05° ◀

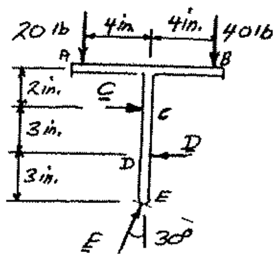


PROBLEM 4.146

The T-shaped bracket shown is supported by a small wheel at E and pegs at C and D. Neglecting the effect of friction, determine the reactions at C, D, and E when $\theta = 30°$.

SOLUTION

Free-Body Diagram:



$$+\uparrow \Sigma F_y = 0: \quad E\cos 30° - 20 - 40 = 0$$

$$E = \frac{60 \text{ lb}}{\cos 30°} = 69.282 \text{ lb} \qquad \mathbf{E} = 69.3 \text{ lb} \measuredangle 60.0° \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_D = 0: \quad (20 \text{ lb})(4 \text{ in.}) - (40 \text{ lb})(4 \text{ in.})$$

$$- C(3 \text{ in.}) + E\sin 30°(3 \text{ in.}) = 0$$

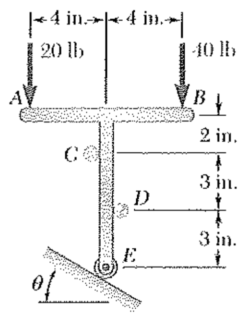
$$-80 - 3C + 69.282(0.5)(3) = 0$$

$$C = 7.9743 \text{ lb} \qquad \mathbf{C} = 7.97 \text{ lb} \longrightarrow \blacktriangleleft$$

$$\Sigma F_x = 0: \quad E\sin 30° + C - D = 0$$

$$(69.282 \text{ lb})(0.5) + 7.9743 \text{ lb} - D = 0$$

$$D = 42.615 \text{ lb} \qquad \mathbf{D} = 42.6 \text{ lb} \longleftarrow \blacktriangleleft$$

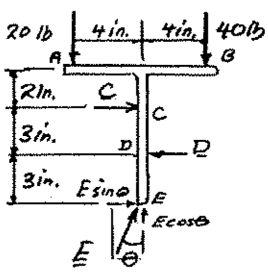


PROBLEM 4.147

The T-shaped bracket shown is supported by a small wheel at E and pegs at C and D. Neglecting the effect of friction, determine (*a*) the smallest value of θ for which the equilibrium of the bracket is maintained, (*b*) the corresponding reactions at C, D, and E.

SOLUTION

Free-Body Diagram:



$$+\uparrow \Sigma F_y = 0: \quad E\cos\theta - 20 - 40 = 0$$

$$E = \frac{60}{\cos\theta} \qquad (1)$$

$$+\circlearrowleft \Sigma M_D = 0: \quad (20 \text{ lb})(4 \text{ in.}) - (40 \text{ lb})(4 \text{ in.}) - C(3 \text{ in.}) + \left(\frac{60}{\cos\theta}\sin\theta\right)3 \text{ in.} = 0$$

$$C = \frac{1}{3}(180\tan\theta - 80)$$

(*a*) For $C = 0$, $\quad 180\tan\theta = 80$

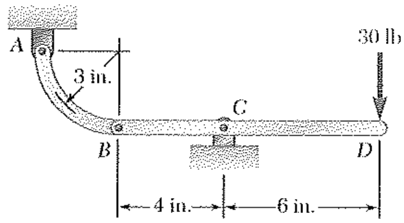
$$\tan\theta = \frac{4}{9} \quad \theta = 23.962^\circ \qquad \theta = 24.0^\circ \blacktriangleleft$$

Eq. (1) $\quad E = \dfrac{60}{\cos 23.962^\circ} = 65.659$

$$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad -D + C + E\sin\theta = 0$$

$$D = (65.659)\sin 23.962 = 26.666 \text{ lb}$$

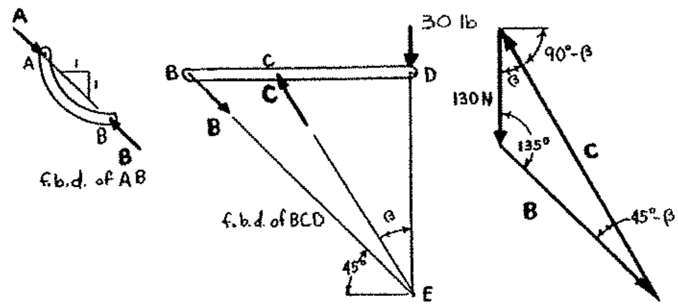
(*b*) $\quad \mathbf{C} = 0 \quad \mathbf{D} = 26.7 \text{ lb} \leftarrow \qquad \mathbf{E} = 65.71 \text{ lb} \angle 66.0^\circ \blacktriangleleft$



PROBLEM 4.148

For the frame and loading shown, determine the reactions at A and C.

SOLUTION



Since member AB is acted upon by two forces, **A** and **B**, they must be colinear, have the same magnitude, and be opposite in direction for AB to be in equilibrium. The force **B** acting at B of member BCD will be equal in magnitude but opposite in direction to force **B** acting on member AB. Member BCD is a three-force body with member forces intersecting at E. The f.b.d.'s of members AB and BCD illustrate the above conditions. The force triangle for member BCD is also shown. The angle β is found from the member dimensions:

$$\beta = \tan^{-1}\left(\frac{6 \text{ in.}}{10 \text{ in.}}\right) = 30.964°$$

Applying of the law of sines to the force triangle for member BCD,

$$\frac{30 \text{ lb}}{\sin(45° - \beta)} = \frac{B}{\sin\beta} = \frac{C}{\sin 135°}$$

or

$$\frac{30 \text{ lb}}{\sin 14.036°} = \frac{B}{\sin 30.964°} = \frac{C}{\sin 135°}$$

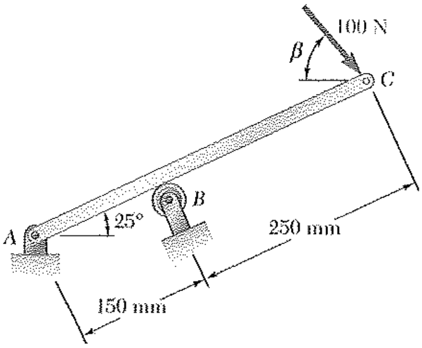
$$A = B = \frac{(30 \text{ lb})\sin 30.964°}{\sin 14.036°} = 63.641 \text{ lb}$$

or **A** = 63.6 lb ⦨ 45.0° ◀

and

$$C = \frac{(30 \text{ lb})\sin 135°}{\sin 14.036°} = 87.466 \text{ lb}$$

or **C** = 87.5 lb ⦪ 59.0° ◀

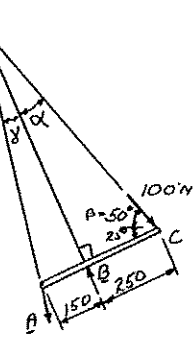


PROBLEM 4.149

Determine the reactions at A and B when $\beta = 50°$.

SOLUTION

Free-Body Diagram: (Three-force body)



Reaction **A** must pass through Point D where 100-N force and **B** intersect

In right ΔBCD

$$\alpha = 90° - 75° = 15°$$

$$BD = 250 \tan 75° = 933.01 \text{ mm}$$

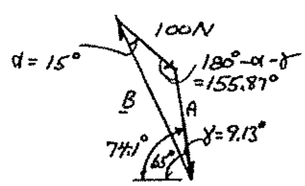
In right ΔABD

$$\tan \gamma = \frac{AB}{BD} = \frac{150 \text{ mm}}{933.01 \text{ mm}}$$

$$\gamma = 9.13$$

Force Triangle

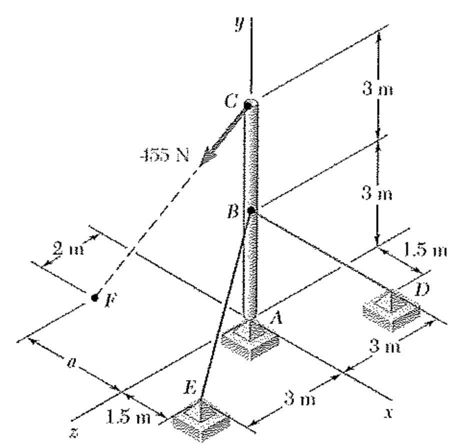
Law of sines



$$\frac{100 \text{ N}}{\sin 9.13°} = \frac{A}{\sin 15°} = \frac{B}{\sin 155.87°}$$

$$A = 163.1 \text{ N}; \quad B = 257.6 \text{ N}$$

A = 163.1 N ⦪ 74.1° **B** = 258 N ⦩ 65.0° ◀

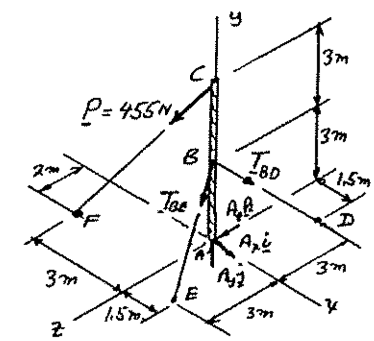


PROBLEM 4.150

The 6-m pole ABC is acted upon by a 455-N force as shown. The pole is held by a ball-and-socket joint at A and by two cables BD and BE. For $a = 3$ m, determine the tension in each cable and the reaction at A.

SOLUTION

Free-Body Diagram:



Five unknowns and six Eqs. of equilibrium, but equilibrium is maintained

$$(\Sigma M_{AC} = 0)$$

$$\mathbf{r}_B = 3\mathbf{j}$$

$$\mathbf{r}_C = 6\mathbf{j}$$

$$\overrightarrow{CF} = -3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k} \quad CF = 7 \text{ m}$$

$$\overrightarrow{BD} = 1.5\mathbf{i} - 3\mathbf{j} - 3\mathbf{k} \quad BD = 4.5 \text{ m}$$

$$\overrightarrow{BE} = 1.5\mathbf{i} - 3\mathbf{j} + 3\mathbf{k} \quad BE = 4.5 \text{ m}$$

$$\mathbf{P} = P\frac{\overrightarrow{CF}}{CE} = \frac{P}{7}(-3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k})$$

$$\mathbf{T}_{BD} = T_{BD}\frac{\overrightarrow{BD}}{BD} = \frac{T_{BD}}{4.5}(1.5\mathbf{i} - 3\mathbf{j} - 3\mathbf{k}) = \frac{T_{BD}}{3}(\mathbf{i} - 2\mathbf{j} - 2\mathbf{k})$$

$$\mathbf{T}_{BE} = T_{BE}\frac{\overrightarrow{BE}}{BE} = \frac{T_{BD}}{3}(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$$

$$\Sigma M_A = 0: \quad \mathbf{r}_B \times \mathbf{T}_{BD} + \mathbf{r}_B \times \mathbf{T}_{BE} + \mathbf{r}_C \times \mathbf{P} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 3 & 0 \\ 1 & -2 & -2 \end{vmatrix}\frac{T_{BD}}{3} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 3 & 0 \\ 1 & -2 & 2 \end{vmatrix}\frac{T_{BE}}{3} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 6 & 0 \\ -3 & -6 & 2 \end{vmatrix}\frac{P}{7} = 0$$

Coefficient of **i**:

$$-2T_{BD} + 2T_{BE} + \frac{12}{7}P = 0 \qquad (1)$$

Coefficient of **k**:

$$-T_{BD} - T_{BF} + \frac{18}{7}P = 0 \qquad (2)$$

PROBLEM 4.150 (Continued)

Eq. (1) + 2 Eq. (2): $-4T_{BD}+\frac{48}{7}P=0 \quad T_{BD}=\frac{12}{7}P$

Eq. (2): $-\frac{12}{7}P-T_{BE}+\frac{18}{7}P=0 \quad T_{BE}=\frac{6}{7}P$

Since $P=445\text{ N} \quad T_{BD}=\frac{12}{7}(455)$ $\quad T_{BD}=780\text{ N}$ ◀

$T_{BE}=\frac{6}{7}(455)$ $\quad T_{BE}=390\text{ N}$ ◀

$$\Sigma\mathbf{F}=0:\quad \mathbf{T}_{BD}+\mathbf{T}_{BE}+\mathbf{P}+\mathbf{A}=0$$

Coefficient of **i**: $\frac{780}{3}+\frac{390}{3}-\frac{455}{7}(3)+A_x=0$

$$260+130-195+A_x=0 \quad A_x=195.0\text{ N}$$

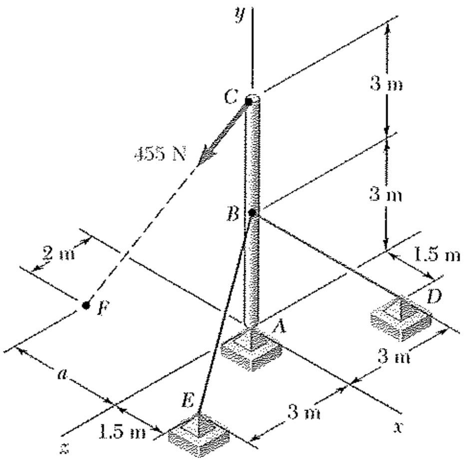
Coefficient of **j**: $-\frac{780}{3}(2)-\frac{390}{3}(2)-\frac{455}{7}(6)+A_y=0$

$$-520-260-390+A_y=0 \quad A_y=1170\text{ N}$$

Coefficient of **k**: $-\frac{780}{3}(2)+\frac{390}{3}(2)+\frac{455}{7}(2)+A_z=0$

$$-520+260+130+A_z=0 \quad A_z=+130.0\text{ N}$$

$\mathbf{A}=-(195.0\text{ N})\mathbf{i}+(1170\text{ N})\mathbf{j}+(130.0\text{ N})\mathbf{k}$ ◀



PROBLEM 4.151

Solve Problem 4.150 for $a = 1.5$ m.

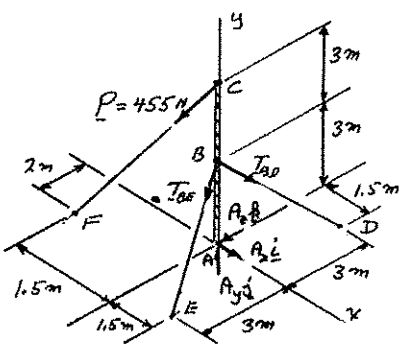
PROBLEM 4.150 The 6-m pole *ABC* is acted upon by a 455-N force as shown. The pole is held by a ball-and-socket joint at *A* and by two cables *BD* and *BE*. For $a = 3$ m, determine the tension in each cable and the reaction at *A*.

SOLUTION

Free-Body Diagram:

Five unknowns and six Eqs. of equilibrium but equilibrium is maintained

$(\Sigma M_{AC} = 0)$



$$\mathbf{r}_B = 3\mathbf{j}$$
$$\mathbf{r}_C = 6\mathbf{j}$$
$$\overrightarrow{CF} = -1.5\mathbf{i} - 6\mathbf{j} + 2\mathbf{k} \quad CF = 6.5 \text{ m}$$
$$\overrightarrow{BD} = 1.5\mathbf{i} - 3\mathbf{j} - 3\mathbf{k} \quad BD = 4.5 \text{ m}$$
$$\overrightarrow{BE} = 1.5\mathbf{i} - 3\mathbf{j} + 3\mathbf{k} \quad BE = 4.5 \text{ m}$$

$$\mathbf{P} = P\frac{\overrightarrow{CF}}{CE} = \frac{P}{6.5}(-1.5\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}) = \frac{P}{13}(-3\mathbf{i} - 12\mathbf{j} + 4\mathbf{k})$$

$$\mathbf{T}_{BD} = T_{BD}\frac{\overrightarrow{BD}}{BD} = \frac{T_{BD}}{4.5}(1.5\mathbf{i} - 3\mathbf{j} - 3\mathbf{k}) = \frac{T_{BD}}{3}(\mathbf{i} - 2\mathbf{j} - 2\mathbf{k})$$

$$\mathbf{T}_{BE} = T_{BE}\frac{\overrightarrow{BE}}{BE} = \frac{T_{BD}}{3}(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$$

$$\Sigma M_A = 0: \quad \mathbf{r}_B \times \mathbf{T}_{BD} + \mathbf{r}_B \times \mathbf{T}_{BE} + \mathbf{r}_C \times \mathbf{P} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 3 & 0 \\ 1 & -2 & -2 \end{vmatrix}\frac{T_{BD}}{3} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 3 & 0 \\ 1 & -2 & 2 \end{vmatrix}\frac{T_{BE}}{3} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 6 & 0 \\ -3 & -12 & +4 \end{vmatrix}\frac{P}{13} = 0$$

Coefficient of **i**:
$$-2T_{BD} + 2T_{BE} + \frac{24}{13}P = 0 \quad (1)$$

Coefficient of **k**:
$$-T_{BD} - T_{BE} + \frac{18}{13}P = 0 \quad (2)$$

PROBLEM 4.151 (Continued)

Eq. (1) + 2 Eq. (2): $$-4T_{BD} + \frac{60}{13}P = 0 \quad T_{BD} = \frac{15}{13}P$$

Eq (2): $$-\frac{15}{13}P - T_{BE} + \frac{18}{13}P = 0 \quad T_{BE} = \frac{3}{13}P$$

Since $$P = 445 \text{ N} \quad T_{BD} = \frac{15}{13}(455) \qquad T_{BD} = 525 \text{ N} ◀$$

$$T_{BE} = \frac{3}{13}(455) \qquad T_{BE} = 105.0 \text{ N} ◀$$

$$\Sigma \mathbf{F} = 0: \quad \mathbf{T}_{BD} + \mathbf{T}_{BE} + \mathbf{P} + \mathbf{A} = 0$$

Coefficient of **i**: $$\frac{525}{3} + \frac{105}{3} - \frac{455}{13}(3) + A_x = 0$$

$$175 + 35 - 105 + A_x = 0 \quad A_x = 105.0 \text{ N}$$

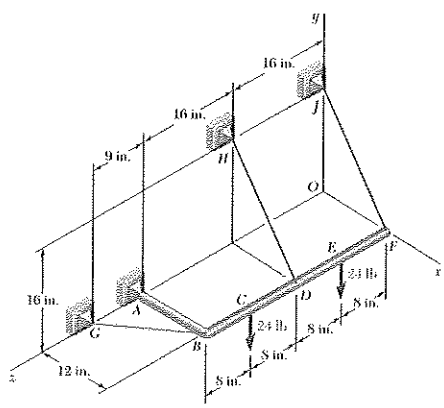
Coefficient of **j**: $$-\frac{525}{3}(2) - \frac{105}{3}(2) - \frac{455}{13}(12) + A_y = 0$$

$$-350 - 70 - 420 + A_y = 0 \quad A_y = 840 \text{ N}$$

Coefficient of **k**: $$-\frac{525}{3}(2) + \frac{105}{3}(2) + \frac{455}{13}(4) + A_z = 0$$

$$-350 + 70 + 140 + A_z = 0 \quad A_z = 140.0 \text{ N}$$

$$\mathbf{A} = -(105.0 \text{ N})\mathbf{i} + (840 \text{ N})\mathbf{j} + (140.0 \text{ N})\mathbf{k} ◀$$

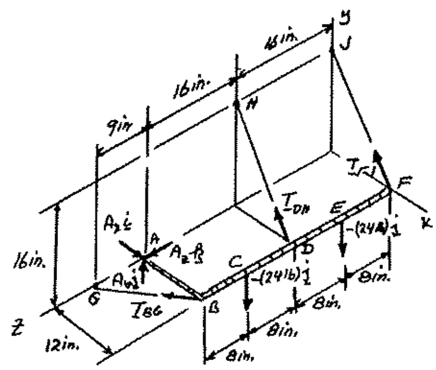


PROBLEM 4.152

The rigid L-shaped member ABF is supported by a ball-and-socket joint at A and by three cables. For the loading shown, determine the tension in each cable and the reaction at A.

SOLUTION

Free-Body Diagram:



$$\mathbf{r}_{B/A} = 12\mathbf{i}$$
$$\mathbf{r}_{F/A} = 12\mathbf{j} - 8\mathbf{k}$$
$$\mathbf{r}_{D/A} = 12\mathbf{i} - 16\mathbf{k}$$
$$\mathbf{r}_{E/A} = 12\mathbf{i} - 24\mathbf{k}$$
$$\mathbf{r}_{F/A} = 12\mathbf{i} - 32\mathbf{k}$$
$$\overrightarrow{BG} = -12\mathbf{i} + 9\mathbf{k}$$
$$BG = 15 \text{ in.}$$
$$\boldsymbol{\lambda}_{BG} = -0.8\mathbf{i} + 0.6\mathbf{k}$$

$$\overrightarrow{DH} = -12\mathbf{i} + 16\mathbf{j}; \quad DH = 20 \text{ in.}; \quad \lambda_{DH} = -0.6\mathbf{i} + 0.8\mathbf{j}$$
$$\overrightarrow{FJ} = -12\mathbf{i} + 16\mathbf{j}; \quad FJ = 20 \text{ in.}; \quad \lambda_{FJ} = -0.6\mathbf{i} + 0.8\mathbf{j}$$

$$\Sigma \mathbf{M}_A = 0: \quad \mathbf{r}_{B/A} \times \mathbf{T}_{BG}\lambda_{BG} + \mathbf{r}_{DH} \times \mathbf{T}_{DH}\lambda_{DH} + \mathbf{r}_{F/A} \times \mathbf{T}_{FJ}\lambda_{FJ}$$
$$+\mathbf{r}_{F/A} \times (-24\mathbf{j}) + \mathbf{r}_{E/A} \times (-24\mathbf{j}) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & 0 \\ -0.8 & 0 & 0.6 \end{vmatrix} T_{BG} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & -16 \\ -0.6 & 0.8 & 0 \end{vmatrix} T_{DH} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & -32 \\ -0.6 & 0.8 & 0 \end{vmatrix} T_{FJ}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & -8 \\ 0 & -24 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & -24 \\ 0 & -24 & 0 \end{vmatrix} = 0$$

Coefficient of **i**: $$+12.8T_{DH} + 25.6T_{FJ} - 192 - 576 = 0 \qquad (1)$$

Coefficient of **k**: $$+9.6T_{DH} + 9.6T_{FJ} - 288 - 288 = 0 \qquad (2)$$

$\frac{3}{4}$ Eq. (1) – Eq. (2): $$9.6T_{FJ} = 0 \qquad T_{FJ} = 0 \blacktriangleleft$$

PROBLEM 4.152 (Continued)

Eq. (1): $12.8T_{DH} - 268 = 0$ $T_{DH} = 60 \text{ lb}$ ◀

Coefficient of **j**: $-7.2T_{BG} + (16 \times 0.6)(60.0 \text{ lb}) = 0$ $T_{BG} = 80.0 \text{ lb}$ ◀

$\Sigma\mathbf{F} = 0$: $\mathbf{A} + T_{BG}\boldsymbol{\lambda}_{BG} + T_{DH}\boldsymbol{\lambda}_{DH} + T_{EJ} - 24\mathbf{j} - 24\mathbf{j} = 0$

Coefficient of **i**: $A_x + (80)(-0.8) + (60.0)(-0.6) = 0$ $A_x = 100.0 \text{ lb}$

Coefficient of **j**: $A_y + (60.0)(0.8) - 24 - 24 = 0$ $A_y = 0$

Coefficient of **k**: $A_z + (80.0)(+0.6) = 0$ $A_z = -48.0 \text{ lb}$

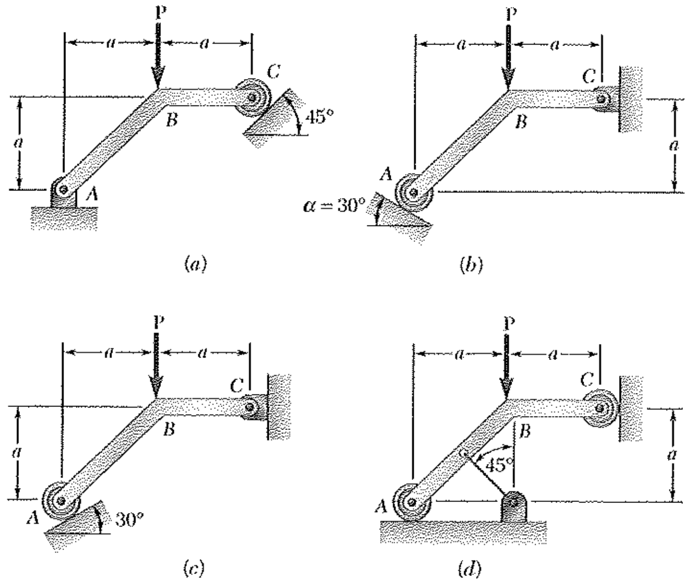
$\mathbf{A} = (100.0 \text{ lb})\mathbf{i} - (48.0 \text{ lb})\mathbf{j}$ ◀

Note: The value $A_y = 0$

Can be confirmed by considering $\Sigma M_{BF} = 0$

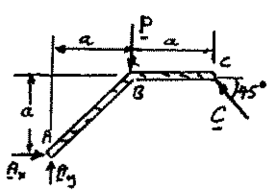
PROBLEM 4.153

A force **P** is applied to a bent rod ABC, which may be supported in four different ways as shown. In each case, if possible, determine the reactions at the supports.



SOLUTION

(*a*)



$$+\circlearrowleft \Sigma M_A = 0: \quad -P_a + (C\sin 45°)2a + (\cos 45°)a = 0$$

$$3\frac{C}{\sqrt{2}} = P \quad C = \frac{\sqrt{2}}{3}P$$

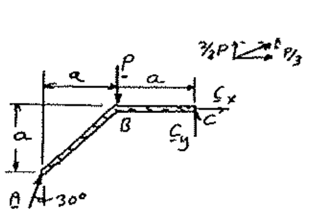
$\mathbf{C} = 0.471P \searrow 45°$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - \left(\frac{\sqrt{2}}{3}P\right)\frac{1}{\sqrt{2}} \quad A_x = \frac{P}{3} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - P + \left(\frac{\sqrt{2}}{3}P\right)\frac{1}{\sqrt{2}} \quad A_y = \frac{2P}{3}\uparrow$$

$\mathbf{A} = 0.745P \angle 63.4°$ ◀

(*b*)



$$+\circlearrowleft \Sigma M_C = 0: \quad +Pa - (A\cos 30°)2a + (A\sin 30°)a = 0$$

$$A(1.732 - 0.5) = P \quad A = 0.812P$$

$\mathbf{A} = 0.812P \angle 60.0°$ ◀

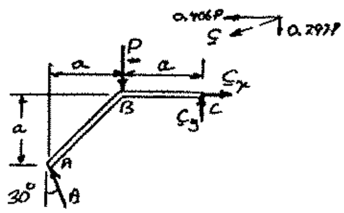
$$\xrightarrow{+} \Sigma F_x = 0: \quad (0.812P)\sin 30° + C_x = 0 \quad C_x = -0.406P$$

$$+\uparrow \Sigma F_y = 0: \quad (0.812P)\cos 30° - P + C_y = 0 \quad C_y = -0.297P$$

$\mathbf{C} = 0.503P \nearrow 36.2°$ ◀

PROBLEM 4.153 (Continued)

(c)



$$+\circlearrowleft \Sigma M_C = 0: \quad +Pa - (A\cos 30°)2a + (A\sin 30°)a = 0$$

$$A(1.732 + 0.5) = P \quad A = 0.448P$$

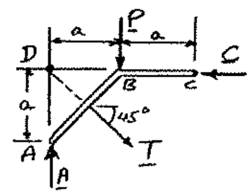
$$A = 0.448P \measuredangle 60.0° \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -(0.448P)\sin 30° + C_x = 0 \qquad C_x = 0.224P \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad (0.448P)\cos 30° - P + C_y = 0 \quad C_y = 0.612P \uparrow$$

$$\mathbf{C} = 0.652P \measuredangle 69.9° \blacktriangleleft$$

(d) Force **T** exerted by wire and reactions **A** and **C** all intersect at Point D.



$$+\circlearrowleft \Sigma M_D = 0: \quad P_a = 0$$

Equilibrium not maintained

Rod is improperly constrained ◀